Scaling Laws and Constraints

- What governs the structure or architecture of systems?
- Are we free to choose; are natural systems free to choose?
- It seems that all systems evolve, grow, or are designed in the presence of contexts, constraints, laws of nature or economics, scarce resources, threats, failure modes
- Can we trace system structure to these externalities?

What Happens as Systems Get Bigger?

- Systems do seem to get bigger over time
- Can they retain the same structure as they do?
- Typical barriers to growth:
  - Natural or biological systems
    - Resistance to mechanical and gravitational loads
    - Energy use and conservation
    - Internal distribution of resources or removal of wastes
    - Fight or flight tradeoffs
  - Man-made systems
    - Engineered products and product families: complexity, cognitive limits
    - Infrastructures: capacity, budgets, complexity, reliability

An Early Explanation: Simon's Fable

- Tempus and Hora make different kinds of watches
  - One kind consists of separable subassemblies while the other consists of highly interconnected parts
- If the makers are interrupted (analogous to environmental stress) the subassemblies survive but the integral units fall apart
- Thus modular systems have survival value and gain prominence
- Magically, they also develop hierarchical structure (presumably analogous to nested subassemblies)

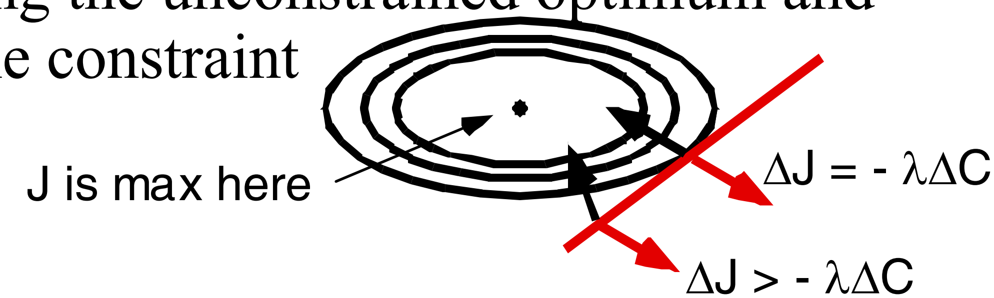
A Less Ambitious Goal

- Instead of trying to explain where entire body plans (read: system structures) came from…
- Let’s just try to explain some repeating patterns in these body plans that support scalability as a factor in survival
- Examples include scaling laws for
  - Size and shape
  - Energy consumption
  - Heating and cooling
  - Distributive systems
- Many of these models rely on network, hierarchy or fractal arguments or metaphors
- A few engineering systems have been analyzed in a similar spirit if not necessarily using the same approaches

Constraints as Drivers of Structure

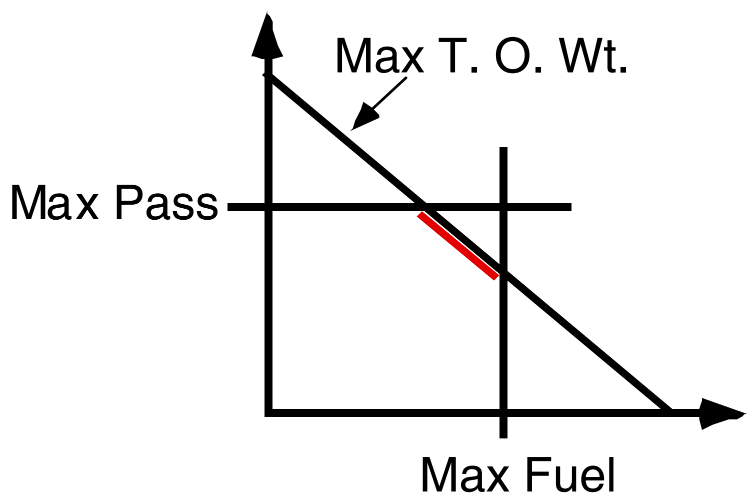
- There is almost always a constraint, a limiting resource, a failure mode
- Systems do not waste resources and can't violate limits on basic processes
  - Bandwidth, pressure drop, congestion, other flow limits
  - Energy balance, heat rejection to avoid temperature rise
  - Energy transfer rates across barriers, diffusion, radiation
  - Information processing, CPU speed, bounded rationality
  - Strength of materials
- Kuhn-Tucker conditions state that constrained optimum balances cost of missing the unconstrained optimum and the cost of violating the constraint

$$\min J_a = J + \lambda C$$



Multiple Constraints

- When there are several constraints, a “balanced” design seeks to operate near several boundaries at once
- Aircraft max passenger load + max fuel load + empty fuselage wt > max takeoff weight



Scaling Laws

- Geometric scaling (starting with Galileo)
  - Proportions are preserved as size increases
- Allometric scaling (Buckingham and others)
  - Proportions are not preserved (baby to adult, shoes, etc.)
  - Instead, different elements of the system scale at different rates
  - Discovering what these rates are, and why they apply, is a research industry of its own in engineering, biology, sociology and economics
- Scaling laws reveal the invariants of a system

Proportions Change During Growth

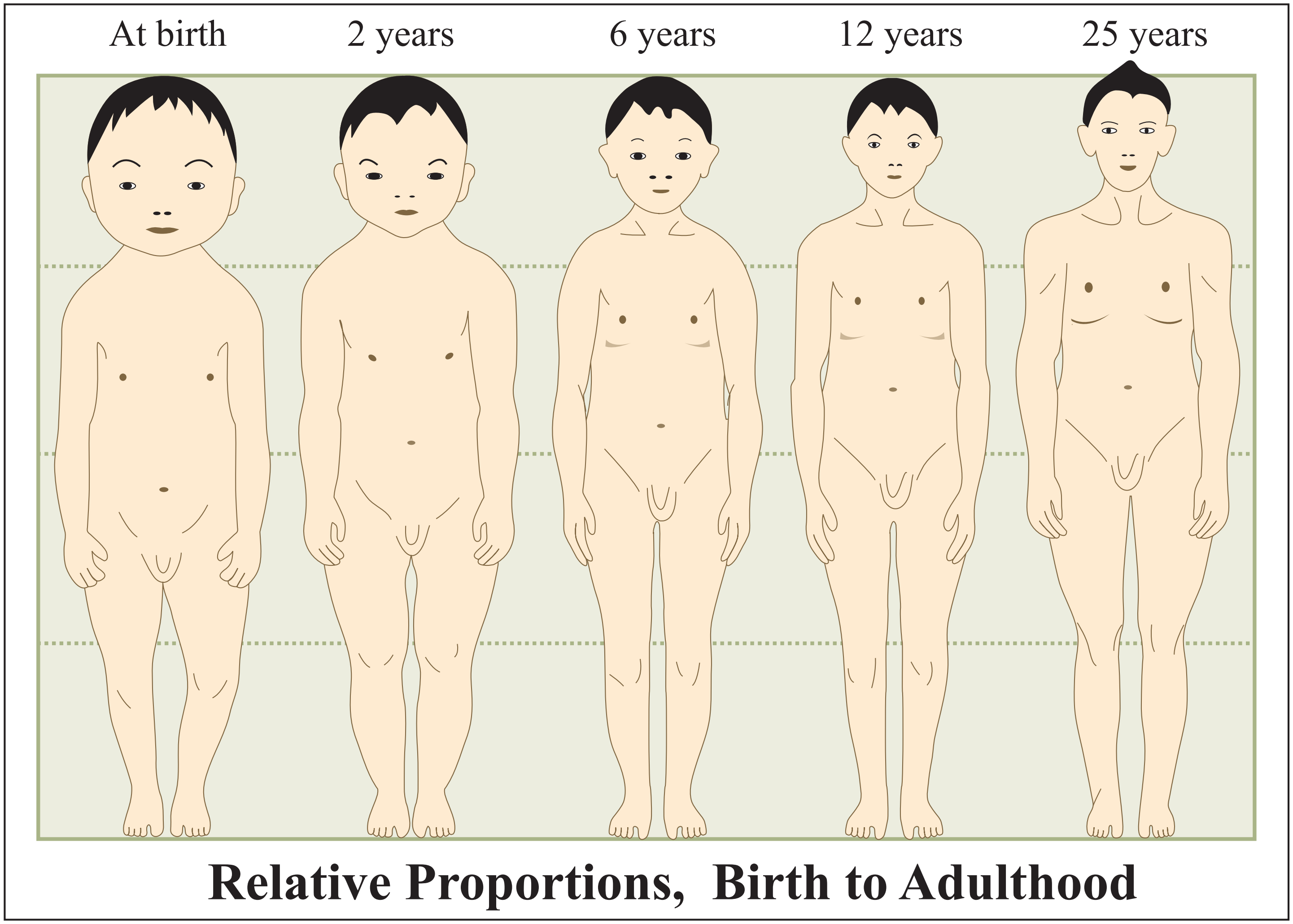


Image by MIT OpenCourseWare.

http://www.carseat.se/images/growthchartcarseat.jpg

Two Uses for Scaling Laws

- Predict how something big will behave using a small model
  - example of wave resistance to ships
- Predict how system behavior changes as size (or some other variable) changes
  - Examples include power needed to run a ship as well as size or other limits on physical systems, including biological

Scaling and Dimensional Analysis

- If physical variables are related by an equation of the form $v_2 = kv_1$ where k is a number, then this is equivalent to $v_2 / v_1 = k$ which is a dimensionless scaling relationship
- For example, when a ship goes through water it makes waves. The wave energy is related to the size of the wave by $mV^2 = mgL$ or $V^2 / gL = k$
- This is called Froude's Law of ships and it shows how energy loss rises as the ship gets bigger
- For low wave energy losses, the Froude number F is kept below 0.4, which describes the wave speed

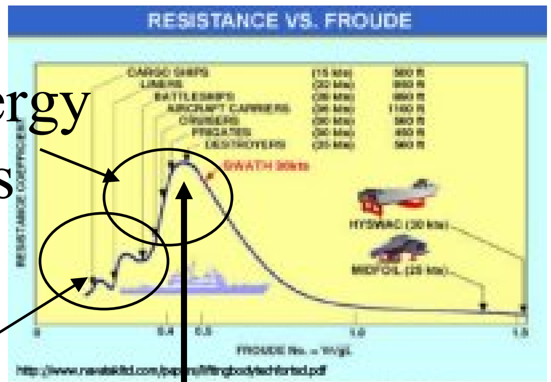
$$V_{wave} = \sqrt{gL / 2\pi} \qquad F = V / \sqrt{gL} = 1/\sqrt{2\pi} = 0.3989$$

Froude Law

$V = F\sqrt{gL}$

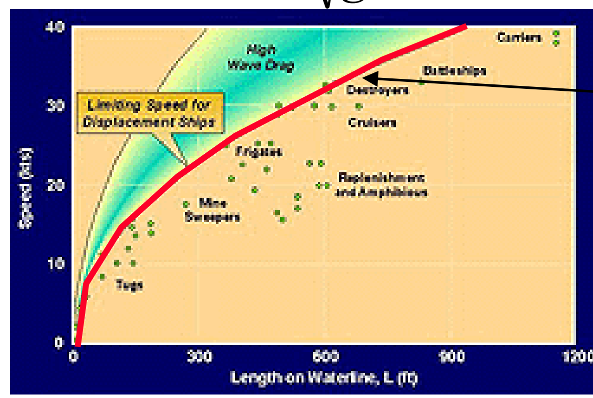
Wave energy dominates

Skin friction dominates



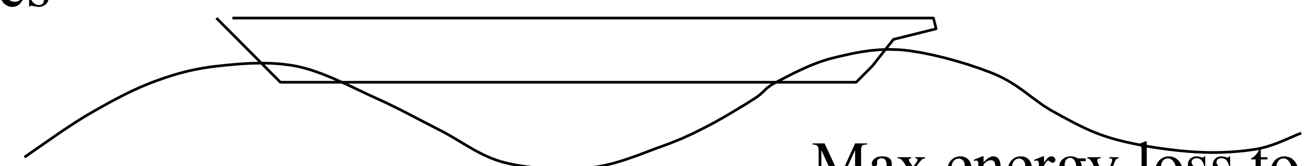
Wave Resistance vs. Froude Number

$F = 0.4$



$F < 0.4$

Short ships cannot go fast



Max energy loss to waves when ship speed = wave speed and ship length = wave length

http://www.globalsecurity.org/military/systems/ship/beginner.htm

Boilers, Coal, and Ship Speed

Adapted from "On Growth and Form" by D'Arcy Thompson

Resistance to flow = R, a force

$$R = R_{\text{skin friction loss}} + R_{\text{wave energy loss}}$$

Froude found empirically that

$$R_{\text{wave energy loss}} = k * Displacement \ \ (D)$$

if the Froude number $F = V / \sqrt{gL}$ was kept constant

$L \propto V^2$ (keep F constant)

$\mathrm{D} \propto \mathrm{L}^3 \propto V^6$

Power $P = R * V \propto V^7 \propto D^{7/6}$

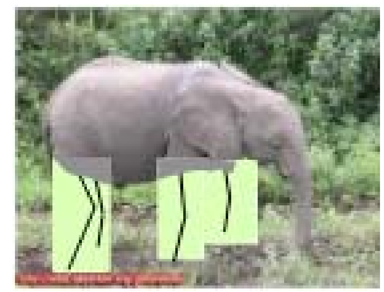
Fuel needed = $P * time = P * dist / V \propto V^6 \propto D$

So as your ship gets bigger, you have enough space for fuel but not for one boiler, so you must have several small boilers, each (2) paired with an engine and a propeller

Scaling Laws in Biological Systems

- Tree height vs diameter (Chave and Levin; Niklas and Spatz; McMahon) elephant legs and daddy long legs
  - Failure mode analysis: buckling
  - Nutrient distribution

$$Height = a * Diameter^{2/3}$$



- Metabolic rate vs body mass (Schimdt-Nielsen; Chave and Levin; West, Brown, and Enquist; McMahon; Bejan)
  - Small animals have so much surface area/mass that they need to generate heat internally much faster than large animals do

$$Metabolic\ \ Rate = a * Body\ \ Mass^{2/3}$$

  - Big animals have an easier time distributing nutrients than small animals do

$$Metabolic\ \ Rate = a * Body\ \ Mass^{3/4}$$

- Network characteristics of ant galleries (Buhl et al)
  - In vitro planar galleries have exponential degree distribution and ~ 0.1 - 0.2 of maximal "meshness" M = ratio of number of facets to maximum = 2*#nodes-5 (M=0 for trees and 1 for fully connected planar graphs) and upper limit on k = 3n-6

Summary of Biological Scaling

Little change with weight

1 Maximum functional capillary diameter changes little with weight
2 Red cell size bears no relationship to weight
3 Haematocrit is pretty constant at about 0.45
4 Plasma protein concentrations vary little with size or species
5 Mean blood pressure is about 100 mmHg more or less independent of size or species
6 Fractional airway dead space (V_D/V_T) is pretty constant at about a third
7 Body core temperature is weight-independent
8 Maximal tensile strength developed in muscle is scale invariant
9 Maximum rate of muscle contraction appears scale invariant
10 Mean blood velocity has been calculated to be proportional to mb^-0.07 (not much variation)

Changes in (almost) direct proportion to weight

1 Heart weight (g) = 5.8 * mb^ 0.98
2 Lung weight (g) = 11.3 * mb^0.98
3 Tidal volume (ml) = 7.69 mb^1.04
4 Vital capacity (ml) = 56.7 mb^1.03
5 Lung compliance (ml/cmH_2O) = 1.56 mb^1.04
6 Blood volume (ml) = 65.6 mb^1.02
7 Muscle mass = 0.40 * mb^1.00
8 Skeletal mass = 0.0608 * mb^1.08

Scaling to the quarter power of weight

1 Heart rate (min^-1) = 241 * mb^-0.25
2 Blood circulation time (seconds) = 17.4 mb^0.25
3 Respiratory rate (min^-1) = 53.5 mb^-0.26

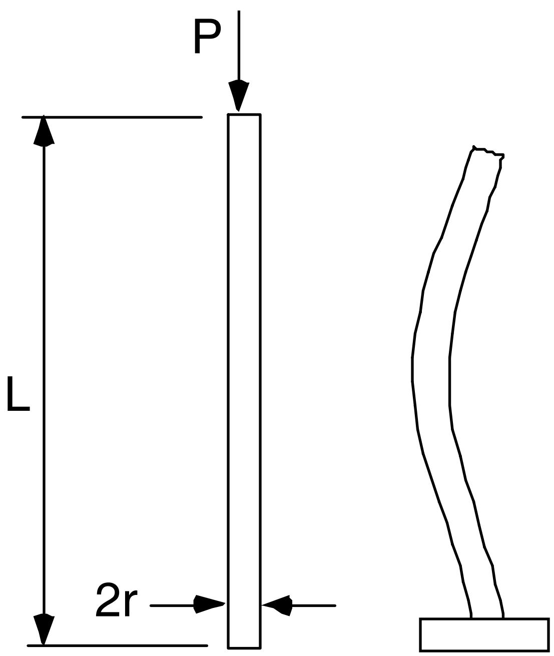
Scaling to the three quarter power of weight

1 Pmet (kcal/day) = 73.3 * mb^0.75 **(KleiberÕs Law)**
2 VO_2max (ml/s) = 1.94 mb^0.79
3 Glucose turnover (mg/min) = 5.59 * mb^0.75
4 In fact, most metabolic parameters vary with the 3/4 power of weight.

http://www.anaesthetist.com/physiol/basics/scaling/Kleiber.htm

Note that many internal organs have their own scaling laws.
So the body law is an average of many internal systems (next slide)

Simplified Buckling Analysis for Tree Height and Leg Length vs Diameter



$$P = 2.05\pi^2 \frac{EI}{L^2}$$

$$I = 0.5\pi r^4$$

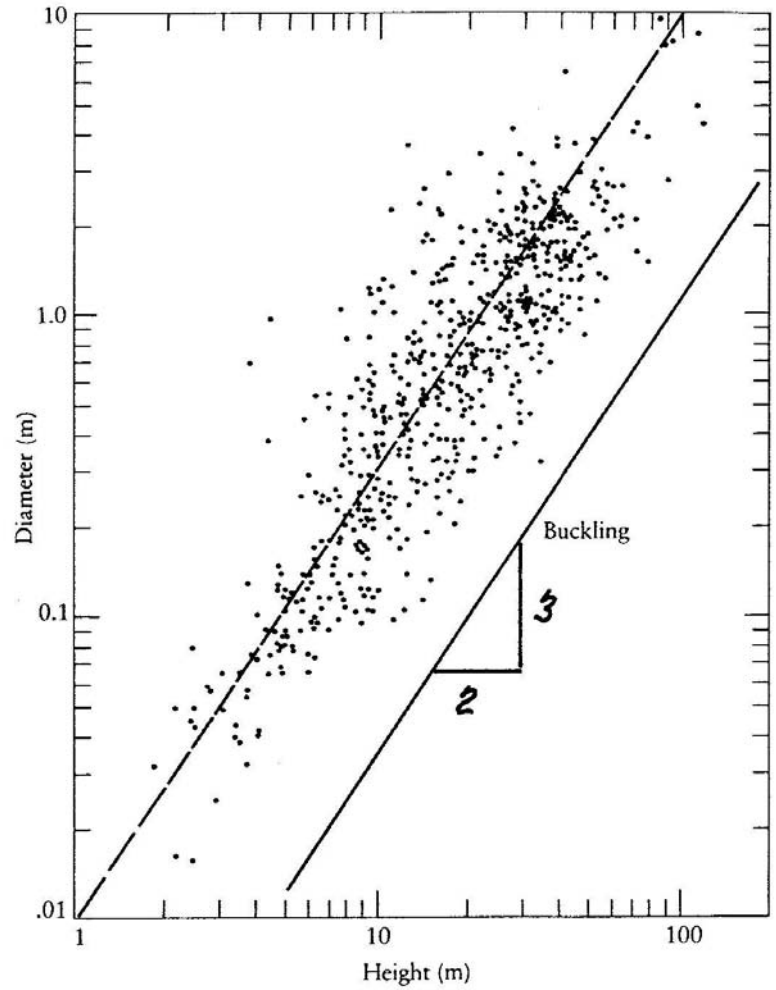
$$P = Mg = \pi r^2 \rho L g$$

$$L^3 = \pi^2 E r^2 / \rho g$$

$$L = k r^{2/3}$$

Solution for first buckling mode for a uniform beam with one end free and the other locked.
Hildebrand *Advanced Calculus*

Tree Diameter vs Height



This shows that trees are over-designed by about 5 - 10x

Source: McMahon and Bonner

Simplified Fluid Flow Analysis for Tree Height

Annual growth G_T is related empirically to leaf mass M_L and total mass M_T allometrically by

$$G_T = k_0 M_L = k_1 M_T^{3/4} = k_1 \left(M_S + M_R + M_L\right)^{3/4}$$

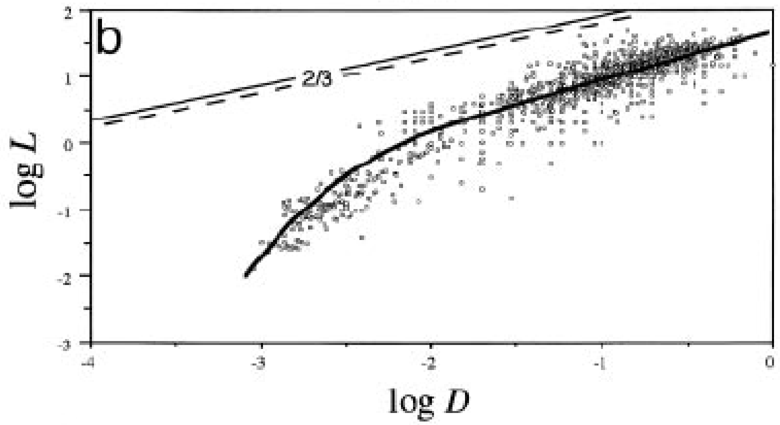
It follows from first principles that the amount of water absorbed by roots per unit time must be conserved as water is transported from roots through stems to leaves, where it is eventually lost. For this reason, M_L must scale isometrically with respect to the hydraulically functional cross-sectional area of stems and roots.

$$M_L = k_2 D^2$$

$$M_R = k_3 M_S$$

$$M_S = k_4 D^2 L$$

$$L = k_5 D^{2/3} - k_6$$



Thus the same answer emerges but the fluid analysis shows that for small diameter the effect is different

Note that trees do not push the height limit, and smaller ones are more conservatively designed than taller ones.

Karl J. Niklas and Hanns-Christof Spatz, "Growth and hydraulic (not mechanical) constraints govern the scaling of tree height and mass," *PNAS* November 2, 2004 vol. 101 no. 44 15661–15663

Scaling of Distribution Systems

- Little systems may not need formal internal distribution systems
  - Villages, single cell organisms, small companies
- Big ones do
  - Cities, animals, big companies
- The distributed thing may be energy, information, "stuff"
- The number of sites needing the distributed thing increases in a dependable way with the size of the system and this number may in fact measure its size
- But the dimensions of the distribution system may not scale in direct proportion to the number of sites
- Scaling rule may be different when there is a single central source (heart) vs when there is not (cascades)
- For mammals (3D), there is an advantage to being big
- To repeat: explaining this has become a thriving enterprise

Basal Metabolic Rate vs Body Mass: The Mouse-Elephant Law

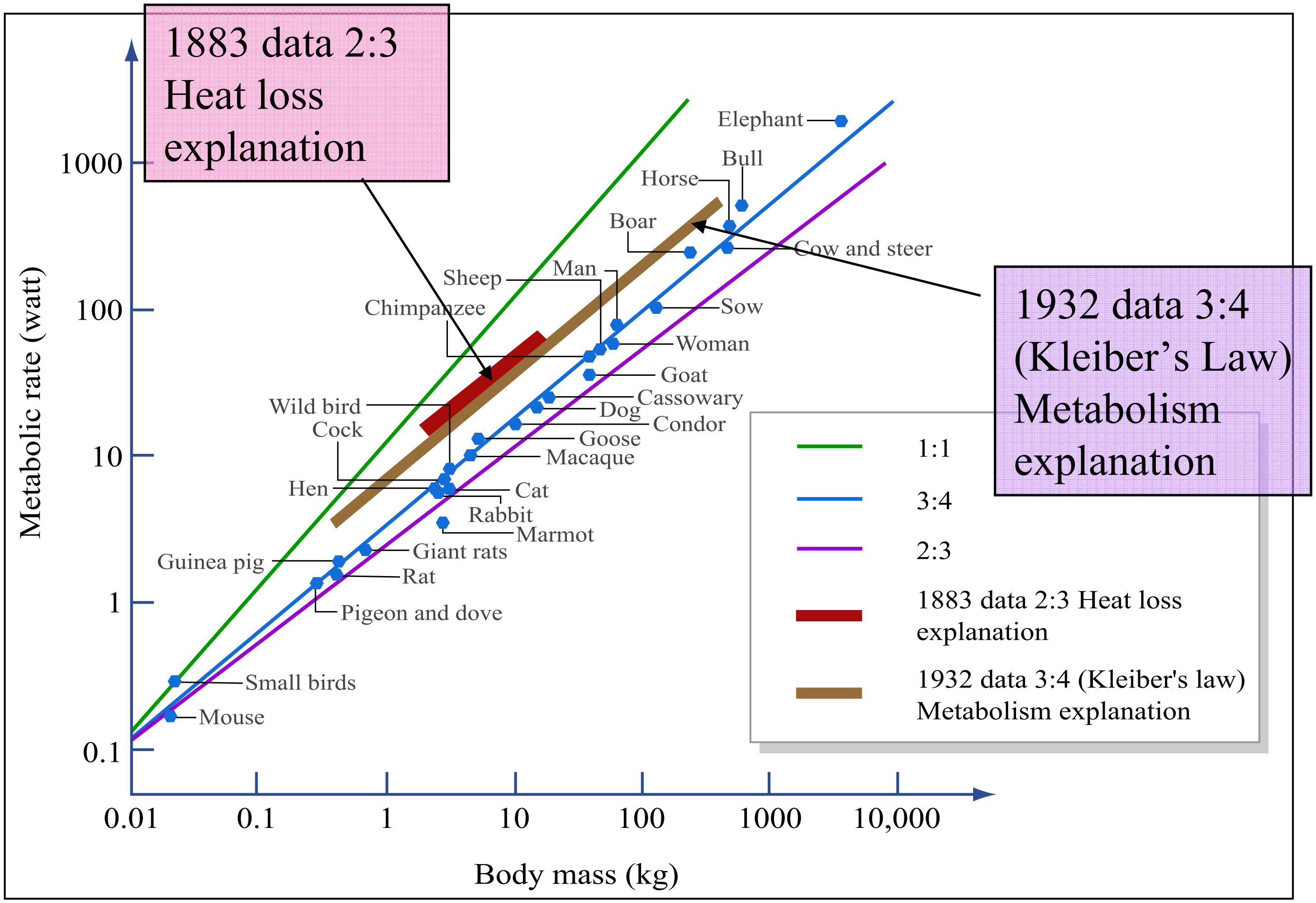


Image by MIT OpenCourseWare.

Calder III, W. A. 1984. *Size, Function, and Life History*. Harvard University Press, Cambridge Mass.

Blood Distribution Explanation for 3/4

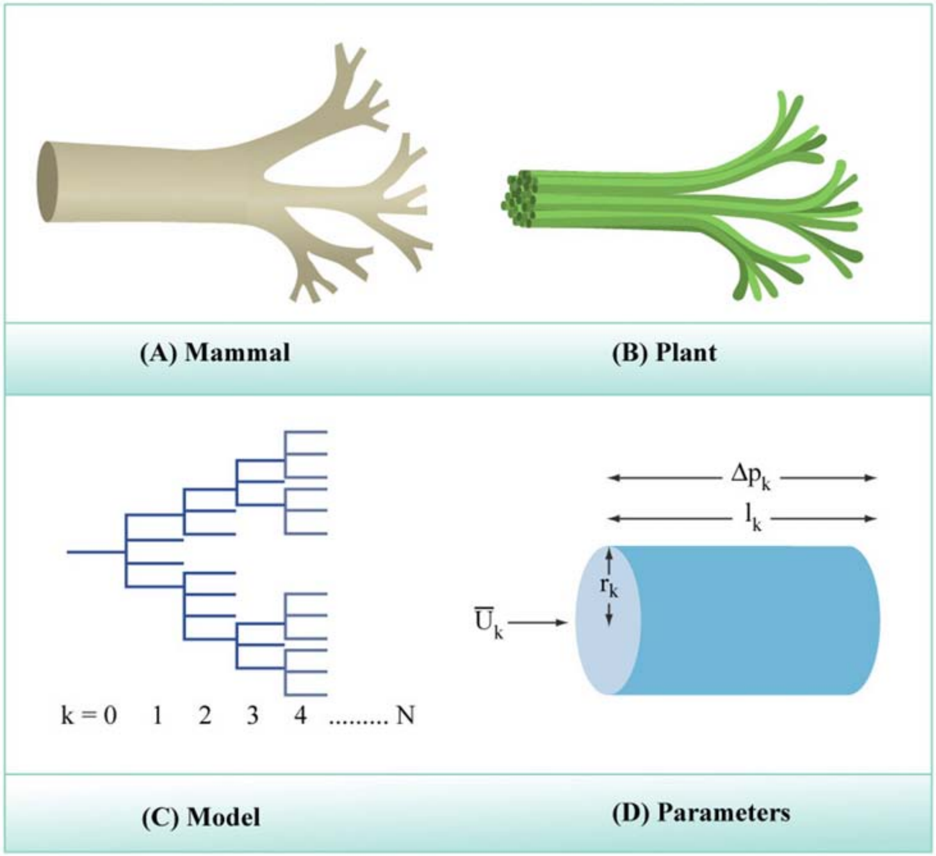


Image by MIT OpenCourseWare.

Geoffrey B. West, James H. Brown, Brian J. Enquist, "A General Model for the Origin of Allometric Scaling Laws in Biology," *Science* Vol. 276 4 April 1997, p 122

Assumptions:

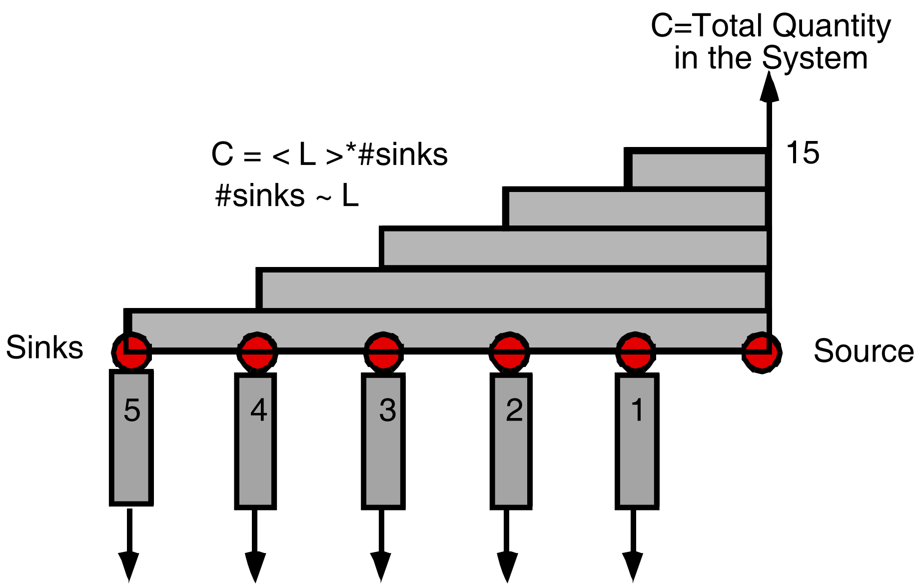
1. There are n_k branches at level k
2. Vessel walls are rigid, radius r_k
3. Fluid volume is conserved
4. Smallest vessel diameter is known
5. Flow pumping power is minimized

Conclusions:

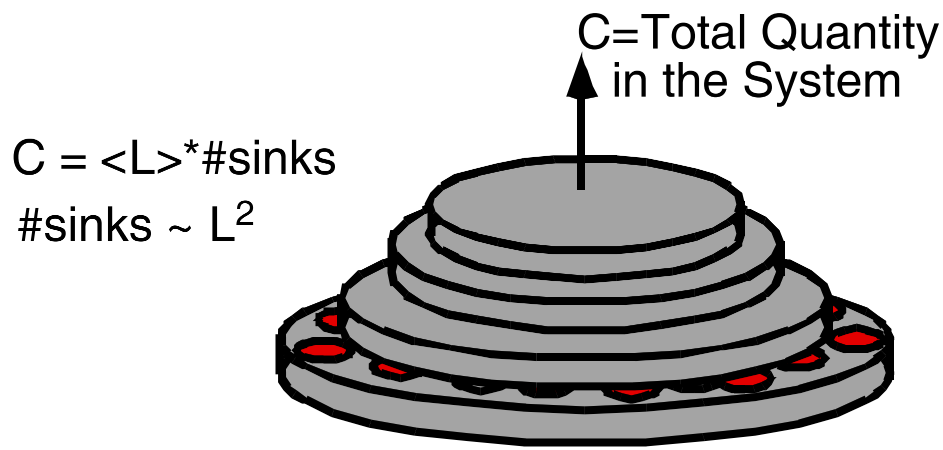
1. Relative radius r_k and length l_k scale with n_k, and n_k = n is the same at every branch level
2. Metabolic rate $B = \alpha M^{.75}$
3. This scaling law drives many others and applies across a huge range of body sizes

Another Network Model for Blood or Other Bulk Transportation Systems

One dimensional system



Two dimensional system



$C =$ total blood volume in the body

$C = k_2 L^{D+1}$

Size and Form in Efficient Transport Networks
Banavar, Maritan and Rinaldo
Nature v 99 p 130 May 1999

Banavar Argument

The total blood volume C for a given organism at any given time depends, in the steady-state supply situation, on the structure of the transportation network. It is proportional to the sum of individual flow rates in the links or bonds that constitute the network. **We define the most efficient class of networks as that for which C is as small as possible.** Our key result is that, for networks in this efficient class, C scales as $L^{(D+1)}$. The total blood volume increases faster than the metabolic rate B as the characteristic size scale of the organism increases. Thus larger organisms have a lower number of transfer sites (and hence B) per unit blood volume. Because **the organism mass scales[1-3,5-7] (at least) as C,** the metabolic rate does not scale linearly with mass, but rather scales as $M^{D/(D+1)}$. In the non-biological context, the number of transfer sites is proportional to the volume of the service region, which, in turn, leads to a novel mass-volume relationship.

Slide 14

$$B = \text{total nutrients delivered per unit time}$$

$$B = \text{metabolic rate}$$

$$B = k_1 L^D$$

$$C = \text{total blood volume in the body}$$

$$C = k_2 L^{D+1}$$

$$M = k_3 C$$

$$M = k_2 k_3 L^{D+1}$$

$$L = k_4 M^{1/(D+1)}$$

$$B = k_5 M^{D/(D+1)}$$

McMahon's Elastic Similarity Law

Assume an animal's body is made of many cylinders. Cylindrical bones fail in bending or buckling.

For bone diameter d and length ℓ

$d = k\ell^{3/2}$ is required to avoid buckling or bending failure

So d increases faster than ℓ whereas if scaling followed geometric rules, d and ℓ would increase in direct proportion

But body mass m is related to d and ℓ by

$m = kd^2\ell$

So

$\ell = m^{1/4}$

and

$d^2 = m^{3/4} = k * \text{muscle force} = k * \text{ metabolic rate}$

Source:
McMahon and Bonner

Problems Noted in These Models

- Incorrect assumptions, for example:
  - The network is not a tree but is re-entrant
  - Blood vessel walls are not rigid
  - Blood is a non-Newtonian "fluid" (next slide)
  - Blood volume does not scale linearly with body mass
  - Surface area is hard to measure and may be much under-estimated due to microscopic crinkles in the surface, for example

$$volume = \pi r^2 L$$

$$area >> 2\pi r L$$

- Other constraints may apply, for example:
  - Heat rejection
  - Impedance matching between source and load
  - Mechanical stress
  - Different organs have different exponents $\neq 0.75$ (next slide)
  - Bones weigh more for larger animals and don't need as much blood as other organs
- Data are too noisy or do not span wide enough range
  - The difference between 2/3 and 3/4 is too hard to see

Different Organs Have Different Exponents

	$K_i = aM^p$	$T_i = bM^q$	$REE_i = K_i \times T_i = abM^{p+q}$
Liver	$K_{liver} = 2861M^{-0.27}$	$T_{liver} = 0.033M^{0.87}$	$REE_{liver} = 94.4M^{0.60}$
Brain	$K_{brain} = 1868M^{-0.14}$	$T_{brain} = 0.011M^{0.76}$	$REE_{brain} = 20.5M^{0.62}$
Kidneys	$K_{kidneys} = 2887M^{-0.08}$	$T_{kidneys} = 0.007M^{0.85}$	$REE_{kidneys} = 20.2M^{0.77}$
Heart	$K_{heart} = 3725M^{-0.12}$	$T_{heart} = 0.006M^{0.98}$	$REE_{heart} = 22.4M^{0.86}$
Remaining tissues	$K_{rem} = 125M^{-0.17}$	$T_{rem} = 0.939M^{1.01}$	$REE_{rem} = 117.4M^{0.84}$

K-M, T-M and REE-M functions for five organ-tissue level components across mature mammals[1-3]

Image by MIT OpenCourseWare.

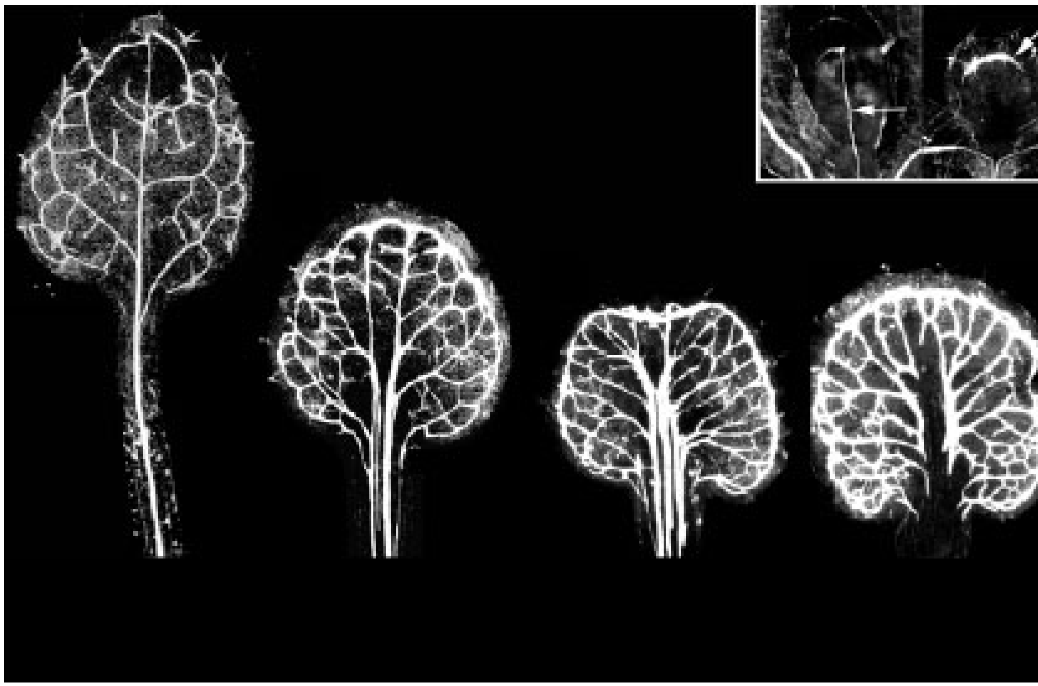
Bones and skin use less energy than heart, brain, etc. Bones are a bigger % of body mass in larger animals.
So you can still get a combined exponent of 0.75 but it means different things for large animals than for small.

http://jn.nutrition.org/cgi/content/full/131/11/2967

J. Nutr. 131:2967-2970, November 2001
The Reconstruction of Kleiber's Law at the Organ-Tissue Level1
ZiMian Wang*, Timothy P. O'Connor, Stanley Heshka* and Steven B. Heymsfield*
Obesity Research Center, St. Luke's-Roosevelt Hospital, Columbia University, College of Physicians and Surgeons, New York, NY 10025 and Department of Biology, City College of New York, CUNY, New York, NY 10025 *

Angiograms

What do we see? Not a tree.



Leaves

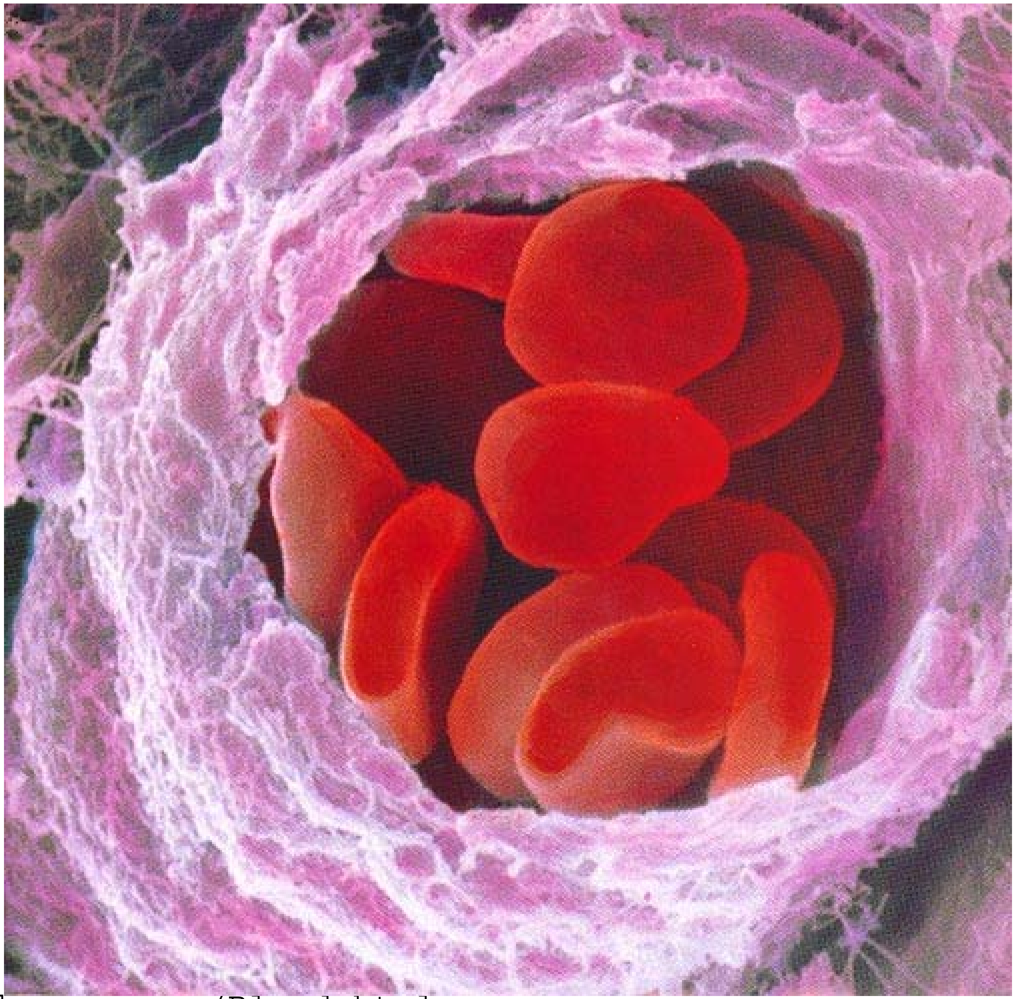
http://labs.csb.utoronto.ca/berleth/auxin_transport.htm

Human Arteries

onyourmind.psychology.washcoll.edu/ OYMSp01/

Red Blood Cells in Small Blood Vessel

What do we see?
Not Newtonian.



tuberose.com/Blood.html

An Observation

- The vascular system seems to have two developmental modes in the embryo for blood vessels and nerves
  – Top-down characterizes the big arteries and veins - branching?
  – Bottom-up characterizes tiny capillaries - many structures
- More generally, many systems have a top/bottom boundary
  – On one side it is top-down
  – On the other side it is bottom-up
  – Standard vs custom parts in products
    - Different for info and power products
  – "Push-pull boundary" in supply chains
  – Transactions between companies, not inside (cf outsourcing)
- The boundary "seems" to be at a good tradeoff point between central planning and local action or optimization
  – Big vs small, now vs later, static vs dynamic, availability vs effectiveness of information

Summary of Metabolic Scaling Models

Authors	Network Model	Limit in the Recursion	What is Optimized
Brown, West, Enquist 1997	Tree with sinks at the leaves	Smallest element has same size	Min power needed to pump blood to leaves of tree
Brown, West, Enquist 1999	None: scaling is rooted in the fixed size of the smallest element	Smallest element has the same size	Max surface area in exchange region, and min time and distance to transport the nutrients
McMahon and Bonner, 1983	Skeleton	N/A	Min mass to avoid structural failure
Painter, 2005 (Max metabolic rate)	Venous blood system for max metabolic rate	N/A	Max flow from heart to lungs
Banavar, Maritan, Rinaldo, 1999	Any outward directed network with sinks at the branch points	Each sink needs the same amount of nutrients per unit time	Min amount of blood in transit -like WIP, JIT, or working capital
Bejan, 1997 and 2000	Two counter-flow trees with sinks at the leaves	Smallest element is a fixed shape “constructal”	Min flow resistance and carry heat to skin

Note: all these models optimize something to reach the 3/4 power law

Engineering Systems

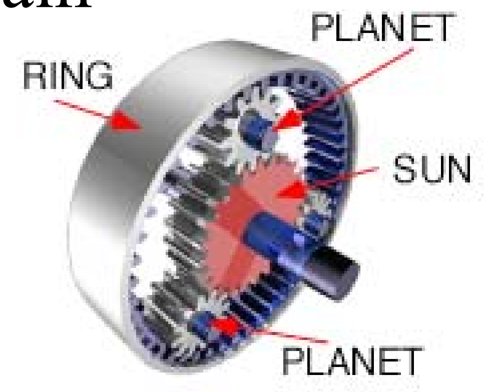
- Their internal structure is not so determined and obvious as it might seem
- Basic S-curve argument for emergence of a new thing
  - Initial turmoil and competition - no dominant architecture
  - A choice emerges from this competition
    - Based on what? No single explanation: "contingent"
  - This choice overwhelms the others, which decline or vanish
  - In hindsight it is not always the best choice
  - But it no longer has serious competitors, so it survives until other pressures force it out
  - There always seems to be some dynamic that changes the rules later and causes success to become failure or causes other mechanizations to gain relative fitness superiority
- This story resembles survival of the fittest motifs in biology

Some Influences on Engineered System Structure

- Need to have static or dynamic balance (symmetry)
- Need to reject heat efficiently or retain heat
- Need to resist the first or dominant failure mode (buckling or shear)
- Accommodating the design driver (weight-limited vs volume-limited submarines and airplanes) (crack propagation: riveted vs welded ships)
- Efficiency of energy conversion or transfer, including impedance matching (arm muscles) (electronics)
- Robustness: based on redundancy or perfection
- Need to keep certain transmission lines short
- Packaging constraints, assembleability, transport, accessibility for repair, ingress/egress
  - To change the plugs on a Jaguar, start by dropping the rear end

Examples

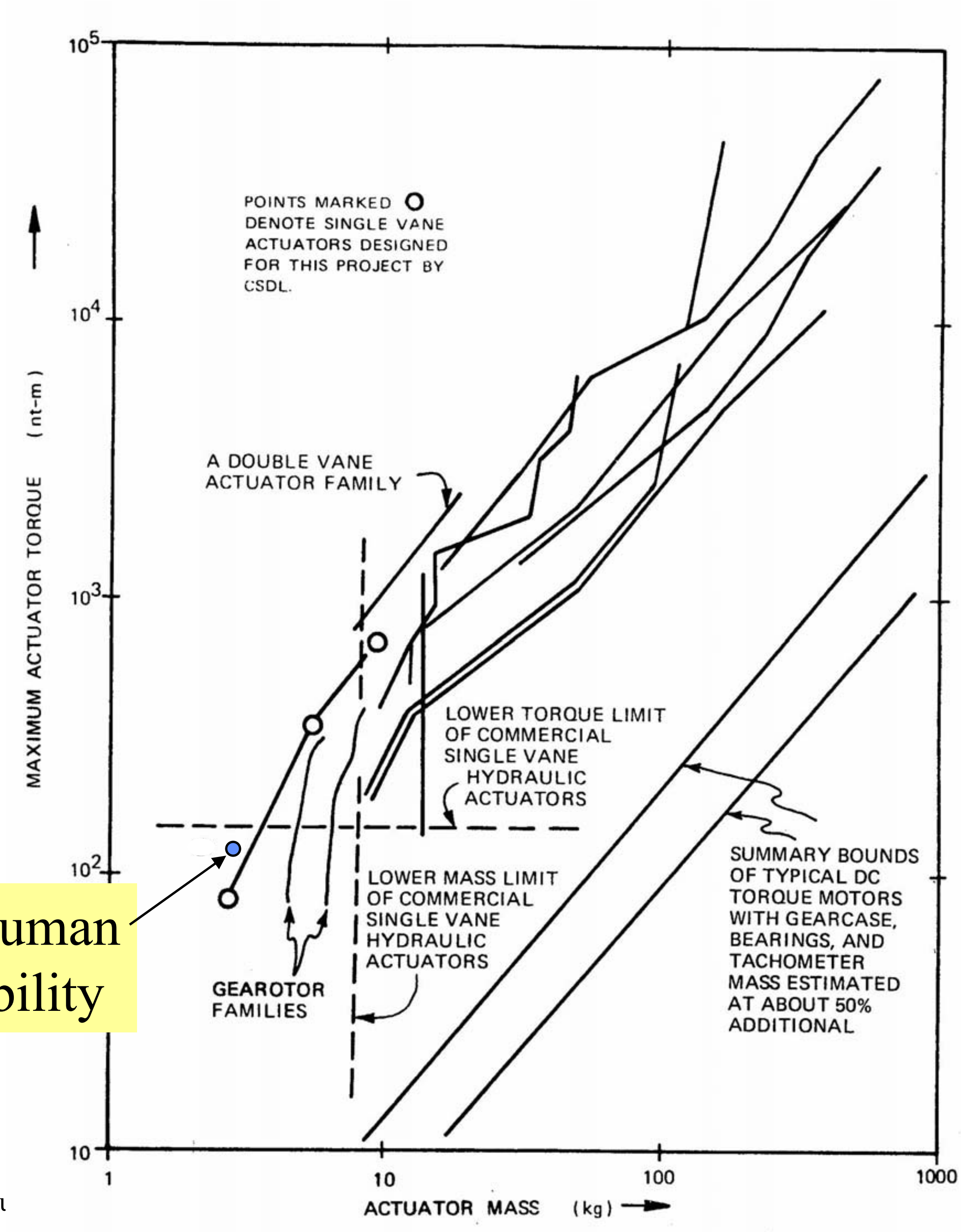
- Swoop shape of Eiffel Tower (still debated 125 years after it was built - approx parabola)
- Number of planet gears in a planetary gear train
- Cost of manual and robot assembly
- Need for gears in engine-wheels propulsion systems
- Positioning of masses in moving assemblies
- Integral and modular architectures (plus migration histories - discussed in an earlier lecture)



Basic Bandwidth Issues and Time-Mass-Distance Scaling Laws for Robot Arms

- Torque required to move a mass M at the end of an arm of length L an angle θ in time T is proportional to
  – $M L^2 \theta / T^2$
- This implies that really fast motions must be really small or use a small arm with small mass
- I estimated
  – my hand's mass = 250g, effective length = 10cm
  – my lower arm + hand's mass = 1700g, effective length = 35 cm
  – ratio arm:hand of $ML^2 = T^2 = 85$ for same θ
- Don't forget: arm mass+payload mass=M

Comparing Actuators of Different Types



Source:
Draper Report
R-837, 1974

Industrial Assembly Cycle Times

- Small parts: 2-3 seconds
- Typical arm movements: 10 seconds
- Automobile final assembly: 60 seconds
- Note: none of these represent max speed due to fatigue and safety constraints

Muscles Located to the Rear

Image of human arm muscle structure removed due to copyright restrictions.
www.anatomy-resources.com/human-anatomy/sh381.htm

Relative Cost of Robot Assembly - Simplified

Unit cost of manual assembly

$$C_{unit-man} = \frac{A\$_p * \# People}{Q}$$

$A\$_p =$ annual cost of a person

$Q =$ annual production volume

$$\# People = \frac{T_p * N * Q}{2000 * 3600}$$

$N =$ number of assembly operations needed

$T_p =$ time to do one operation

$C_{unit-man} \propto A\$_p * T_p =$ "price - time product"

Unit cost of robot assembly

$$C_{unit-robot} = \frac{f_{ac} * I}{Q}$$

$f_{ac} =$ annual cost recovery factor

$I =$ required investment in robots

$I = \# robots * \cos t / robot$

$$\# Robots = \frac{T_r * N * Q}{2000 * 3600}$$

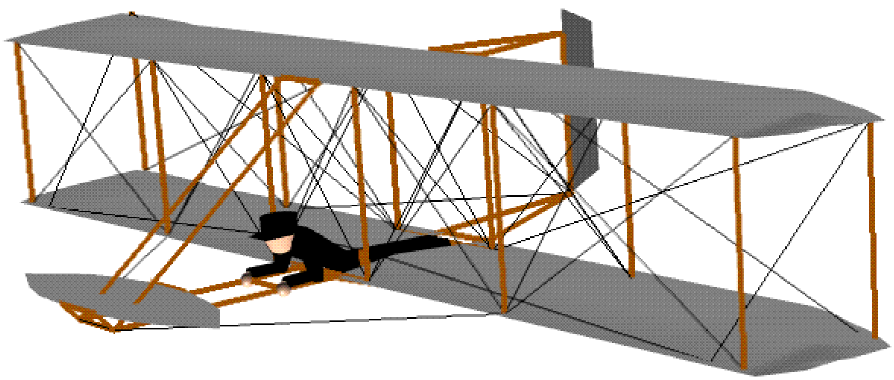
$f_{ac} * \cos t / robot =$ annual cost of a robot $= A\$_r$

$C_{unit-robot} \propto A\$_r * T_r$

$$\frac{C_{unit-robot}}{C_{unit-man}} = \frac{f_{ac} * T_r * \cos t / robot}{A\$_p * T_p} = \frac{T_r * A\$_r}{T_p * A\$_p} > 1 \quad \text{most of the time}$$

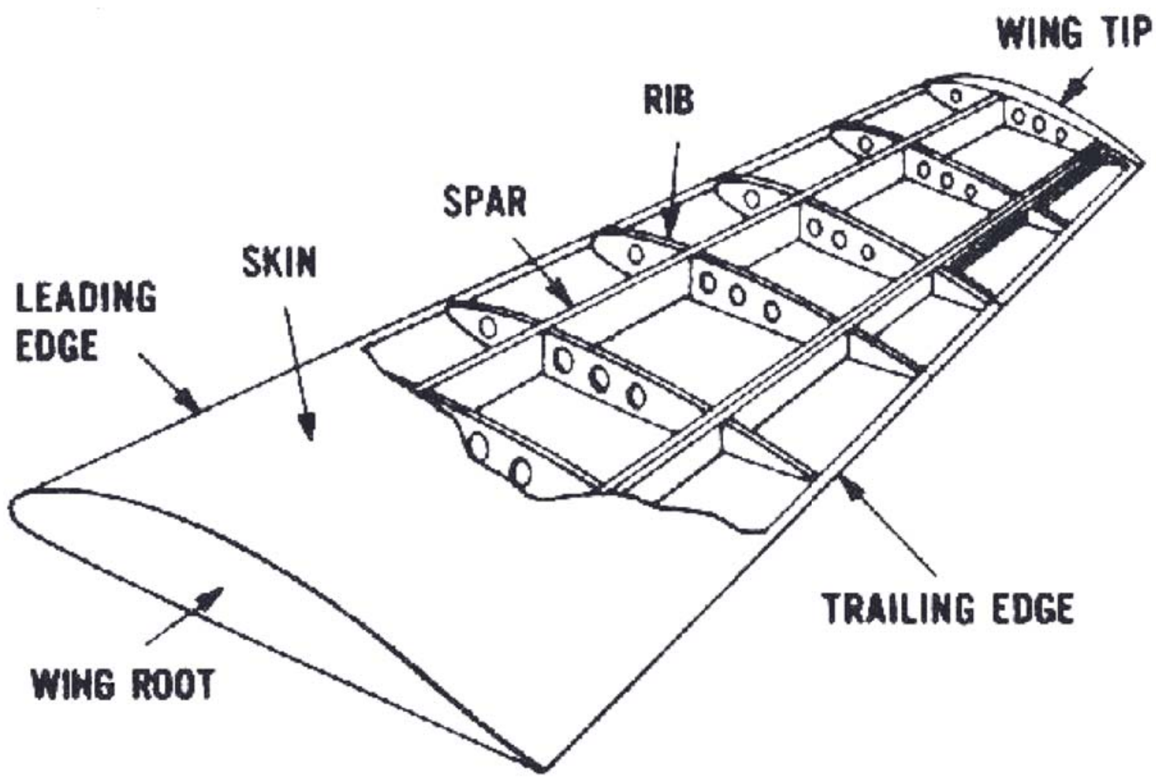
Modular Airplane Wing

Image of Wright Brothers' airplane removed due to copyright restrictions



http://wright.nasa.gov/airplane/Images/rola.gif

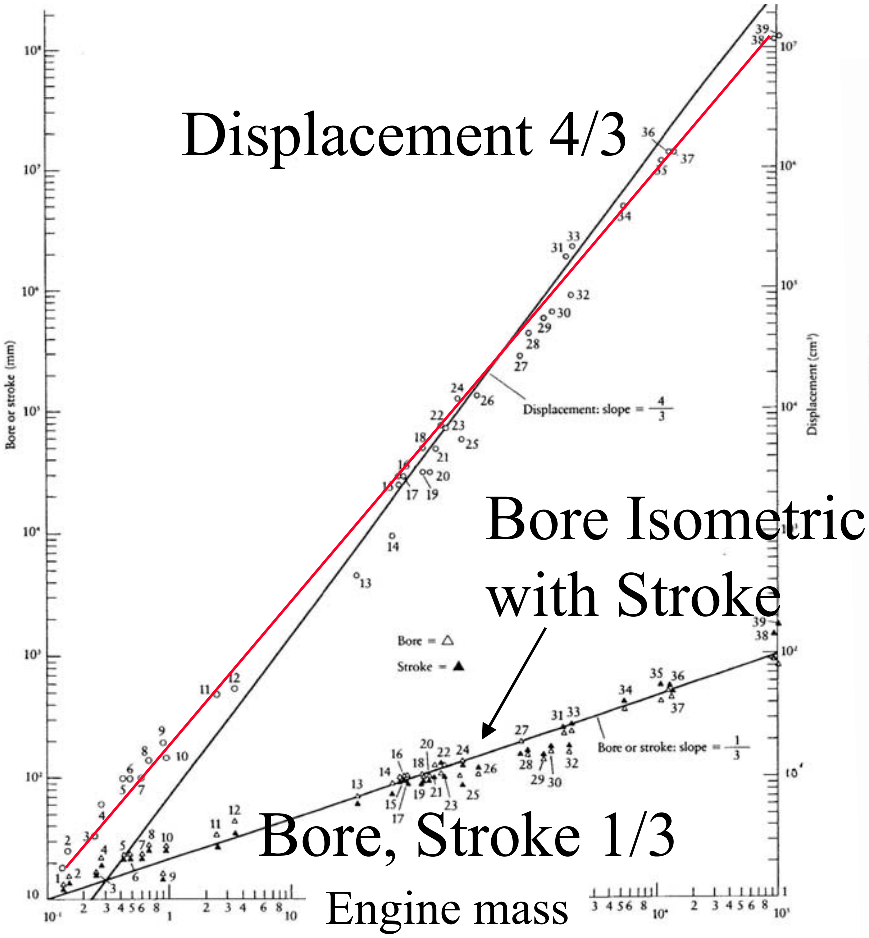
Integral Airplane Wing

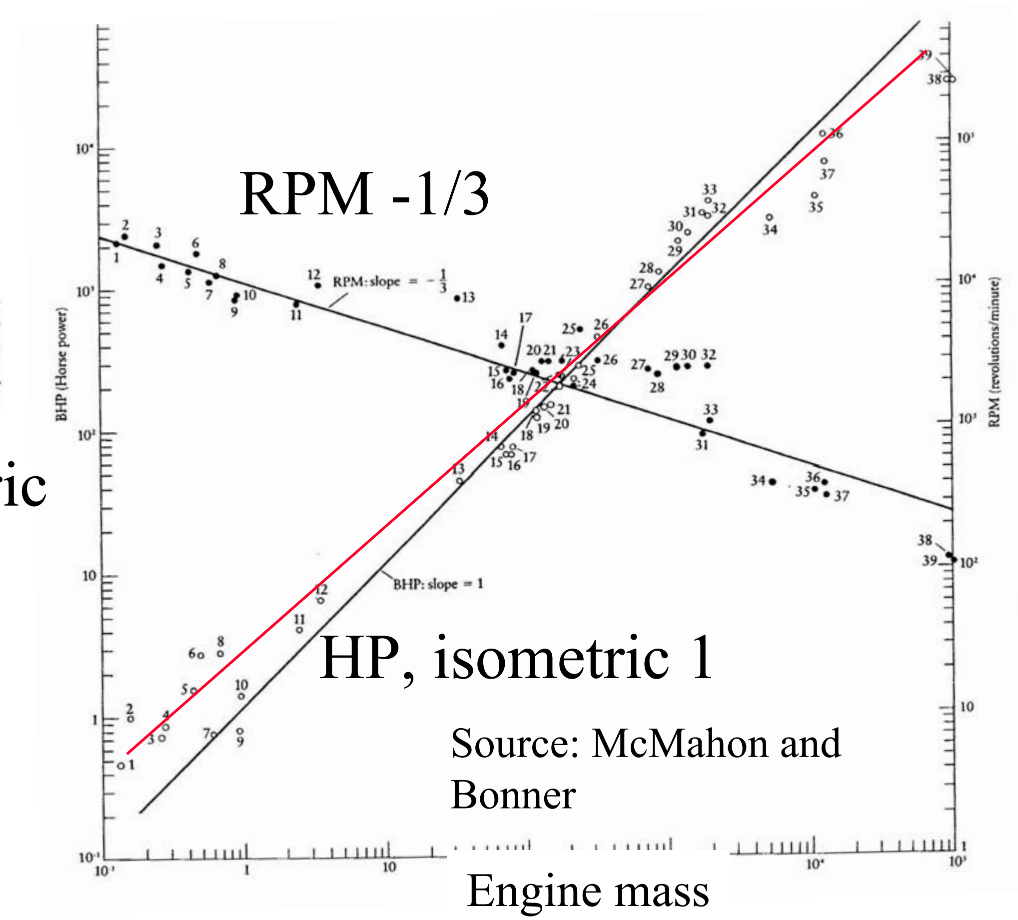


Propulsion Systems

- Efficient speed of steam turbines is 3600 RPM
- Efficient speed of ship propellers is 360 RPM
- Thus there are 10:1 reduction gears
- In cars you can change the ratio over about 300%
- In steam ships there are many small boilers rather than one big one because one boiler would be bigger than the ship once the speed of the ship exceeds a certain value (D'Arcy Thompson "On Growth and Form")

Aircraft Piston Engine Bore, Stroke, Displacement, HP, and RPM vs Wt





Source: McMahon and Bonner

Stroke*ω = peak piston velocity = const

Peak specific strain energy = const

Taylor: Constraint is cooling

Common Thread of "Economics"

	Biological Systems	**Economic Entities and Systems**	**Engineering Products, Systems, and Enterprises**
Pressures (Exogenous)	Temperature, moisture, and chemicals in excess or lack, or rapid change Other species competing for the same resources Too many predators, too few prey, or both too skillful	Competition from other individuals, firms, and nations Technological, environmental, or political change	Physical environment Economic environment Regulatory environment Competing technologies or enterprises Differing goals of stakeholders Changes in the above
"Incentives" or Motivations (Endogenous priorities)	Reproduction, survival	Economic returns, survival Greed, fear A sense of safety, hegemony, or confidence Feeling justly rewarded	Technical excellence or leadership Economic returns and survival Prestige, "winning," pride, dominance
How the Systems Respond (A lot of the response is structural or behavioral)	Fight or flight decision Niche carving and occupation Defense mechanisms Elaborated structure Efficient use of resources Navigating the flexibility-efficiency frontier	Developing efficient processes like open markets, division of labor, and comparative advantage Forming alliances, teams, firms, covert activities Seeking virtuous circles Calculating future value	Technical innovation Process, managerial, and organizational improvement Patents, legal attack, dirty tricks Associations and joint ventures Efficient use of resources Navigating the flexibility-efficiency or diversification-fortification trade

Some Research Questions

- Do evolved systems actually optimize something and if so what?
- Can evolved systems be models for designed systems?
- Do systems seek to optimize several things at once or hew to several constraints at once?
- How to design a system if one set of constraints appears to be operative now but perhaps others will become operative later?
- Where to put the top-down/bottom-up boundary?
- How to deal with systems like social ones where there are no necessary behavior laws like those imposed by nature?

Some Takeaways

- How many different fields have these questions
- How many different disciplines have addressed them
- How recently new work has been done on problems some of which were first addressed 100+ years ago
- How similar the questions are
- How incomplete the answers seem to be
- How often the explanation is governed by success with the main function or pressure from a non-negotiable constraint, rather than any ilities

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