Logic I
Fall 2009
Session 14 Handout

The vocabulary of PL

- The symbols in the language PL, the basic building blocks, are:
  - *Sentence letters of PL*: A, B, C_3, ...
  - *Predicates of PL*: A′, B′, C''_2, ...
  - *Individual terms of PL*: Individual constants of PL and variables of PL
    - *Individual constants of PL*: a, b_4, c_2, ..., v
    - *Variables of PL*: w, x, y_3, z_1, ...
  - *Quantifiers of PL*: $(\forall \mathbf{x})$, $(\forall \mathbf{z}_2)$, $(\exists \mathbf{x})$, ...
  - And, of course, parentheses and the truth-functional connectives from SL.

Essentials of PL syntax

- *Sentence*: An expression **P** is a sentence of PL iff **P** is a formula of PL and no variable occurs free in **P**.
- *Formula*: An expression is a formula of PL iff it can be formed by (possibly repeated) applications of the following rules:
  1. Every atomic formula of PL is a formula of PL.
  2. If **P** is a formula of PL, so is $\neg\mathbf{P}$.
  3. If **P** and **Q** are formulas of PL, so are $(\mathbf{P}\&\mathbf{Q})$, $(\mathbf{P}\vee\mathbf{Q})$, $(\mathbf{P}\supset\mathbf{Q})$, and $(\mathbf{P}\equiv\mathbf{Q})$.
  4. If **P** is a formula of PL that contains at least one occurrence of **x** and no **x**-quantifier, then $(\forall x)\mathbf{P}$ and $(\exists x)\mathbf{P}$ are both formulas of PL.
- *Atomic formula*: An expression is an atomic formula of PL iff it is either a sentence letter of PL or an **n**-place predicate of PL followed by **n** individual terms (i.e. variables or individual constants) of PL.

Other important definitions

- *Scope*: The scope of a quantifier is the (sub)formula of which the quantifier is the main logical operator.
- *Bound variable*: An occurrence of a variable **x** is bound iff **x** is within the scope of an **x**-quantifier.

- *Free variable*: An occurrence of a variable is free iff it is not bound.
- *Substitution instance of* ***P***: If P is a sentence of PL of the form $(\forall x)\mathbf{Q}$ or $(\exists x)\mathbf{Q}$, and **a** is an individual constant, then $\mathbf{Q}(\mathbf{a}/\mathbf{x})$ is a substitution instance of **P**. The constant **a** is the *instantiating constant*.
- $\mathbf{P}(\mathbf{a}/\mathbf{x})$ is the expression just like **P** except that it contains the individual constant **a** wherever **P** contains the individual variable **x**.

Essentials of PL semantics

- The main semantic notion is that of an *interpretation* of PL.
- An interpretation includes:
  - A specification of a UD (universe of discourse)
  - An assignment of TVs to the sentence letters of PL
  - An assignment of a member of the UD to each individual constant of PL
  - An interpretation of each predicate of PL (formally, for all n, an assignment of a set of n-tuples of members of the UD to each n-place predicate of PL)
- Interpretations, like TVAs, officially cover *every* individual constant, predicate, etc., but we need not represent all that detail.
- By "interpretation", we will unofficially mean a *partial interpretation* that specifies a UD and makes assignments to all elements of the set of sentences we are considering.
- E.g., if we are dealing with sentences about dogs, and specifically about Fido, Sparky, Davy, and about which dogs growl or chase each other:
  - UD: The set of all dogs
  - Gx: x growls (formally, $\{x \in UD \mid x \text{ growls}\}$)
  - Cxy: x chases y (formally, $\{\langle x, y\rangle \in UD^2 \mid x \text{ chases } y\}$)
  - f: Fido
  - s: Sparky
  - d: Davy

24.241 Logic I
Fall 2009