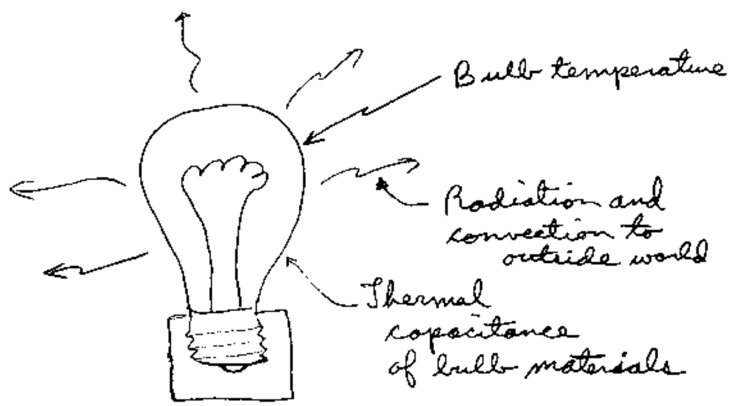

Figure 1.15: Sketch of bulb and relevant thermal elements.

1.1.4 Thermal first-order system

For an example thermal system we study the desk lamp shown in the picture (to be added). This lamp bulb is electrically heated via the bulb filament. The resulting bulb temperature is measured with the infrared sensor shown in the figure (to be added). A sketch of the light bulb in the lamp is shown in the line drawing of Figure 1.15.

We left the lamp on for a long enough time to reach steady state, and then turned off the lamp and measured the decay of temperature back to ambient. Data taken from this system is shown in tabular and graphical form in Figure 1.16. By inspection of this data, the bulb system is well-fit by a first-order model of the form of (1.1). An estimate of the associated time constant is about 3 minutes. But we need to have τ in seconds, so the system time constant is formally given as $\tau = 180$ sec.

An abstraction to a lumped model of this system is shown in Figure 1.17. Here the *thermal capacitance* of the bulb is summarized by the block of material labeled with the capacitance C_b with units of [J/°K]. The block is assumed to have a uniform temperature T_b [°K]. This block has a total stored thermal energy $W_b = C_b T_b$ [J]. The change of thermal stored energy happens via heat flow

$$q_b = \frac{dW_b}{dt} = C_b \frac{dT_b}{dt}. \tag{1.23}$$

Here q_b in units of watts represents heat flow *into* the bulb. As shown in the figure, we assume that the block is insulated on three sides, and so the

Hot light bulb cooling

t (min)	T (deg C)
0	101.2
1	81.6
2	66.2
3	54.4
4	47.6
5	41.8
6	39.4
7	35.8
8	34.4
9	32.6
10	32.2
12	29.4
14	29.4

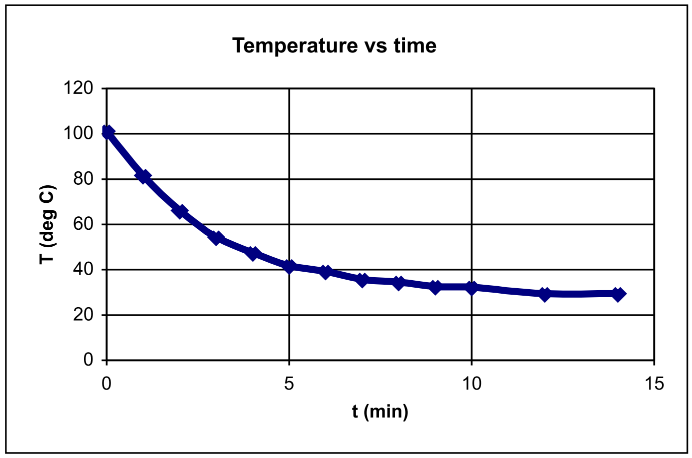

Figure 1.16: Data from light bulb cooling experiment.

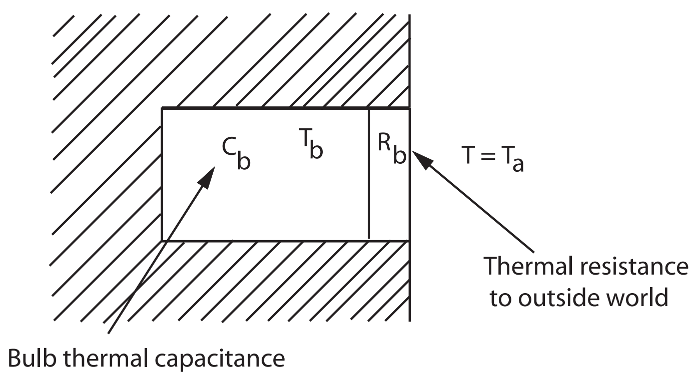

Figure 1.17: Lumped model for bulb cooling experiment.

heat flow through those sides is zero. The block is connected to the outside ambient temperature via the thermal resistance R_b, such that

$$q_b = \frac{T_a - T_b}{R_b}. \tag{1.24}$$

This resistance represents the flow of heat into the bulb as a linear function of the temperature difference[4] between the ambient and the bulb temperatures.

Setting equality between the last two equations gives

$$C_b \frac{dT_b}{dt} = \frac{T_a - T_b}{R_b}. \tag{1.25}$$

Now, it's convenient to define a variable to represent the temperature difference between the bulb and ambient: $T \equiv T_b - T_a$. Since the ambient temperature is constant, $dT/dt = dT_b/dt$. Making these substitutions and multiplying (1.25) through by R_b yields

$$R_b C_b \frac{dT}{dt} + T = 0. \tag{1.26}$$

If we define $\tau = R_b C_b$, this is in the form of (1.1). The natural response is thus as calculated in section 1.1, with its associated figures. Specifically, if the initial temperature difference of the bulb is defined as $T(0) = T_0$, then the temperature difference as a function of time varies as

$$T(t) = T_0 e^{-t/R_b C_b} \quad [\mathrm{K}]. \tag{1.27}$$

If you want to convert back to the absolute temperature of the bulb, remember that $T_b = T + T_a$.

1.1.5 Fluidic first-order system

A fluidic system which can be modeled with a first-order differential equation is shown in Figure 1.18. Here a tank filled with liquid drains through a long, thin pipe. The height of the liquid above the pipe inlet is defined as h. If we assume that the liquid has a density of ρ [kg/m^3], then the pressure P_t at

[4]In real systems, more exact and likely nonlinear models can apply, but a linear model gives a first understanding of this system response, and is well able to match the measured behavior. For example, pure radiative cooling varies as temperature difference to the fourth power, which is highly nonlinear. There will certainly be radiative heat flow in this system, however, the experimental data fits well to a linear heat flow model which suggests that radiative cooling is not highly significant at the bulb envelope temperatures of 100 $^\circ C$ and down to ambient.

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