2.004 MODELING DYNAMICS AND CONTROL II Spring 2003

Problem Set No. 2

Problem 1

A fender is mounted to a vehicle via two shock absorbers as depicted in the sketch.

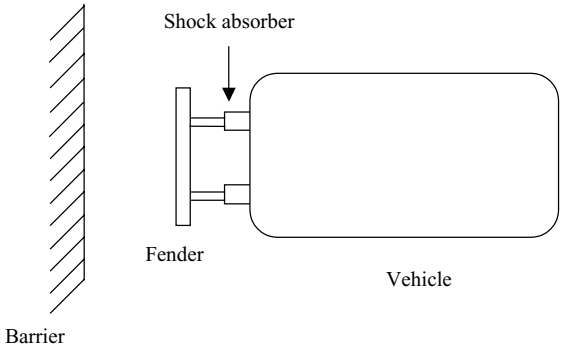

Figure 1

Each shock absorber can be approximately described as a combination of a linear spring and linear damper in parallel (i.e., subject to the same deflection). The stiffness of each shock absorber is k and its damping coefficient is b. The mass of the vehicle is m.

In one test the vehicle is crashed into a stationary barrier. Results show that while its fender is in contact with the barrier the motion of the vehicle can be approximately described by a linear second-order system.

(a) Write algebraic expressions for the damping ratio and undamped natural frequency of this model in terms of the parameters m, b, and k.
(b) A second stationary-barrier collision is performed at twice the impact velocity of the first. Is the

(i) damping ratio
(ii) undamped natural frequency
(iii) collision duration
greater, lesser or equal?

In another test (intended to study vehicle-to-vehicle collisions) a second, identical (target) vehicle is positioned against the barrier so that it doesn't move but its fender engages the fender of the bullet (impacting) vehicle.
(c) Formulate a simple model to describe the force exerted on the bullet vehicle while its fender is in contact with that of the stationary vehicle.
(d) Write algebraic expressions for the damping ratio and undamped natural frequency of this new model in terms of the parameters m, b, and k.

Problem 2

A block of mass $m_1 = 100$kg is at rest on a very long frictionless table, one end of which is terminated in a wall. Another block of mass m_2 is placed between the first block and the wall and set in motion to the left with constant speed v_{2i} as shown below. Assuming that all collisons are perfectly elastic, find the value of m_2 for which both blocks move with the same velocity after m_2 has collided with m_1, and once with the wall. (The wall has infinite mass effectively.)

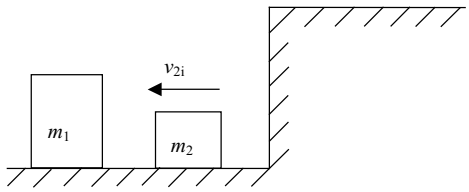

Figure 2

Problem 3

A bullet of mass m, travelling with a horizontal speed v, strikes a wooden block of mass M hanging vertically by two cords (see sketch below). Following this collison, the bullet is immediately stuck in the wooden block which, in turn, swings like a pendulum to a maximum height y.

Your job is to relate the initial velocity of the bullet v to the height y.

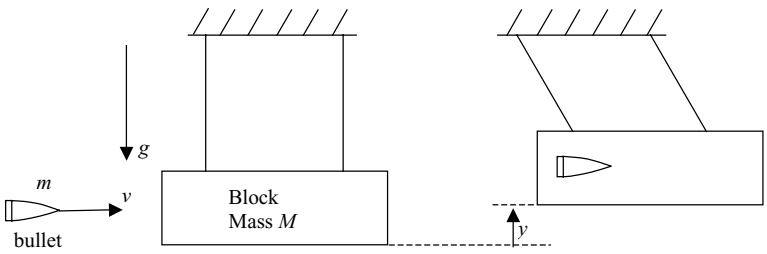

Figure 3