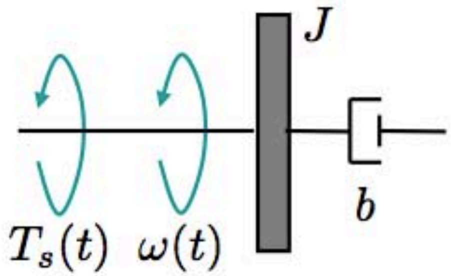
Goals for today

- Transfer function
  - Flywheel example
  - Other examples: car suspension system

- Poles and zeros in complex s-plane
  - pole, zero definitions
  - the significance of poles and zeros:
    from *s*-domain representation to transient characteristics

Transfer Functions

- Consider again the motor-shaft system :



$J\dot{\omega}(t) + b\omega(t) = T_s(t),$

where now $T_s(t)$ is an arbitrary function,

but still $\omega(t=0)=0$ (no spin–down).

Proceeding as before, we can write

$$\Omega(s) = \frac{T_s(s)}{Js+b} \Leftrightarrow \frac{\Omega(s)}{T_s(s)} = \frac{1}{Js+b}.$$

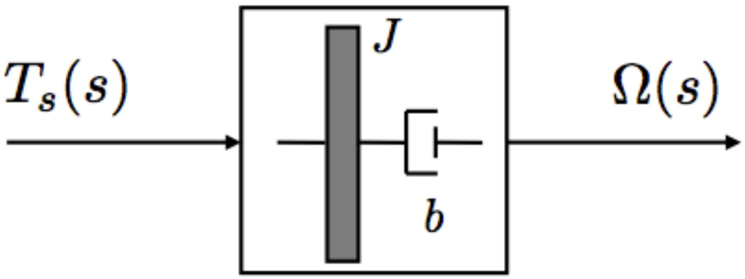
Generally, we define the ratio

$$\frac{\mathcal{L}\left[\text{output}\right]}{\mathcal{L}\left[\text{input}\right]} = \textbf{Transfer Function}\text{; in this case, } \mathrm{TF}(s) = \frac{1}{Js+b}.$$

We refer to the $(\mathrm{TF})^{-1}$ of a single element as the **Impedance** $Z(s)$.

Transfer Functions in block diagrams

$$\text{Transfer Function}(s) := \frac{C(s)}{R(s)}.$$

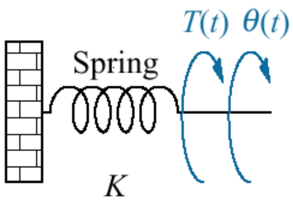
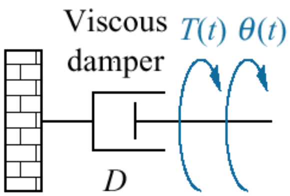
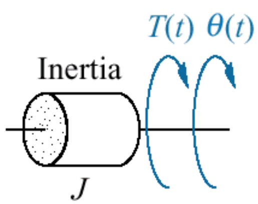


$$\text{TF}(s) := \frac{\Omega(s)}{T_s(s)} = \frac{1}{Js+b}.$$

Important: To be able to define the Transfer Function, the system ODE must be linear with constant coefficients.

Such systems are known as **Linear Time-Invariant**, or **Linear Autonomous**.

Impedances: rotational mechanical

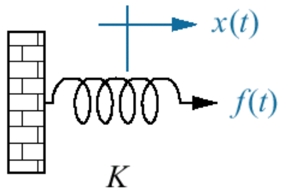
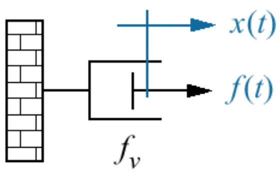
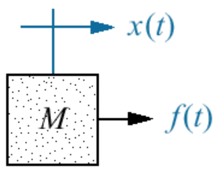
Component	Torque-angular velocity	Torque-angular displacement	Impedance $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)\,d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\dfrac{d\theta(t)}{dt}$	Ds
	$T(t) = J\dfrac{d\omega(t)}{dt}$	$T(t) = J\dfrac{d^2\theta(t)}{dt^2}$	Js^2

Note: The following set of symbols and units is used throughout this book: $T(t)$ = N-m (newton-meters), $\theta(t)$ = rad (radians), $\omega(t)$ = rad/s (radians/second), K = N-m/rad (newton-meters/radian), D = N-m-s/rad (newton-meters-seconds/radian), J = kg-m^2 (kilogram-meters^2 = newton-meters-seconds^2/radian).

(In the notes, we sometimes use b or B instead of D.)

Nise Table 2.5

Impedances: translational mechanical

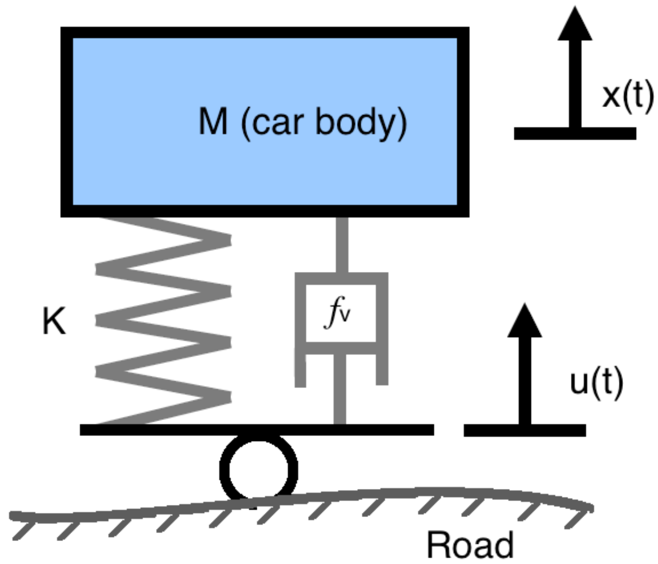
Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)\,d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).

(In the notes, we sometimes use b or B instead of f_v.)

Nise Table 2.4

Transfer Functions: On car suspension system



System ODE: $M\ddot{x}(t) + f_v\dot{x} + Kx = b\dot{u} + Ku$

$$U(s) \rightarrow \boxed{\frac{f_v s + K}{M s^2 + f_v s + K}} \rightarrow X(s)$$

Summary

- Basic Laplace transform

$$\mathcal{L}\left[f(t)\right] \equiv F(s) = \int_{0-}^{+\infty} f(t)\mathrm{e}^{-st}\mathrm{d}t.$$

$$\mathcal{L}\left[u(t)\right] \equiv U(s) = \frac{1}{s}.$$

$$\mathcal{L}\left[\mathrm{e}^{-at}\right] = \frac{1}{s+a}.$$

$$\mathcal{L}\left[\dot{f}(t)\right] = sF(s) - f(0-).$$

$$\mathcal{L}\left[\int_{0-}^{t} f(\xi)\mathrm{d}\xi\right] = \frac{F(s)}{s}.$$

- Obtain transfer functions

$$\text{Known:} \quad M\ddot{x} + b\dot{x} + kx = u(x)$$

$$\text{With 0 initial conditions:} \quad (Ms^2 + bs + k)X(s) = F(s)$$

$$\Longrightarrow \quad \boxed{\frac{X(s)}{F(s)} \equiv TF(s) = \frac{1}{Ms^2 + bs + k}}$$

Definition of poles and zeros

- Transfer function can usually be written as a numerator divided by a denominator (both are functions of s):

$$TF(s) = \frac{N(s)}{D(s)}$$

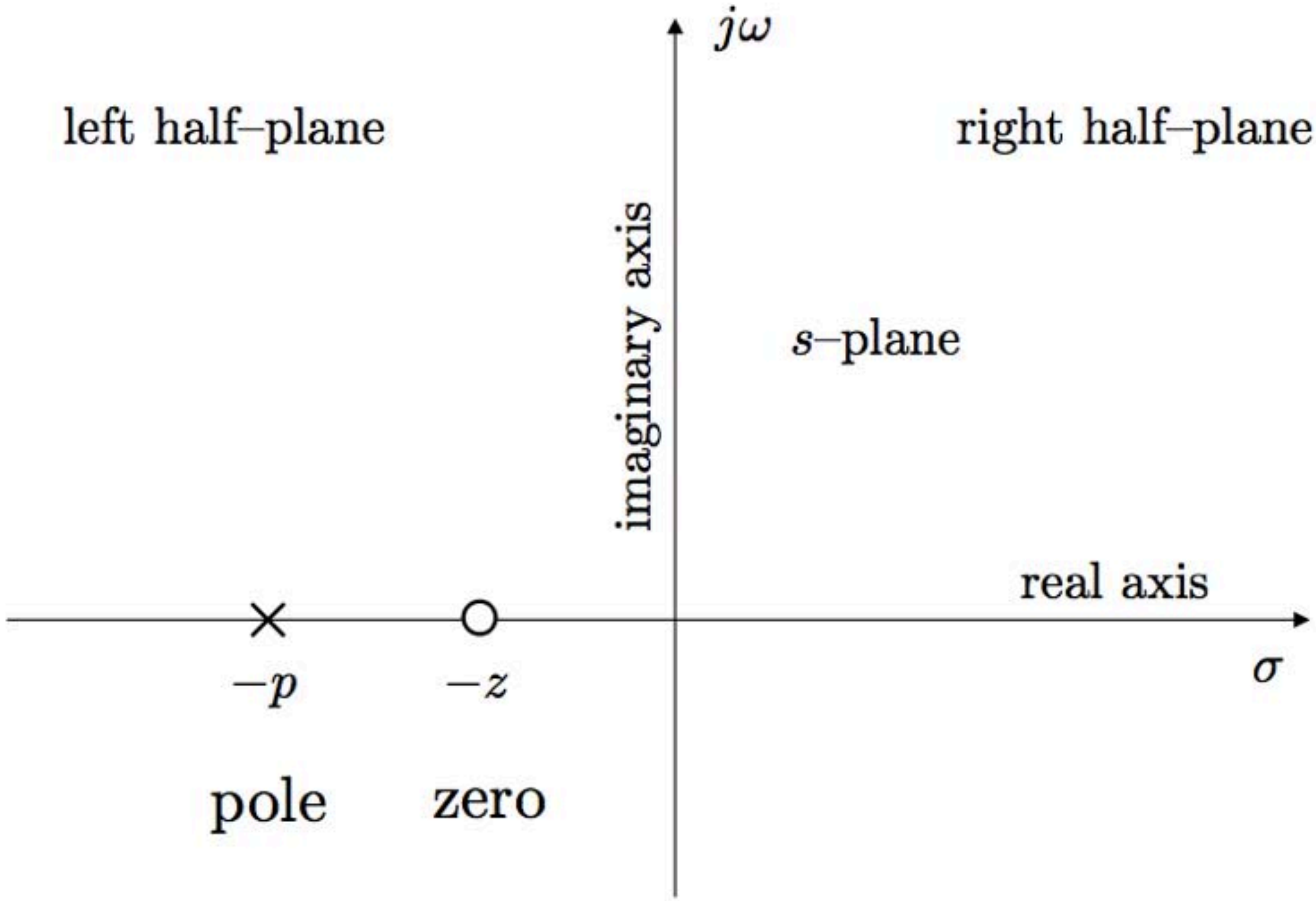
- Poles are **all complex solutions** to

$$D(s) = 0$$

- Zeros are **all complex solutions** to

$$N(s) = 0$$

Representation of poles and zeros on the *s*-plane



Lab assignment p.1

- Derive the flywheel TF for one, two, three magnets, using the values of moment if inertia *J* and viscous damping *b* from the previous lab

- How many zeros and/or poles are there in the flywheel TF? Plot their location(s) on the complex plane for the case of three magnets

Lab assignment p.2

- If you remove two of the three magnets, will the pole(s) move to the left or to the right? Explain the relationship of your answer to the change you observe in the time domain.

- Derive the step response of the flywheel in the Laplace domain

Lab assignment p.3

- In the presence of the Instructor(s) only, connect the CD motor to the flywheel and obtain the step response with one, two, three magnets. Explain the difference based on your Laplace-domain derivation in the previous question.