21L.017 The Art of the Probable: Literature and Probability
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II. An Argument for Divine Providence, taken from the Constant Regularity observed in the Births of both Sexes. By Dr. John Arbuthnot, *Physician in Ordinary to her Majesty, and Fellow of the College of Physicians and the Royal Society.* [1]

AMONG innumerable Footsteps of Divine Providence to be found in the Works of Nature, there is a very remarkable one in the exact Ballance that is maintained between the Numbers of Men and Women; for by this means it is provided, that the Species may never fail, nor perish, since every Male may have its Female, and of a proportional Age. This Equality of Males and Females is not the Effect of chance but Divine Providence, working for a good End, which I thus demonstrate :

Let there be a Die of Two sides, M and F, which denote Cross and Pile), now to find all the Chances of any determinate Number of such Dice, let the Binome $\mathrm{M} + \mathrm{F}$ be raised to the Power, whose Exponent is the Number of Dice given; the Coefficients of the Terms will show all the Chances sought. For Example, in Two Dice of Two sides, M + F the chances are $\mathrm{M}^2 + 2\mathrm{MF} + \mathrm{F}^2$, that is One Chance for M double, One for F double, and Two for M single and F single; in Four such Dice there are Chances $\mathrm{M}^4 + 4\mathrm{M}^3\mathrm{F} + 6\mathrm{M}^2\mathrm{F}^2 + 4\mathrm{MF}^3 + \mathrm{F}^4$; that is, One Chance for M quadruple, One for F quadruple, Four for triple M and single F,Four for single M and triple F. and Six for M double: and F double: and universally,: if the Number of Dice be n, all their Chances will be expressed in this Series,

$\mathrm{M}^n + \frac{\mathrm{n}}{1} \times \mathrm{M}^{\mathrm{n}-1}\mathrm{F} + \frac{\mathrm{n}}{1} \times \frac{\mathrm{n}-1}{2} \times \mathrm{M}^{\mathrm{n}-2}\mathrm{F}^2 + \frac{\mathrm{n}}{1} \times \frac{\mathrm{n}-1}{2} \times \frac{\mathrm{n}-2}{3} \times \mathrm{M}^{\mathrm{n}-2}\mathrm{F}^2 +, \&c.$

It appears. plainly, that when the Number of Dice is even, there are as many M's as F's, in the middle Term of this Series, and in all the other Terms there are most M's or most F's.

If therefore a Man undertake, with an even Number of Dice, to throw as many M's as F's, he has all the Terms but the middle Term against him; and his lot is the Sum of all the Chances, as the coefficient of the middle Term, is to the power of 2 raised to an exponent equal to the number of Dice: so in Two Dice, his Lot is $\frac{2}{4}$ or $\frac{1}{2}$, in Three Dice $\frac{6}{16}$ or $\frac{3}{8}$, in Six Dice $\frac{20}{64}$ or $\frac{5}{16}$, in Eight Dice $\frac{70}{256}$ or $\frac{35}{128}$, &c.

To find this middle Term in any given Power or Number of Dice, continue the Series $\frac{\mathrm{n}}{1} \times \frac{\mathrm{n}-1}{2} \times \frac{\mathrm{n}-2}{3}$, &c. till the number of terms are equal to $\frac{1}{2}$n. For Example, the coefficient of the middle Term of the tenth Power $\frac{10}{1} \times \frac{9}{2} \times 83 \times \frac{7}{4} \times \frac{6}{5} = 252$, the tenth Power of Two is 1024, if therefore A undertake to throw with Ten Dice in one throw an equal Number of M's and F's, he has 252 chances out of 1024 for him, that is his Lot is $\frac{259}{1024}$ or $\frac{63}{256}$, which is less than $\frac{1}{2}$.

It will be easy by the help of Logarithms, to extend this Calculation to a very great Number, but that is not my present Design. It is visible from what has been said, that with a very great Number of Dice, A's Lot would become very small; and consequently (supposing M to denote Male and F Female) that in the vast Number of Mortals, where would be but a small part of all the possible Chances, for its happening at any assignable time, that an equal Number of Males and Females should be born.

It is indeed to be confessed that this Equality of Males and Females is not Mathematical but Physical, which alters much the foregoing Calculation; for in this Case the

[1]From: *Philosophical Transactions of the Royal Society of London* **27** (1710), 186–190, reprinted in M G Kendall and R L Plackett (eds), *Studies in the History of Statistics and Probability Volume II*, High Wycombe: Griffin 1977, pp. 30–34.

middle Term will not exactly give A's Chances, but his chances will take in some of the Terms next the middle one, and will lean to one side or the other. But it is very improbable (if mere Chance govern'd) that they would never reach as far as the Extremities: But this Event is happily prevented by the wise Oeconomy of Nature; and to judge of the wisdom of the Contrivance, we must observe that the external Accidents to which Males are subject (who must seek. their Food with danger) make a great havock of them, and that this loss exceeds far that of the other Sex occasioned by Diseases incident to it, as Experience convinces us. To repair that Loss, provident Nature, by the Disposal of its wise Creator, brings forth more Males than Females; and that in almost a constant proportion. This appears from the annexed Tables, which contain Observations for 82 years of the births in *London*. Now, to reduce the Whole to a Calculation, I propose this

Problem. A lays against B. that every Year there shall be born more Males than Females: To find A's Lot, or the Value of his Expectation.

It is evident from what has been said, that A's lot for each year is less than $\frac{1}{2}$ (but, that the Argument might be stronger) let his Lot be equal to $\frac{1}{2}$ for one year. If he undertakes to do the same thing 82 times running, his Lot will be $\overline{\frac{1}{2}}|^{82}$, which will be easily found by the Table of Logarithms to be $\frac{1}{4\ 8360\ 0000\ 00000\ 00000\ 00000\ 0000}$. But if A wager with B, not only that the Number of Males shall exceed that of Females, every Year, but that this Excess shall happen in a constant Proportion, and the Difference lie within fix'd limits; and this not only for 82 Years, but for Ages of Ages, and not only at *London*, but all over the World; which it is highly probable is the Fact, and designed that every Male may have a Female of the same Country and suitable Age; then A's Chance will be near an infinitely small Quantity, at least less than any assignable fraction. From whence it flows, that it is Art, not Chance, that governs.

There seems no more probable Cause to be assigned in Physics for this Equality of the Births, than that in our 'first Parents Seed there were at first formed an equal Number of both Sexes.

Scholium. From hence it follows, that Polygamy is contrary to the Law of Nature and Justice, and to the Propagation of the Human Race; for where Males and Females are in equal number, if one *M*an take Twenty Wives, Nineteen Men must live in Celibacy, which is repugnant to the Design of Nature; nor is it probable that Twenty Women will be so well impregnated by one Man as by Twenty.

	Christened.			Christened.	
Anno.	*Males.*	*Females.*	*Anno.*	*Males.*	*Females.*
1629	5218	4683	1648	3363	3181
30	4858	4457	49	3079	2746
31	4422	4102	50	2890	2722
32	4994	4590	51	3231	2840
33	5158	4839	52	3220	2908
34	5035	4820	53	3196	2959
35	5106	4928	54	3441	3179
36	4917	4605	55	3655	3349
37	4703	4457	56	3668	3382
38	5359	4952	57	3396	3289
39	5366	4784	58	3157	3013
40	5518	5332	59	3209	2781
41	5470	5200	60	3724	3247
42	5460	4910	61	4748	4107
43	4793	4617	62	5216	4803
44	4107	3997	63	5411	4881
45	4047	3919	64	6041	5681
46	3768	3536	65	5114	4858
47	3796	3536	66	4678	4319

	Christened.			Christened.	
Anno.	*Males.*	*Females.*	*Anno.*	*Males.*	*Females.*
1667	5616	5322	1689	7604	7267
68	6073	5560	90	7909	7302
69	6506	5829	91	7662	7392
70	6278	5719	92	7602	7316
71	6449	6061	93	7676	7483
72	6443	6120	94	6985	6647
73	6073	5822	95	7263	6713
74	6113	5738	96	7632	7229
75	6058	5717	97	8062	7767
76	6552	5847	98	8426	7626
77	6423	6203	99	7911	7452
78	6568	6033	1700	7578	7061
79	6247	6041	1701	8102	7514
80	6548	6299	1702	8031	7656
81	6822	6533	1703	7765	7683
82	6909	6744	1704	6113	5738
83	7577	7158	1705	8366	7779
84	7575	7127	1706	7952	7417
85	7484	7246	1707	8239	7623
86	7575	7119	1708	8239	7623
87	7737	7214	1709	7840	7380
88	7487	7101	1710	7640	7288