3.35 – Fracture and Fatigue
Problem Set 5 – Solutions
November 13, 2003

2.1 Cyclic slip differs from monotonic slip in that during fully reversed cyclic slip there is no rotation of the slip system. For this reason, the primary slip system remains the most highly stressed, confining slip to this plane

2.2 Vein structures and PSB structures in FCC crystals are composed mainly of edge dislocations because edge dislocations of opposite character on parallel planes form dipoles, whereas screw dislocations can easily cross slip and annihilate each other, leaving only edge dislocations.

(3) ΔK_{th} corresponds to $\frac{da}{dN} \leq 10^{-11}$ m/cycle

Smallest crack advance detectable = 0.03 mm
$f = 2\,Hz = 2$ cycles/sec

How many cycles will it take to propagate a crack a distance, da, of 0.03mm, such that $\frac{da}{dN} = 10^{-11}$ m/cycle?

$$dN = \frac{da}{10^{-11}\,m/cycle} = \frac{3 \times 10^{-5}\,m}{10^{-11}\,m/cycle} = 3 \times 10^{6}\ cycles$$

At 2Hz, how many cycles per day?

$$\left(\frac{2\ cycles}{sec}\right)\left(\frac{3600\,sec}{hour}\right)\left(\frac{24\ hour}{day}\right) = \frac{172800\ cycles}{day}$$

How many days?

$$\frac{3 \times 10^{6}\ cycles}{172800\ \frac{cycles}{day}} = \boxed{17.36\ days}$$

(4) Piston: $d_i = 9\times10^{-2}\,m$ $\quad d_o = 11\times10^{-2}\,m$

$\sigma_y = 550\,MPa \quad K_{Ic} = 30\,MPa\sqrt{m}$

Semi-circular flaw: $r = 1\times10^{-3}\,m$

$\frac{da}{dN} = 3\times10^{-10}(\Delta K)^{2.5}$ $\quad L = 20cm$

$R = \frac{r_o + r_i}{2} = 0.05m$

$t = 0.01m$

The "Pressure Vessel Design Handbook" by H.H. Bednar states that a shell is treated as thin if $R/t > 10$. In this case, $R/t = 5$, so we must treat this as a thick pressure vessel

Hoop Stress: $\sigma_\theta = \left(\frac{P_{max}\, r_i^2}{r_o^2 - r_i^2}\right)\left[1 + \frac{r_o^2}{R^2}\right]$

$$= \left(\frac{(55MPa)(0.045m)^2}{(0.055m)^2 - (0.045m)^2}\right)\left(1 + \frac{(0.055m)^2}{(0.05m)^2}\right)$$

$\sigma_\theta = 246.14\,MPa$

For a radial flaw: $K_I = \frac{1.12\,\sigma\sqrt{\pi a}}{\sqrt{Q}}$

Let $Q \approx \psi^2 - 0.212\left(\frac{\sigma^2}{\sigma_y^2}\right)$ $\quad \sigma_y = 550MPa$

$\psi = \frac{3\pi}{8} + \frac{\pi}{8}\left(\frac{a^2}{c^2}\right) = \frac{4\pi}{8}$

$$Q = \left(\frac{\pi}{2}\right)^2 - 0.212\left(\frac{(246.14\,MPa)^2}{(550MPa)^2}\right) = 2.42$$

$\therefore K_I = 1.27\sigma\sqrt{a}$ where a is the radius of the circular flaw

$$\frac{da}{dN} = 3\times10^{-10}(\Delta K)^{2.5} = 3\times10^{-10}\,(1.27)^{2.5}\,(246.14\,MPa)^{2.5}\,a^{2.5/2}$$

Note: Since we are loading $0 \rightarrow 55MPa \rightarrow 0$ $(R=0)$, $K_{max} = \Delta K$

$$a_c = \frac{K_{Ic}^2}{1.27^2\sigma^2} = \frac{(30MPa\sqrt{m})^2}{97717\,MPa^2} = 9.21\,mm = 9.21\times10^{-3}\,m$$

Integrate Paris law

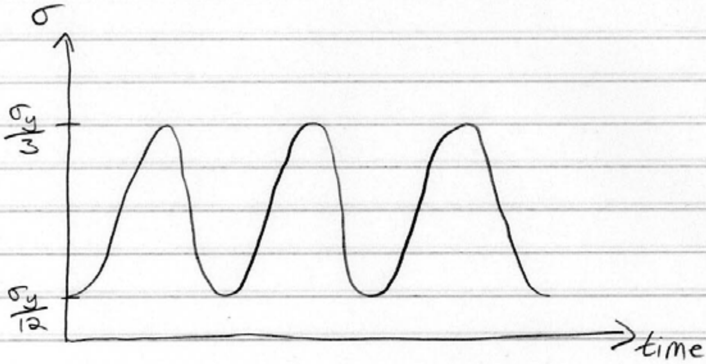
$$\int_{1\times10^{-3}\,m}^{9.21\times10^{-3}\,m} a^{-2.5/2} = 3\times10^{-10}\,(1.27)^{2.5}\,(246.14)^{2.5}\int_{0}^{N_f} dN$$

$$-4a^{-0.25}\Big|_{1\times10^{-3}\,m}^{9.21\times10^{-3}\,m} = 5.182\times10^{-4}\,N_f$$

$$-4\left(3.228 - 5.623\right) = 5.182\times10^{-4}\,N_f$$

$$\boxed{N_f = 18{,}487 \text{ cycles}}$$

(5) $\sigma_{min} = \frac{\sigma_y}{12}$, $\sigma_{max} = \frac{\sigma_y}{3}$, $K_I = \sigma^{\infty}\sqrt{\pi a}$, $\frac{da}{dN} = C(\Delta K)^m$



(i) Find $a_{critical}$:

$$\sigma_{max} = \frac{1}{3}\sigma_y$$

$$K_{I_{max}} = \sigma_{max}\sqrt{\pi a_{cr}} = \frac{1}{3}\sigma_y\sqrt{\pi a_{cr}}$$

Fracture occurs when $K_{I_{max}} = K_{IC}$

$$\frac{1}{3}\sigma_y\sqrt{\pi a_{cr}} = K_{IC}$$

$$\boxed{a_{cr} = \frac{9}{\pi}\left(\frac{K_{IC}}{\sigma_y}\right)^2}$$

(ii) Derive N_f:

$$\frac{da}{dN} = C(\Delta K)^m$$

$$\Delta K = (\sigma_{max} - \sigma_{min})\sqrt{\pi a}$$
$$= \left(\frac{1}{3}\sigma_y - \frac{1}{12}\sigma_y\right)\sqrt{\pi a}$$
$$= \frac{1}{4}\sigma_y\sqrt{\pi a}$$

$$\frac{da}{dN} = C\left(\frac{1}{4}\sigma_y\sqrt{\pi a}\right)^m$$
$$= C\left(\frac{\sigma_y}{4}\right)^m(\pi a)^{m/2}$$

$$\frac{da}{dN} = C\left(\frac{\sigma_y\sqrt{\pi}}{4}\right)^m a^{m/2}$$

$$\int_{a_i}^{a_f} a^{-m/2}\,da = \int_0^{N_f} C\left(\frac{\sigma_y\sqrt{\pi}}{4}\right)^m dN$$

For $m \neq 2$

$$\left.\frac{a^{1-m/2}}{1-m/2}\right|_{a_i}^{a_f} = C\left(\frac{\sigma_y\sqrt{\pi}}{4}\right)^m N_f$$

$$\frac{1}{1-m/2}\left\{a_f^{\,1-m/2} - a_i^{\,1-m/2}\right\} = C\left(\frac{\sigma_y\sqrt{\pi}}{4}\right)^m N_f$$

From (i): $a_f = \frac{9}{\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2$

$a_i = 0.01$ in (limit of resolution for NDE method)

$$\therefore\ N_f = \frac{1}{C(1-m/2)}\left(\frac{4}{\sigma_y\sqrt{\pi}}\right)^m\left\{\left[\frac{9}{\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2\right]^{1-m/2} - [0.01]^{1-m/2}\right\}$$

Make sure units are consistent in the end. This will depend on the units of K_{Ic}, σ_y, etc.

For $m=2$

$$\ln\left(\frac{a_f}{a_i}\right) = C\left(\frac{\sigma_y\sqrt{\pi}}{4}\right)^2 N_f$$

$a_f = \frac{9}{\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2$, $a_i = 0.01$

$$N_f = \frac{1}{C}\left(\frac{4}{\sigma_y\sqrt{\pi}}\right)^2 \ln\left(\frac{900}{\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2\right)$$