7 Fourier Series Calculations

Compute the Fourier series coefficients A_0, A_n, and B_n for the following signals on the interval $T = [0, 2\pi]$:

1. $f(t) = 2\sin(t + \pi/4) + \cos(5t + \pi/3)$
Solution: use trigonometric identities to rewrite this as
$f(t) = 2\sin(t)\cos(\pi/4) + 2\cos(t)\sin(\pi/4) + \cos(5t)\cos(\pi/3) + \sin(5t)\sin(\pi/3)$. *Thus,* $A_0 = 0$, $A_1 = 2\sin(\pi/4) = \sqrt{2}$, $B_1 = 2\cos(\pi/4) = \sqrt{2}$, $A_5 = \cos(\pi/3) = 1/2$, $B_5 = \sin(\pi/3) = \sqrt{3}/2$, *and all the other coefficients are zero.*

2. $$f(t) = \begin{cases} 1, t < T/2 \\ 0, t \geq T/2 \end{cases} \text{ (biased square wave)}$$

Solution: A_0 *is the mean value of the signal, or* $A_0 = 1/2$. *Applying the formulas for the coefficients, we get*

$$\begin{aligned} A_n &= \frac{1}{\pi}\int_0^{2\pi} f(t)\cos(nt)dt = \frac{1}{\pi}\int_0^{\pi}\cos(nt)dt = \frac{1}{n\pi}\sin(nt)|_0^{\pi} = 0 \\ B_n &= \frac{1}{\pi}\int_0^{2\pi} f(t)\sin(nt)dt = \frac{1}{\pi}\int_0^{\pi}\sin(nt)dt = -\frac{1}{n\pi}\cos(nt)|_0^{\pi} = z(n), \end{aligned}$$

where z *is zero if* n *is even, and* z *is* $2/n\pi$ *if* n *is odd. Try this out in MATLAB!*

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