6.849	Class 14	Oct. 25, 2012

- Hinged dissection software: just specific examples

$\boxed{\text{PROJECT}}$: hinged dissection animator
- implement slender adornments (refinement + expansive motion)
- implement general algorithm?
- implement polyform algorithm

$\boxed{\text{PROJECT}}$: design elegant hinged dissections

Polyform = n copies of one shape glued together along corresp. edges

Inductive construction: [Demaine, Demaine, Eppstein, Frederickson, Friedman 2005]
- base case: hinge-dissect 1 copy such that every edge has incident hinge
- step: take spanning tree of copies
  remove leaf copy
  induct on n−1 remaining copies
  rotate base case to meet them
  reconnect ~ get same hinging

⇒ folded states (use slender for motion)

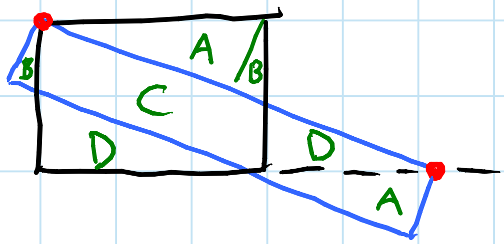
Also: poly△ → poly□, etc.

3D [Demaine, Demaine, Lindy, Souvaine 2005]

Physical:

- in liquid
- DNA
- Macrobot/Decibot
- related: reconfigurable robots

- Rectangle → rectangle [Montucla 1778]
  - superposing strips method



  - same method for Dudeney's △ → □
  - more stable table [Frederickson 2008]

$\boxed{\text{PROJECT}}$: build reconfigurable furniture

- \# pieces doubles? at least, in worst case

- Pseudopolynomial: say integer

  if polygon vertices lie on common grid,

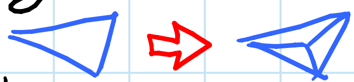
  \# pieces = poly(n, r)

  ↳ \# grid positions = $\frac{\text{size}}{\text{cell size}}$

- idea: ensure constant-depth recursion

  ① triangulate polygons with grid vertices

  ⇒ matching Δ areas of $\frac{1}{2}$ [Pick's Theorem]

  ② chainify 

  ⇒ vertices on $\frac{1}{3}$ grid

  ③ fix which vertices connect which Δs

  by only modifying parent in subtree move

  ④ $\Delta \to \Delta$ by overlaying 3 constructions:

  Ⓐ $\Delta \to$ ▭ ← actually done last

  Ⓑ □ $\to$ ▭ ε

  Ⓒ ▭ ε $\to$ ▭ ε

  FedEx

  ... using pseudocuts

  ↳ simulate cut overlays

- 3D dissection:
  - volumes must match
  - insufficient by Dehn's solution [1901] to Hilbert's third Problem [1900]
  - Dehn invariants must match:

$$\sum_{\text{edge } e} \ell(e) \otimes \left[\theta(e) + \underbrace{\mathbb{Q}\cdot\pi}\right]$$

ignore added rational multiples of $\pi \Rightarrow$ "irrational part"

tensor product

  - tensor product space: linear combination of pairs $\ell \otimes \theta$ where

$$\ell_1 \otimes \theta + \ell_2 \otimes \theta = (\ell_1 + \ell_2) \otimes \theta$$
$$\ell \otimes \theta_1 + \ell \otimes \theta_2 = \ell \otimes (\theta_1 + \theta_2)$$
$$c(\ell \otimes \theta) = (c\ell) \otimes \theta = \ell \otimes (c\theta) \quad \forall c \in \mathbb{Q}$$

  - Dehn's Theorem: invariant under dissection
    - e.g.: cut edge $(\ell_1+\ell_2)\otimes\theta \to \ell_1\otimes\theta + \ell_2\otimes\theta$
      slice angle $\ell\otimes(\theta_1+\theta_2) \to \ell\otimes\theta_1 + \ell\otimes\theta_2$
    - $\Rightarrow$ no dissection of cube $\to$ regular tetrahedron

$12(1 \otimes \underbrace{90°}_{=\emptyset})$ $\quad$ $6(2.04\cdots \otimes \underbrace{70.5288°\cdots}_{\arccos(1/3)})$

- 3D dissection exists $\Leftrightarrow$ volumes & Dehn Invariants match [Sydler 1965]
- ditto in 4D [Jessen 1968]

$\boxed{\text{OPEN}}$: 5D & higher?

$\boxed{\text{OPEN}}$: efficient algorithm to check Dehn match
- decidable [Kreinovich–Geomb. 2008]

$\boxed{\text{OPEN}}$: algorithm to find dissection

- refinement into hinged dissection still works [Abel et al.]

6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
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