

18.03 Problem Set 8, first half

The second half will be available by Monday, April 26.

| | | |
|-----|-----------|-----------------------------------------------|
| L29 | F 16 Apr | The pole diagram: SN 22, 23. |
| L30 | W 21 Apr | Fourier and Laplace: a tale of two transforms |
| R20 | Th 22 Apr | Review. |
| L31 | F 23 Apr | Hour Exam III |

IV. First order systems

| | | |
|-----|----------|-------------------------------------------------------------|
| L32 | M 28 Apr | Linear systems and matrices: EP 5.1–5.3, SN 25, Notes LS.1. |
|-----|----------|-------------------------------------------------------------|

Part I.

29. (F 16 Apr) (a) Find the Laplace transform of $f(t) = (u(t) - u(t - 2\pi)) \sin(t)$ by use of the t -shift rule.

(b) For each of the following functions $f(t)$, find the pole diagram of $F(s)$. (i) $f(t) = 1$. (ii) $f(t) = e^{-t} + 3e^{-3t}$. (iii) $f(t) = \cos(2t) + e^{-t} \sin t$.

30. Nothing

31. Hour exam

Part II.

29. (F 16 Apr) [Poles] (a) For each of the pole diagrams below:

(i) Describe common features of all functions $f(t)$ whose Laplace transforms have the given pole diagram.

(ii) Write down two examples of such $f(t)$ and $F(s)$.

The diagrams are: (1) $\{1, i, -i\}$. (2) $\{-1 + 4i, -1 - 4i\}$. (3) $\{-1\}$. (4) The empty diagram.

(b) A mechanical system is discovered during an archaeological dig in Ethiopia. Rather than break it open, the investigators subjected it to a unit impulse. It was found that the motion of the system in response to the unit impulse is given by $w(t) = u(t)e^{-t/2} \sin(3t/2)$.

(i) What is the characteristic polynomial of the system? What is the transfer function $W(s)$?

(ii) Sketch the pole diagram of the system.

(ii) The team wants to transport this artifact to a museum. They know that vibrations from the truck that moves it result in vibrations of the system. They hope to avoid circular frequencies to which the system response has the greatest amplitude. What frequency should they avoid?

(iv) Invoke the Mathlet **Amplitude Response and Pole Diagram**, and set the system parameters b and k to the values you determined for the Ethiopian system. Check to see that the amplitude response curve shows a maximum where you predicted it would. Grab the 3D image and move it to view the graph of $|W(s)|$ from different angles. Explain in words what each of the following graphical features in the 3D window represents: The yellow box-like figure; the green box-like figure; the yellow curve forming the base of the yellow boxlike figure; the red arrows; the yellow diamonds.

30. Nothing

31. Hour exam

Part I.29 solutions. (a) $f(t) = g(t) - g_{2\pi}(t)$ where $g(t) = u(t) \sin t$. $G(s) = \frac{1}{s^2+1}$
 $F(s) = G(s) - e^{-2\pi s}G(s) = \frac{1-e^{-2\pi s}}{s^2+1}$.

(b) (i) $\{0\}$. (ii) $\{-1, -3\}$. (iii) $\{2i, -2i, -1 - i, -1 + i\}$.

MIT OpenCourseWare
<http://ocw.mit.edu>

18.03 Differential Equations
Spring 2010

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.