

JOEL LEWIS: Hi. Welcome to recitation.

In lecture you've learned how to compute derivatives of polynomials, and you've learned the relationship between derivatives and tangents lines. So let's do a quick example that puts those two ideas together.

So here I have a question on the board: compute the tangent line to the curve y equals x cubed minus x at the point $(2, 6)$.

So why don't you take a minute, work on that yourself, pause the video, we'll come back and we'll do it together.

All right. Welcome back.

So we have this function, y equals x cubed minus x . Let's just draw a quick sketch of it. So looks to me like it has zeros at 0 , 1 , minus 1 . And it sort of does something like this in between. Very rough sketch there. And way over-- well, OK. We'll call that the point $(2, 6)$. Feel a little sketchy, but all right.

OK. So we want to know what the tangent line to the curve at that point is. So in order to do that, we need to know what its derivative is, and then that'll give us the slope. And then with the slope, we have the slope and we have a point, so we can slap that into, say, your point-slope formula for a line.

So, all right, so the derivative of this function is y prime. So, OK, so here we have a sum of two things, and they're both powers of x . And so we learned our rules for a power of x that the derivative of x to the n is n times x to the n minus 1 . And so we also learned that the derivative of a sum of two things is the sum of the derivatives. So in this case, so the derivative of x cubed minus x is $3x$ squared minus 1 .

OK. So this is the slope of the function in terms of x . But in order to compute the tangent line, we need the slope at the particular point in question. Right? This is really important. So we aren't going to use $3x$ squared plus 1 as our slope. Right? We want the slope at the point x equals 2 . Right?

OK. So what we want for the slope of the tangent line is y prime of 2 . Right? We want it at this point when x is equal to 2 . So that's equal to, well, 3 times 2 squared is 12 , minus 1 is 11 . So

this is the slope, this is the slope of the tangent line.

I just want to say this one more time for emphasis. This is a really common mistake that we see on lots of homework and exams when teaching calculus. You have to remember that when you compute the slope of the tangent line, you compute the derivative and then you need to plug in the value of x at the point in question. Or the value of x and y . You know, you need to plug in the values of the point that you have. So here, the derivative is $3x^2 - 1$, so the slope at the point $(2, 6)$ is 11. It's just a number. The slope at that point is that particular number.

OK, so now, to compute the tangent line, we have a point, $(2, 6)$, and we have a slope, 11. So we can plug into point-slope form. So the equation of the-- on the-- tangent line is $y - 6$ is equal to 11 times $x - 2$. Right? So it's $y - y_0$ is equal to the slope times $x - x_0$. If you like, some people prefer to write their equations of their lines in slope-intercept form. So if you wanted to do that, you could just multiply through by 11 and then bring the constants together. So we could say, or y equals $11x - 22 + 6$ is -16 .

So either of those is a perfectly good answer to the question. So that's that.