1)

$u = ax^2 + bxy + cy^2 \qquad w = 0$

find $v \qquad v(y=0) = 0$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \cancel{\frac{\partial w}{\partial z}} = 0 \Rightarrow \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$

$\cancel{\frac{\partial w}{\partial z}} \to 0$

$\frac{\partial u}{\partial x} = 2ax + by + 0 = -\frac{\partial v}{\partial y}$

$\int \partial v = -\int(2ax + by)\,\partial y \Rightarrow v = -\left(2axy + \frac{by^2}{2}\right) = -2axy - \frac{by^2}{2}$

$$\boxed{v = -2axy - \frac{by^2}{2}}$$

2) Total acceleration of $\vec{u}$ => $\frac{D\vec{u}}{Dt} = \frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\vec{\nabla}\vec{u}$

$\vec{u}\cdot\vec{\nabla}\vec{u} = ?$ let $\vec{u} = 2y\hat{\imath} + xt\hat{\jmath} + xy^2\hat{k}$ [m/sec]

if we let $\vec{u}_1 = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$

then $\vec{u}_1\cdot\vec{\nabla}\vec{u}_1 = \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)\hat{\imath} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)\hat{\jmath} + \left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)\hat{k}$

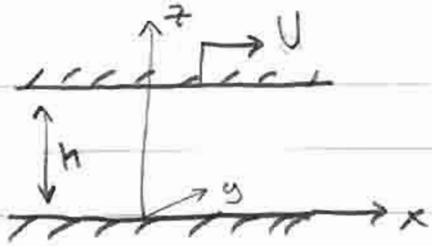
now substitute

$\vec{u}\cdot\vec{\nabla}\vec{u} = (2y\cdot 0 + xt\cdot 2 + xy^2\cdot 0)\hat{\imath} + (2y\cdot t + xt\cdot 0 + xy^2\cdot 0)\hat{\jmath} + (2y\cdot y^2 + xt\cdot 2xy + xy^2\cdot 0)\hat{k}$

$\vec{u}\cdot\vec{\nabla}\vec{u} = 2xt\,\hat{\imath} + 2yt\,\hat{\jmath} + (2y^3 + 2x^2yt)\hat{k}$

$\frac{\partial \vec{u}}{\partial t} = x\hat{\jmath}$ => $\frac{D\vec{u}}{Dt} = 2xt\,\hat{\imath} + (x + 2yt)\hat{\jmath} + (2y^3 + 2x^2yt)\hat{k}$

3)



$\boxed{v = 0}$ $\boxed{w = 0}$

$$\boxed{u = \frac{Uz}{h} \quad \begin{cases} u = 0 \ @\ z = 0 \\ u = U \ @\ z = h \end{cases}}$$

a) by continuity equation $\frac{\partial u}{\partial x} + \underbrace{\cancel{\frac{\partial v}{\partial y}}}_{0\ v=0} + \underbrace{\cancel{\frac{\partial w}{\partial z}}}_{0\ w=0} = 0$

$\boxed{\therefore \frac{\partial u}{\partial x} = 0}$ Plug into navier stokes equations

ignoring corriollis forces and gravity.

$$\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{0 \text{ steady}} + u\underbrace{\cancel{\frac{\partial u}{\partial x}}}_{\frac{\partial u}{\partial x}=0} + \underbrace{\cancel{v}}_{v=0}\frac{\partial u}{\partial y} + \underbrace{\cancel{w}}_{w=0}\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\underbrace{\cancel{\frac{\partial^2 u}{\partial x^2}}}_{\frac{\partial u}{\partial x}=0} + \underbrace{\cancel{\frac{\partial^2 u}{\partial y^2}}}_{\text{no change in } y} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\underbrace{\cancel{\frac{\partial v}{\partial t}}}_{0} + \underbrace{u\cancel{\frac{\partial v}{\partial x}}}_{0} + \underbrace{v\cancel{\frac{\partial v}{\partial y}}}_{0\ v=0} + \underbrace{w\cancel{\frac{\partial v}{\partial z}}}_{0} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\underbrace{\cancel{\frac{\partial^2 v}{\partial x^2}}}_{v=0} + \underbrace{\cancel{\frac{\partial^2 v}{\partial y^2}}}_{v=0} + \underbrace{\cancel{\frac{\partial^2 v}{\partial z^2}}}_{v=0}\right)$$

$$\frac{\partial p}{\partial y} = 0$$

$$\underbrace{\cancel{\frac{\partial w}{\partial t}}}_{0} + \underbrace{u\cancel{\frac{\partial w}{\partial x}}}_{0} + \underbrace{v\cancel{\frac{\partial w}{\partial y}}}_{0\ w=0} + w\cancel{\frac{\partial w}{\partial z}} = -\frac{1}{\rho}\frac{\partial p}{\partial w} + \nu\left(\cancel{\frac{\partial^2 w}{\partial x^2}} + \underbrace{\cancel{\frac{\partial^2 w}{\partial y^2}}}_{w=0} + \underbrace{\cancel{\frac{\partial^2 w}{\partial w^2}}}_{0}\right)$$

$$\frac{\partial p}{\partial w} = 0$$

let $\mu = \rho \cdot \nu$

$\therefore \frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2}$ $\quad u = \frac{Uz}{h}$ $\quad \frac{\partial^2 u}{\partial z^2} = 0$

$\frac{\partial p}{\partial x} = 0$ $\therefore$ $P = \text{constant}!$

b) suppose $\frac{\partial P}{\partial x} = k$

what is the velocity distribution?

$$\frac{\partial P}{\partial x} = k = \mu \frac{\partial^2 u}{\partial z^2} \quad \Rightarrow \quad \partial z^2 \cdot k = \mu \partial^2 u$$

$$\partial^2 u = \frac{k}{\mu} \partial z^2$$

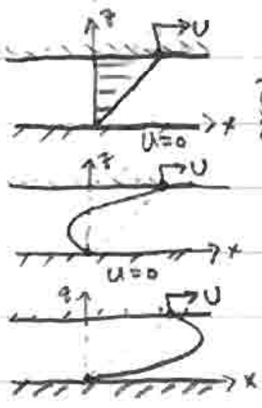
$$\frac{\partial u}{\partial z} = \frac{k}{\mu} z + c_1$$

$$u = \frac{k}{\mu} \frac{z^2}{2} + c_1 z + c_2$$

$$u = 0 \ @ \ z = 0 \Rightarrow c_2 = 0$$
$$\vec{u} = U \ @ \ z = h \Rightarrow c_1 = \frac{U}{h} - \frac{kh}{2\mu}$$

$$u = \frac{k z^2}{2\mu} + \left(\frac{U}{h} - \frac{kh}{2\mu}\right) z \quad \Rightarrow \quad \vec{v} = \left[\frac{k z^2}{2\mu} + \left(\frac{U}{h} - \frac{kh}{2\mu}\right) z\right] \hat{\imath}$$

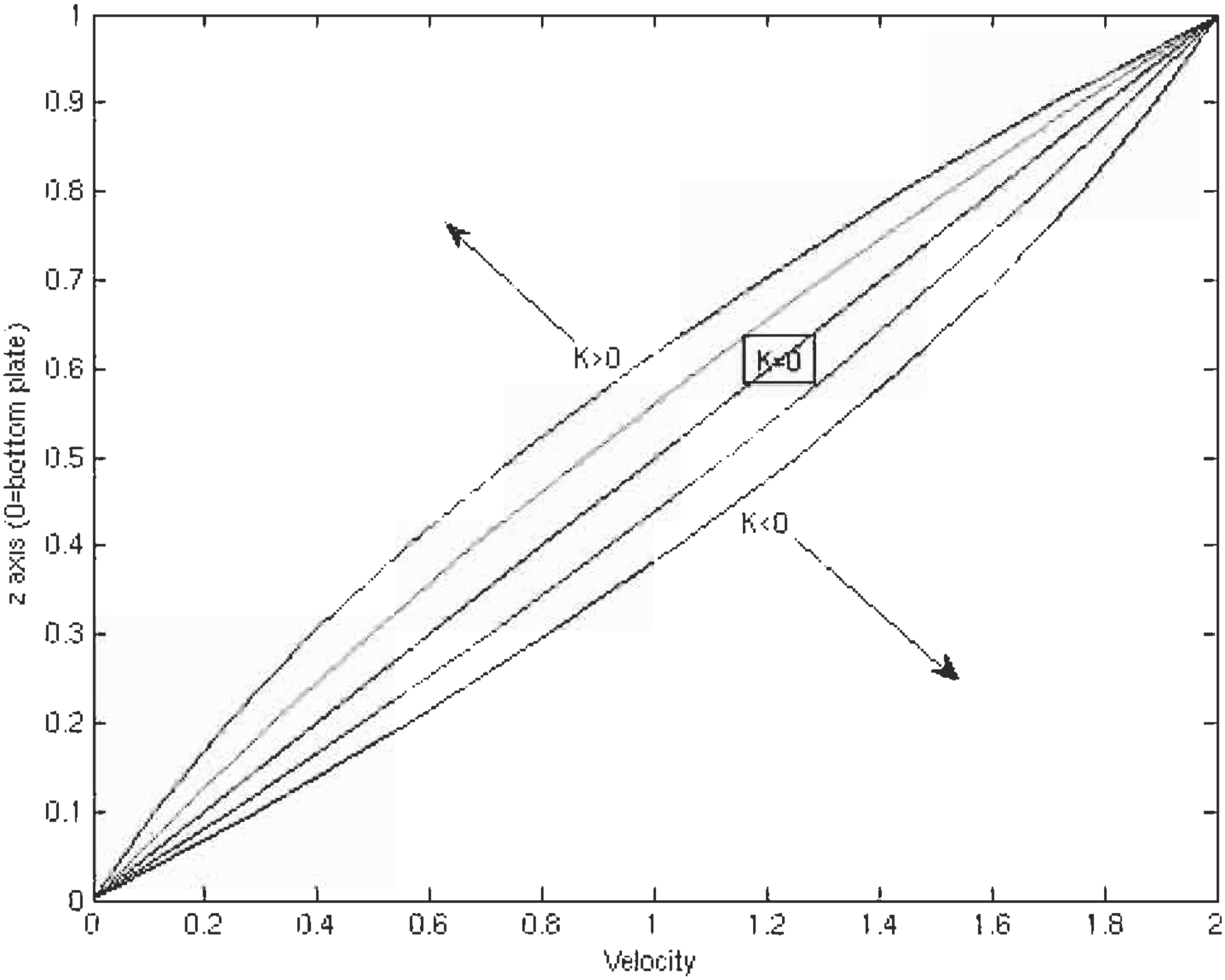
sketch for several values



$\frac{\partial P}{\partial x} = 0 \quad k = 0$

$k > 0$

$k < 0$



z axis (0=bottom plate)
Velocity
K>0
K=0
K<0

c) what is τ_{xz} on each plate

$$\tau_{xz} = \mu \frac{\partial u}{\partial z} = \mu \left[\frac{\cancel{2} k z}{\cancel{2}\mu} + \frac{U}{h} - \frac{kh}{2\mu} \right] = kz + \frac{U\mu}{h} - \frac{kh}{2}$$

$$\tau_{xz} = kz + \frac{U\mu}{h} - \frac{kh}{2}$$

top plate $z = h$ $\quad \tau_{xz} = \frac{U\mu}{h} + \frac{kh}{2}$

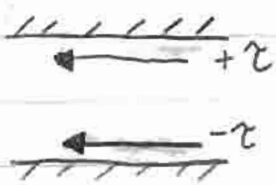
bottomplate $z = 0$ $\quad \tau_{xz} = \frac{U\mu}{h} - \frac{kh}{2}$

letting $\tau_{xz} = 0$

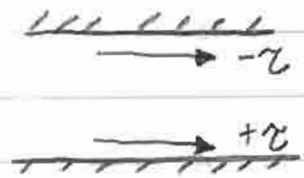
top plate $\quad k = -\frac{2U\mu}{h^2}$ $\qquad$ bottomplate $\quad k = {}^{+}\frac{2U\mu}{h^2}$

sketch direction of τ_{xz} for positive & negative values of k.

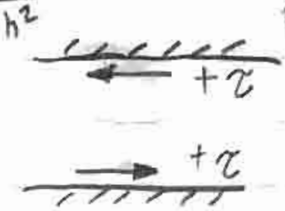
$k > \frac{2U\mu}{h^2}$



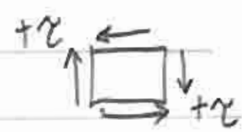
$k < -\frac{2U\mu}{h^2}$



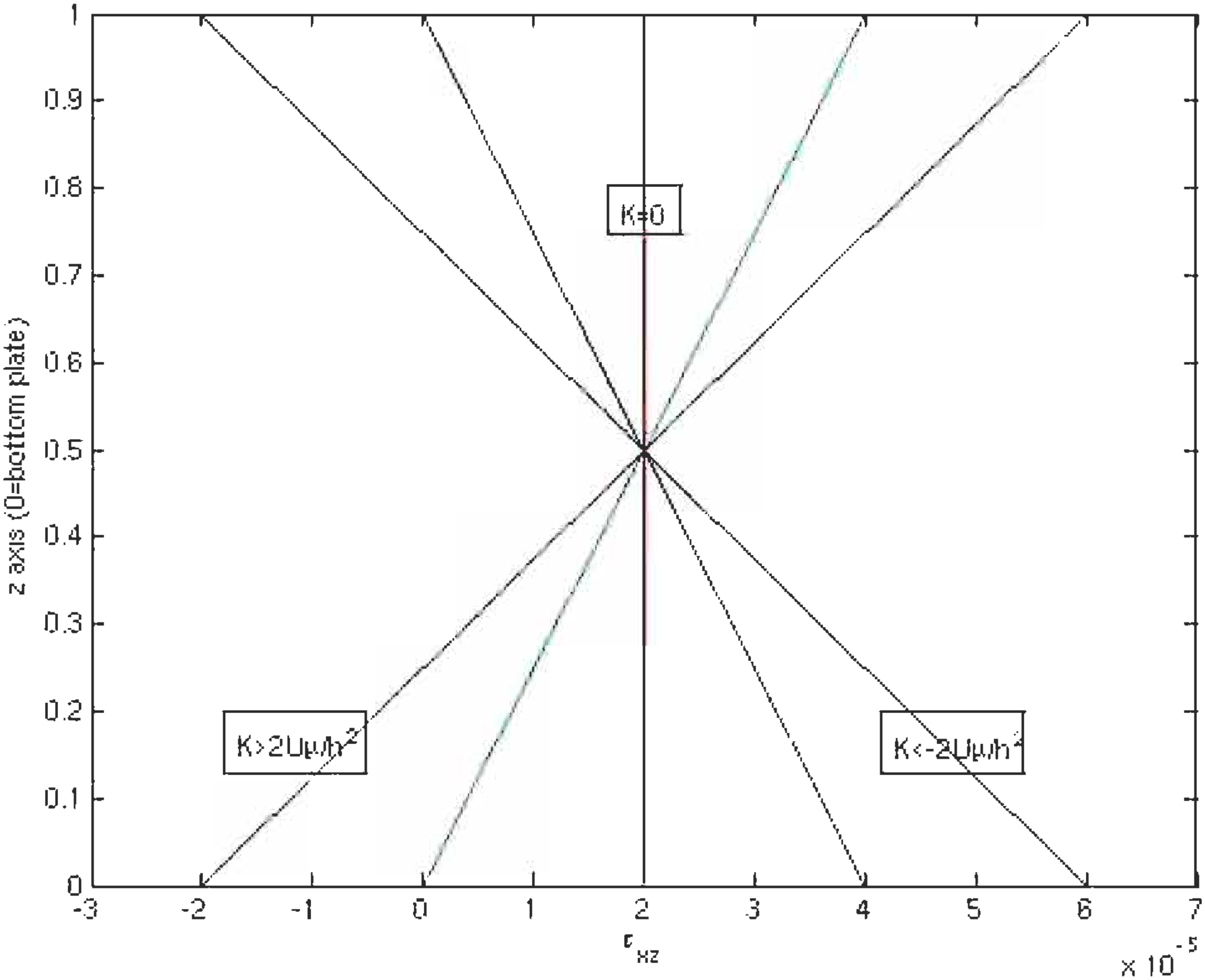
$-\frac{2U\mu}{h^2} < k < \frac{2U\mu}{h^2}$



I define
τ positive on a face
looks like

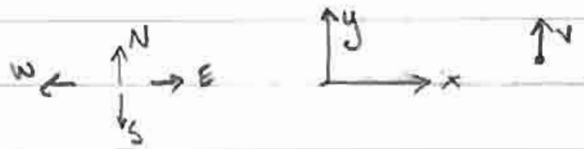


There are 2 points where shear stress goes to zero, once on the top face $k = -\frac{2U\mu}{h^2}$
once on bottom face $k = \frac{2U\mu}{h^2}$.



K=0
K>2Uμ/h²
K<-2Uμ/h²
z axis (0=bottom plate)
τ_xz
x 10^-5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-3
-2
-1
0
1
2
3
4
5
6
7

4-) a) $\phi = 42°$ lat. $v = 75$ m/hr $\partial y = 60$ feet



Use coriollis acceleration: $\dfrac{\partial u}{\partial t} = 2 \Omega v \sin\phi$

$$\frac{\partial u}{\partial t} = 2 \cdot 7.292\times10^{-5}\,[\text{rad/sec}] \cdot 75\,[\text{mi/hr}] \cdot \sin(42°)$$

$$= 2 \cdot 7.292\times10^{-5}\,[\text{rad/s}] \cdot 33.53\,[\text{m/s}] \cdot \sin(47°)$$

$$\frac{\partial u}{\partial t} = 0.00327\,[\text{m/s}^2] = a_0$$

$$x(t) = x_0 + v_0(t) + \frac{1}{2} a_0 (t^2) \qquad x_0 = 0 \quad v_0 = 0$$

$$x(t) = \frac{a_0}{2} t^2$$

time of flight $= 60 \text{ feet} / 75 \text{ mi/hr} = 0.545\,[\text{sec}]$

$= 60 \text{ ft} \cdot \dfrac{1 \text{ hr}}{75 \text{ mi}} =$

$$x(0.595) = \frac{0.545^2 \cdot .00327}{2} = 0.000486\,[\text{m}]$$

$$\boxed{\text{deflection} = 0.486\,[\text{mm}] \text{ to the east}}$$

b) Thrown east $\Rightarrow \dfrac{\partial v}{\partial t} = -2 \Omega u \sin\phi$ - same magnitude but deflects south!

Thrown south $\Rightarrow \dfrac{\partial u}{\partial t} = 2 \Omega v \sin\phi$ - same magnitude but deflected west!

Thown west $\Rightarrow \dfrac{\partial v}{\partial t} = -2 \Omega u \sin\phi$ - same magnitude deflected north!

c) if thrown in Wellington New Zealand $\phi = -42°$

Therefore, ball thrown North will deflect 0.486 mm West instead!

d) at equator $\sin(0) = 0$ No deflection

e) if ball velocity = 100 m/hr direction is still eastward but magnitude goes to 0.365 [mm]

$100 \text{ mi/hr} = 44.7 \text{ m/s}$

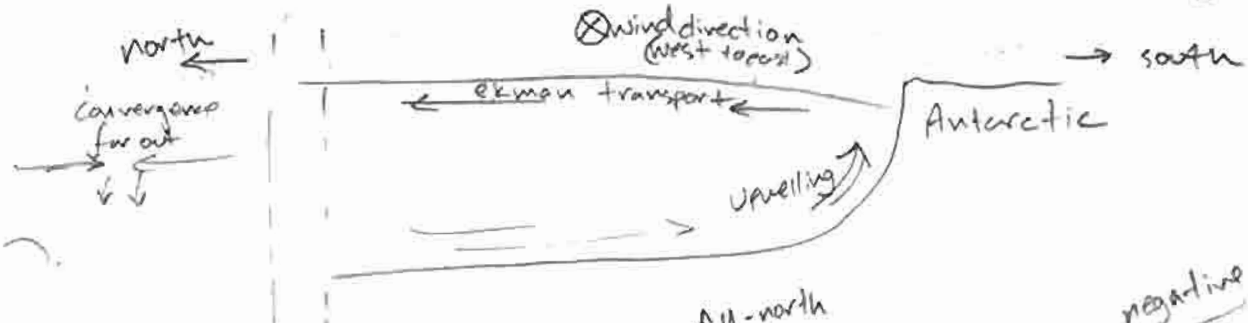
$$t = \frac{60 \text{ ft}}{100 \text{ mi/hr}} = 0.409 \text{ s}$$

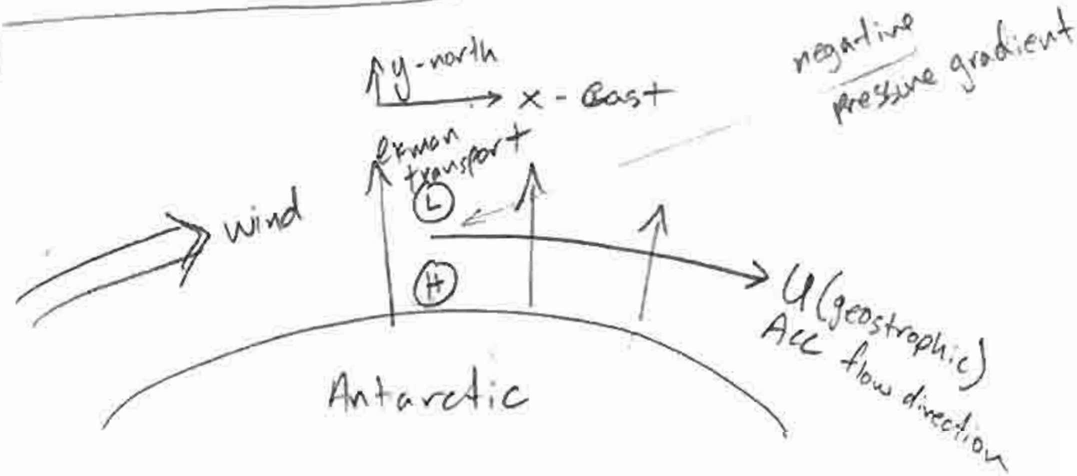
$$\frac{\partial u}{\partial t} = a_0 = 2 \cdot 7.292 \times 10^{-5} [\text{rad/s}] \cdot 100 [\text{mi/hr}] \cdot \sin(42°) = 0.00436 [\text{m/s}^2]$$

$$x(t) = \frac{a_0}{2} t^2 = \frac{0.00436}{2} (.409)^2 = 0.000365 [\text{m}]$$

Antartic Circumpolar Current

ekman transport occurs @ 90° to wind direction near antartica. Wind is from ~~the~~ west and therefore ekman transport carries fluid away from Antartica. Conservation of mass enables cooler deeper water near the coast to replace water moving out to sea. This cooler water is more dense because it is also more salty.





geostrophic flow east west $u = -\frac{1}{f\rho}\frac{\partial P}{\partial y}$

For Antartic Circumpolar Current is a negative pressure gradient in the Northern direction (y). This makes flow from west to east.

see pg 232 in text.