PROFESSOR: Welcome back to recitation. In this video what I'd like us to do is practice Taylor series. So I want us to write the Taylor series for the following function, f of x equals 3 x cubed plus 4 x squared minus 2x plus 1. So why don't you pause the video, take some time to work on that, and then I'll come back and show you what I get.

All right, welcome back. Well, we want to find the Taylor series for this polynomial f of x equals 3 x cubed plus 4 x squared minus 2x plus 1. So what I'm going to do is I'm just going to write down Taylor's-- or the expression we have for the sum, for the Taylor series in general and then I'm going to start computing what I need and I'm going to see what I get. So what do I need to remember? Well let's remind ourselves what the formula is. We should get f of x is equal to the sum from n equals 0 to infinity of the nth derivative of f at 0 over n factorial times x to the n. So that's what we want.

So what I obviously need to start doing is figuring out derivatives of f at 0. And so what I'm going to do is I'm just going to make myself a little table. So let's see, we're going to say f 0 at 0, f 1 at 0, f 2 at 0, f 3 at 0, f 4 at 0. And I'm getting tired of writing, so I'm going to stop there. OK, so let's take the function-- the zeroth derivative of f is just the function itself, so let's come back here. What is the function if I evaluate it at x equals 0? 0, 0, 0, 1. I get 1.

All right. What is the first derivative? So I'm going to write out the first derivative and then I'm going to say I'm evaluating it at x equals 0. So the first derivative looks like it's 9 x squared plus 8 x minus 2. So I'm gonna write this down. 9 x squared plus 8 x minus 2. Evaluate it at x equals 0. 0, 0. I get negative 2.

All right, well, let me take the second derivative. OK let's see what I get here. I get 18x plus 8. And I'm going to evaluate that at x equals 0. This is just a way to write, I'm going to evaluate what's here at x equal this number, so if you haven't seen that before. So I get 8. OK and then the third derivative is 18, oh just 18. Evaluate it at x equals 0. I get 18. And then the fourth derivative. What's the derivative of a constant function? It's 0. What do you think the fifth derivative is evaluated at 0? Looks like it'll be 0. You take the sixth derivative. Looks like everything bigger than 3-- so the nth derivative at 0 is equal to 0 for n bigger than 3.

So it looks like we should only have 4 terms in this. So that maybe seems a little weird, but let's keep going and see what happens. Let's start plugging things in. So again, let's remember the formula. I'm going to walk over here to the right and I'm going to start using that formula and

using these numbers that I have and writing it out. So the first term is going to be the function evaluated at 0 divided by 0 factorial times 1. 0 factorial is 1, so it's just going to be the function evaluated at 0 times 1. The function evaluated at 0, we said was 1, so that's the first term in the Taylor series.

OK what's the next term? The next term, remember, is the first derivative evaluated at 0 divided by 1 factorial, which is still 1, times x. So the first derivative, if I come back over here, evaluated at 0, I get negative 2. So I'm going to get minus 2x. The next term, so I had zeroth derivative, first derivative, now I'm at the second derivative. Now it's getting confusing. I'm going to start writing the things above. The second derivative evaluated at 0 divided by 2 factorial times x squared. That's what I should have here.

Let's come over here. Second derivative evaluated at 0 was 8. So it's going to be 8 over 2, 'cause 2 factorial is 2, x squared. So it's going to be plus 4 x squared. And then I have to have the third derivative evaluated at 0 divided by 3 factorial times x cubed. What's 3 factorial? 3 factorial is 6. What was the third derivative evaluated at 0? It was 18. 18 divided by 6 is 3. So I get plus 3 x cubed. And all the other terms were 0, so I'll just stop writing them.

OK now if you watched the video all the way through here, at some point maybe you said "Christine, this is madness." Well why is it madness? Because what is this? Well this is the function again, right? It's exactly what we started with. The order is opposite of what it was before 'cause now the powers go up instead of down, but it's the same polynomial.

OK we talked about this briefly, I think, when we were doing some quadratic approximations. And I mentioned way back that quadratic approximations of polynomials at x equals 0 are just the polynomials again. This is the exact same kind of thing happening. Because what is the Taylor series? It's just better and better approximations as n gets larger and larger. So if I wanted to have a fourth order approximation of this function f of x at x equals 0, I would get the same function back.

That's really the idea of what's happening here. So maybe you saw the sort of trick in this question, and when you saw this problem you laughed at me and you said, "Well I'm just going to write down the function again and I'll be done." Maybe you didn't see that right away, and if you didn't see that right away that's OK. I bet you're in good company. And it's totally fine because now you've seen this. You've seen how it works out. And you know, hey, now any time I see a polynomial and I want to do the Taylor series for this polynomial, I just have to

write down the polynomial again.

So that was the main goal of this video. It took us a long time to get there, but I think we got it. So the answer to the ultimate answer to the question of write the Taylor series of this function, it's just this function again. All right, that's where I'll stop.