| 6.890 | Lecture 16 | Oct. 28, 2014 |
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Today: video games & PSPACE. First some NP:

**Metatheorem 1:** [Viglietta – FUN 2012 & arXiv:1201.4995]

- player traversing planar environment from start
- location traversal & single-use paths ⇒ NP-hard
  - ↳ player must visit some locations
  - ↳ player can traverse only once
  - reduction from Planar Max-deg-3 Hamiltonicity
  - vertex → location traversal
    - ⇒ visit each vertex ≥ once
  - edge → single-use path
    - max. degree 3 ⇒ never revisit vertex
- applications
  - Boulderdash
  - Lode Runner
  - Zelda II [Aloupis, Demaine, Guo, Viglietta 2014]

**Metatheorem 2:** [Viglietta – FUN 2012 & arXiv:1201.4995]

- location traversal & tokens + toll roads ⇒ NP-hard
  - tokens: can pick (one) up
  - toll roads: ↳ need token to traverse
  - vertex → location traversal + token
  - edge → toll road
    - traversing twice ⇒ stranded without token
- application: Pac-Man

# Recall from L1:

PSPACE = {problems solvable in polynomial space}
- $\subseteq$ EXP: only exponentially many states
- $\supseteq$ NP: simulate all executions, take running OR
- = NPSPACE [Savitch 1970]

## Base PSPACE-complete problems:
- simulate linear-space algorithm (e.g. Turing machine)
- QSAT: (AKA QBF & TQBF)
  given (fully) quantified Boolean formula, is it true?
  - e.g. $\forall x \exists y : (\bar{x} \vee y) \wedge (x \vee \bar{y})$ $(x \equiv y)$
  - can assume quantifiers in front (prenex) & alternate $\forall/\exists$ ($\exists$ only $\Rightarrow$ SAT $\Rightarrow$ NP-comp.)
- Schaefer-style dichotomy theorem:
  - $\in$ P $\Leftrightarrow$ Horn, dual-Horn, 2-CNF, or X(N)OR (not if satisfied by all true/all false)
  - PSPACE-complete otherwise [Chen-C. Surveys 2009]
- planar Q3SAT [Schaefer-SICOMP 1981] [L7]
  - add $\exists$ for new variables at end of quantifiers
- planar 1-in-3 Q3SAT (as in L7)

## Metatheorem 3: [Viglietta - FUN 2012 & arXiv:1201.4995]

- player traversing planar environment from specified start to specified goal
- door + open pressure plate + close pressure plate ⇒ PSPACE-hard
  - ↳ traversable only if open
  - ↳ walk on it ⇒ open specific door
  - → ditto, close
  - reduction from Q3SAT
  - clause gadget
  - existential quantifier gadget
  - universal quantifier gadget
  - one plate of each type for each door
- applications:
  - many FPSs e.g. Doom, Quake, Heretic, Hexen, ...
  - many RPGs e.g. Eye of the Beholder
  - many adventure games e.g. SCUMM engine (Maniac Mansion, Monkey Island, Space Quest, ...)
  - Prince of Persia

## Metatheorem 4: buttons instead of pressure plates

↳ optional: can press or not
↳ activates 3 doors at once

- pressure plate gadget

- in fact 2 doors per button suffice
  [Bodlaender & van der Zanden – unpublished 2014]
- applications: MANY
  - Sonic the Hedgehog (Sega Genesis)
  - The Lost Vikings (Super NES; PC) "Erik the Swift"
  - Tomb Raider (Sega Saturn & PS1; PC)

## Metatheorem 5: [Alanpis, Demaine, Guo, Viglietta 2014]

- door with traverse, open, close paths ⇒ PSPACE-hard
  traverse: only if open; open: can open; close: ↳ must close

- applications:
  - Legend of Zelda: A Link to the Past
    (Ocarina of Time, Majora's Mask, Oracle of Seasons, The Minish Cap, Twilight Princess ⊇ PushPush-1)
  - Donkey Kong Country 1, 2, 3
  - Super Mario Bros. [Demaine, Viglietta, Williams – unpublished, 2014]
  - Lemmings [Viglietta – FUN 2014]

# 6.890 Algorithmic Lower Bounds: Fun with Hardness Proofs

Fall 2014