

HST.584 / 22.561 Exam Solutions

Marking Scheme: a, b, c, d, e, f, h, i, j, k, l, m – 5 marks; g – 20 marks; Total = 70 marks
as students could omit 2 questions.

- a) As stated, we assume all magnetization is completely relaxed (i.e. along the z-axis) at the start of the pulse sequence. Following a right-handed rotation of 45° about the $-y$ axis, we have $\vec{M} = \frac{M_0}{\sqrt{2}}(-1,0,1)$.

- b) Given the constraints, after 99 ms any transverse components will have experienced T_2^* dephasing of the form e^{-t/T_2^*} . Therefore,

$$\vec{M} = \frac{M_0}{\sqrt{2}}(-e^{-97/50}, 0, 1) = M_0(-0.102, 0, 1/\sqrt{2}).$$

- c) We desire the same signal amplitude after our second pulse as we had after our first pulse. This means we require the same transverse component (transverse component determines signal amplitude) i.e. $M_x(97\text{ms}) = M_x(2\text{ms})$. If $T_1 \gg 101$ ms (i.e. no longitudinal relaxation has occurred), just prior to the second pulse we will assume that all existing transverse magnetization has been dephased (T_2^* decay) and our longitudinal component is the same as after our initial pulse. Therefore, to generate the same M_x component following our second pulse, we need a 90° pulse. In other words:

i. $\vec{M}(2\text{ms}) = \frac{M_0}{\sqrt{2}}(-1,0,1)$

ii. $\vec{M}(99\text{ms}) = \frac{M_0}{\sqrt{2}}(0,0,1)$

iii. $\vec{M}(101\text{ms}) = \frac{M_0}{\sqrt{2}}(-1,0,0)$ following 90° pulse about $-y$ axis

- d) If we account for T_1 recovery of the longitudinal magnetization, $M_z(99\text{ms})$ will be larger than $M_z(2\text{ms})$ and thus we will require a smaller flip angle than in the previous question.

$$M_z(t) = M_0 + (M_z(0) - M_0)e^{-t/T_1}$$

$$M_z(99\text{ms}) = M_0 + \left(\frac{M_0}{\sqrt{2}} - M_0\right)e^{-97/100} = 0.889M_0$$

We require $M_x(101\text{ms}) = M_z(99\text{ms})\sin\theta_2 = 0.889M_0 \sin\theta_2 = M_x(2\text{ms}) = \frac{-M_0}{\sqrt{2}}$.

Therefore, $\theta_2 = 52.3^\circ$ about the $-y$ axis.

- e) Our bandwidth determines the slice thickness; our frequency offset from the Larmor frequency determines the slice position along the z-axis.

i. Bandwidth: $\Delta z \gamma G_z = 0.1\text{cm}(4000\text{G}^{-1}\text{s}^{-1})(1\text{Gcm}^{-1}) = 400\text{Hz}$

ii. Offset: $z_{\text{off}} \gamma G_z = 1\text{cm}(4000\text{G}^{-1}\text{s}^{-1})(1\text{Gcm}^{-1}) = 4000\text{Hz}$

f) This is simply a Fourier relationship. Sinc RF pulses produce rectangular excitation profiles. Gaussian RF pulses produce Gaussian excitation profiles, with an inverse relationship (i.e. A wide Gaussian in time will excite a narrow Gaussian slice, a narrow Gaussian in time will produce a wide Gaussian slice). I am not going to attempt to draw Gaussians in Word – see me if you'd like an example.

g) The basic formula for determining k-space position is $k(t) = \gamma \int_0^t G(\tau) d\tau$. Our unit of δk corresponds to $\delta k = \gamma G t = 4000 G^{-1} s^{-1} (1 G cm^{-1}) (0.5 \times 10^{-3} s) = 2 cm^{-1}$. To determine our trajectory, we simply integrate our gradient waveforms.

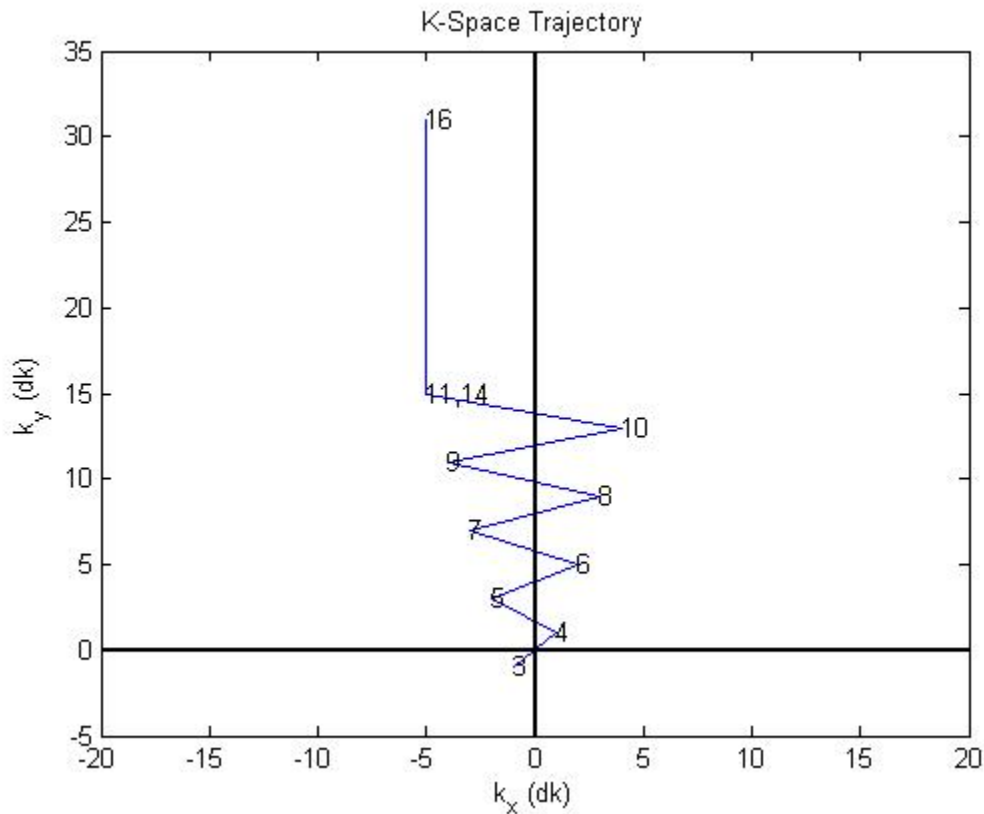


Fig. 1 – k-space trajectory for the gradient waveforms between $t = 0$ ms and $t = 100$ ms. The numbers on the actual trajectory corresponds to times in ms.

h) If we read off the trajectory we traced out in the previous question, we produce the following ordered pairs (in units of δk):

Time (ms)	(k_x, k_y) (Units of δk)
0	(0, 0)
11	(-5, 15)
14	(-5, 15)
16	(-5, 31)
99	(-5, 31)
110	(15, -5)

- i) This large G_y pulse is a spoiler or crusher gradient meant to dephase the residual transverse magnetization.
- j) No, we did not use a rectilinear approach to cover k-space (constant spacing along k-space lines \rightarrow think EPI or 2D-FT imaging approach).
- k) After the second series of gradient manipulations, we have traced out two zig-zag patterns about the $+k_y$ axis and $+k_x$ axis. While we can use Fourier symmetry relations to reduce the k-space coverage for a real object (and most objects are real), we have not covered enough of k-space yet to employ one of the relations. At minimum, we need to cover $\frac{1}{2}$ of k-space to reconstruct a real object (and in practice slightly more, such as $\frac{9}{16}$ or $\frac{5}{8}$, in order to make appropriate phase corrections – for more detail on this take HST.587 with Dr. Elfar Adelsteinsson)
- l) Resolution = $1 / k_{\max}$ in both dimensions.

Taking $k_{x\max} = 15 \delta k = 30 \text{ cm}^{-1}$. Therefore, $\text{res}_x = 1/30 \text{ cm}$

Taking $k_{y\max} = 15 \delta k = 30 \text{ cm}^{-1}$. Therefore, $\text{res}_y = 1/30 \text{ cm}$

- m) k_x is sampled most densely near the origin (between 3 & 4 ms) & least densely on the final leg of the trajectory (between 10 & 11 ms).

$$\text{FOV}_x = 1 / \Delta k_x, \text{FOV}_y = 1 / \Delta k_y$$

If our FOV is based on the least densely sampled region, we cover $9 \delta k$ in 16 samples. Therefore, $\Delta k_x = 9 (2 \text{ cm}^{-1}) / 16 = 9/8 \text{ cm}^{-1}$. Thus $\text{FOV}_x = 8/9 \text{ cm}$.

Similarly, $\Delta k_y = 9 (2 \text{ cm}^{-1}) / 16 = 9/8 \text{ cm}^{-1}$. Thus $\text{FOV}_y = 8/9 \text{ cm}$.