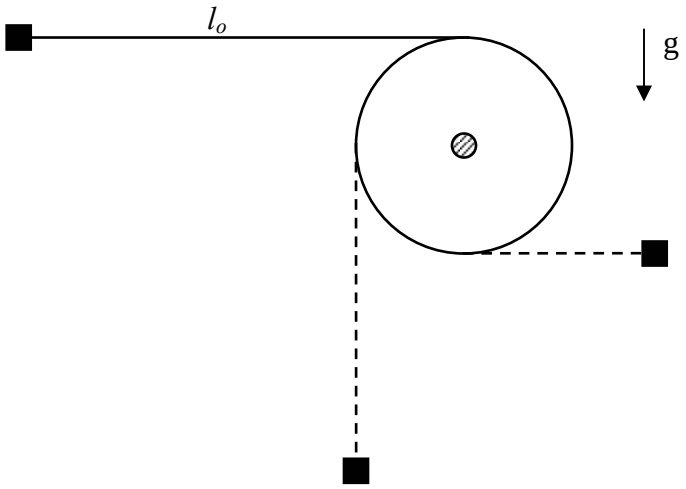
2.003J/1.053J Dynamics and Control I
Fall 2007

Problem Set 5

Out: Wednesday, 10 October, 2007
Due: Monday, 15 October, 2007

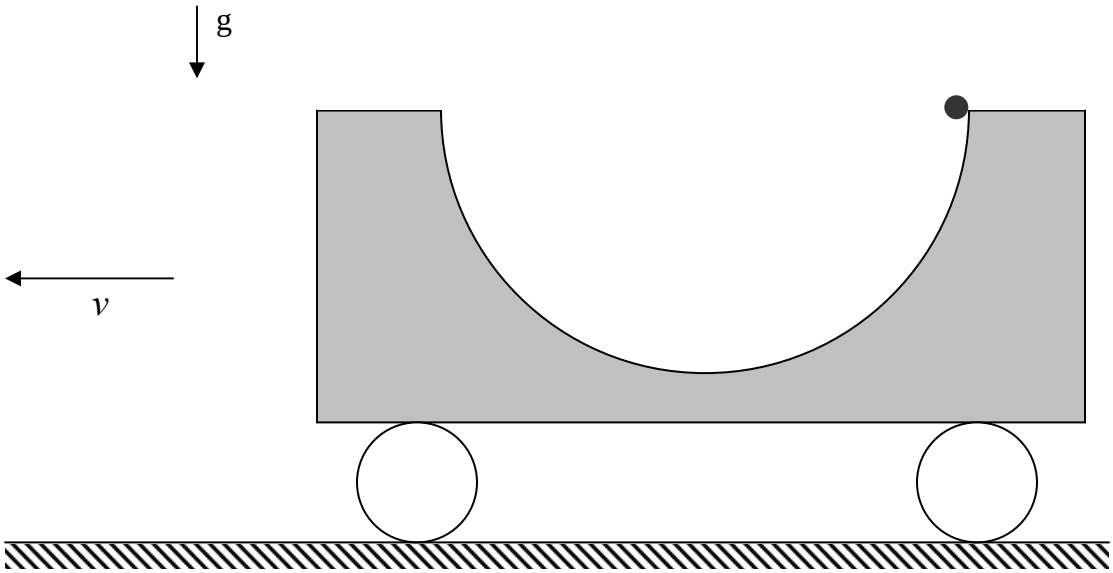
1. Pulley problem

Recall the pulley problem we solved so far again and again. A pulley of radius R is *rigidly* fixed to the wall as shown in the figure. A point mass is attached to one end of a *non-elastic* string of length l_o and the other end of the string is fixed to the top of the pulley as shown in the figure. Initially the mass is held such that the string is tight and horizontal and is suddenly released. Note that gravity acts. In one of the problem sets we found the equation of motion of the mass using Newton's laws and the work-energy principle. Now find the equation of motion of the mass using the equation that relates the torque and the angular momentum about an arbitrary point (the one with the "pesky term").



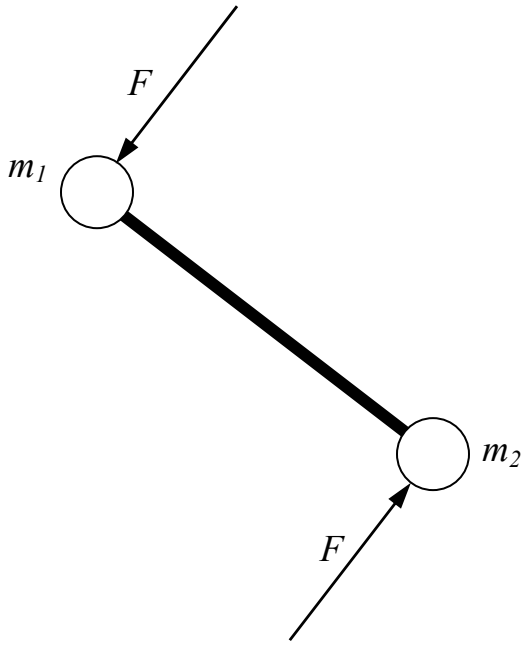
2. Semicircular platform

A semicircular platform of radius R is shown in the figure below. A particle of mass m is held at the tip of the platform. At $t = 0$, the platform is powered such that it starts moving to the left with the constant velocity v and simultaneously the point mass is released. Assume that the platform continues to move with the constant velocity. Note that gravity acts. Now find the equation of motion of the mass using the equation that relates the torque and the angular momentum about an arbitrary point (the one with the "pesky term").



3. Dumbbell Problem

Two particles of masses m_1 and m_2 are connected by a massless rigid rod of length L to form a dumbbell. Imagine that the dumbbell is lying on a frictionless table-top. A massless rocket is attached to each mass. At $t = 0$ both the rockets are ignited. Each rocket exerts a force that is always perpendicular to the massless rod. Both the rockets are positioned such that the forces they exert are equal and opposite with magnitude F. Find the equation(s) of motion of the dumbbell.



4. The center of mass and the moment of inertia of a 2D rigid body

A 2D rigid body shown in the figure below has constant density ρ kg/m^2. It is a thin circular disc of radius R centered at point P that contains a circular cavity centered at point Q. The distance $PQ = R/3$.

a. Find the center of mass of the rigid body
b. Find its moment of inertia about the vertical axis coming out of page through point P
c. Find its moment of inertia about the vertical axis coming out of the page through the center of mass