Examples of Series

Example: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

We'll now look at some other interesting series and will return to the (very important) geometric series later.

The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ turns out to be very similar to the improper integral $\int_1^\infty \frac{dx}{x^2}$, which is convergent. The series is also convergent.

It's easy to calculate that $\int_1^\infty \frac{dx}{x^2} = 1$. It's hard to calculate that:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

This was computed by Euler in the early 1700's.

Example: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$

Notice that the series in these examples start with $n = 1$; we can't start with $n = 0$ because a_0 would be of the form $\frac{1}{0}$. That's ok — we can start these series at any positive value of n and it won't make a difference to whether they converge or diverge.

The series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ is similar to the integral $\int_1^\infty \frac{dx}{x^3}$, which converges to $\frac{1}{2}$. The series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ was only recently proved to converge to an irrational number; there is no elementary way of describing its value.

We've been loosely comparing series to integrals; we can make this formal by using a Riemann sum with $\Delta x = 1$.

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