Subject 24.242. Logic II. Sample problems from the last homework, due Thursday, May 13

Recall that a *normal modal system* for the modal sentential calculus is a set of formulas Γ that meets the following conditions:

(TC) Every tautological consequence of Γ is in Γ.
(Nec) If ϕ is in Γ, so in $\Box\phi$.
(K) All instances of the schema $(\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi))$ are in Γ.

1. A binary relation *R* on a set *W* is *symmetric* iff, for every *v* and *w* in *W*, if *Rwv* then *Rvw*. Let KB be the smallest normal modal system that contains all instances of the schema
(B) $(\Diamond\Box\phi \rightarrow \phi)$
Show that a sentence is in KB if and only if it's valid for the class of frames $<W,R,I>$, with *R* symmetric..

2. Prove de Jongh's theorem that all instances of schema
(4) $(\Box\phi \rightarrow \Box\Box\phi)$
are elements of the smallest normal modal system that includes all instances of the schema:
(L) $(\Box(\Box\phi \rightarrow \phi) \rightarrow \Box\phi)$.
[Hint: The instance of schema (L) that you'll use is $(\Box(\Box(\phi \wedge \Box\phi) \rightarrow (\phi \wedge \Box\phi)) \rightarrow \Box(\phi \wedge \Box\phi))$.]