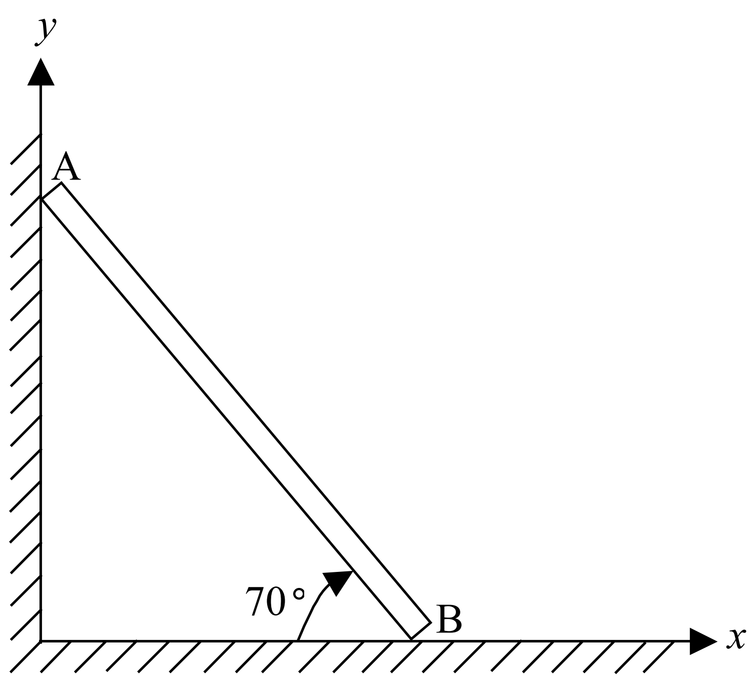
2.004 MODELING DYNAMICS AND CONTROL II Spring 2003

Problem Set No. 4

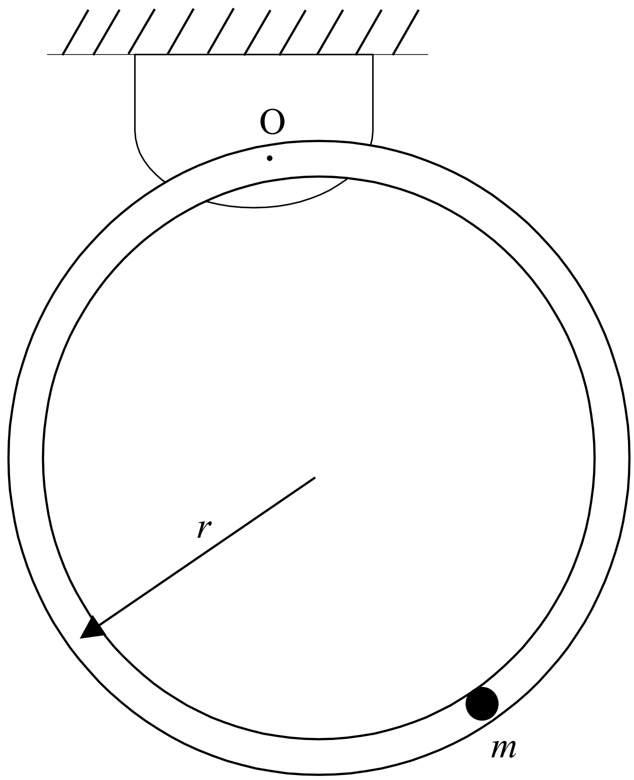
Problem 1

A ladder AB of length 1m slides down a wall. At the instant shown below, the velocity of point B touching the floor is $v_B = 2$ m/s. Determine the velocity of point A touching the wall at the same instant.



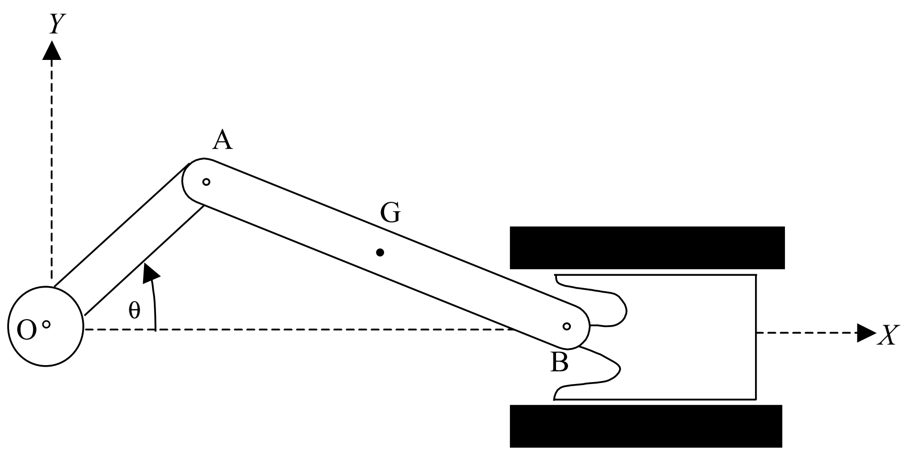
Problem 2

A small tube is bent in the form of a circle of radius r and is pivoted about a fixed point O on its circumference. A particle of mass m can slide inside the tube. Select a complete and independent set of generalized coordinates for this system and express the velocity of the mass m in terms of these.



Problem 3

Crank and Connecting Rod. Internal combustion engines make wide use of the crank and connecting-rod mechanism. The crank OA rotates about the Z-axis through O with the angular velocity $\omega_{crank} = \frac{d\theta}{dt}$. The connecting rod AB is a rigid body which is constrained to move in the XY plane perpendicular to the Z-axis through O in such a way that the end A remains pinned to the crank at A and the end B remains pinned to the piston which is constrained to translate back and forth along the X-axis. Without constraints it takes six coordinates to completely locate two rigid bodies in a plane. With the constraints of the crank and connecting rod mechanism, it takes only one generalized coordinate to completely locate both the crank and the connecting rod. Several choices are possible: the crank angle θ, the distance x from O to the piston pin B, the angle between the connecting rod AB and the X-axis, etc. Experience has shown that θ is the most convenient generalized coordinate.



In the mechanism shown let the length of the crank throw OA be a and length of the connecting rod AB be $2a$. The point G is at the midpoint of the connecting rod AB.

(a) Determine the x- and y- coordinates of the linear velocity of the point G in terms of a and $\dot{\theta}$ when θ has the particular value $\theta = 0$.

(b) Determine the angular velocity of the connecting rod AB in terms of a and $\dot{\theta}$ when θ has the particular value $\theta = 0$.

(c) Determine the x- and y- coordinates of the linear velocity of the point G in terms of a and $\dot{\theta}$ when θ has the particular value $\theta = \pi/2$.

(d) Determine the angular velocity of the connecting rod AB in terms of a and $\dot{\theta}$ when θ has the particular value $\theta = \pi/2$.

Problem 4

The three-degree-of-freedom system sketched below consists of a block of mass M which is free to slide on the floor, a rigid rod of length L pivoted to the block at point O, and a rigid disk of radius r that is pivoted to the rod at the end point A. An independent and complete set of generalized coordinates (x, ϕ, ψ) for this system is indicated in the sketch below.

Your job is to

(a) Find the angular velocity of the disk (with respect to ground) in terms of the generalized coordinates x, ϕ and ψ.

(b) Express the X- and Y- components of the velocity of point A (the XY coordinate system is fixed on ground) in terms of the generalized coordinates x, ϕ and ψ.