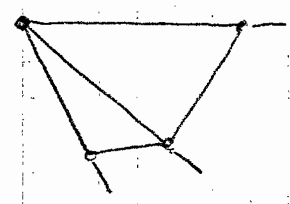
Photogrammetry (making measurement using images)

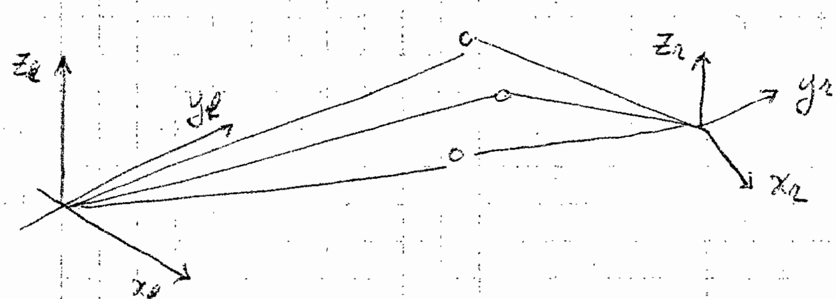
1. absolute orientations — points ⟷ points
2. exterior orientation (given a 3D map) — points ⟷ rays



3. interior orientation — rays ⟷ points
4. relative orientation — rays ⟷ rays <u>(stereo)</u>

* avoid singularities
* avoid coord. syst. dependencies
* prefer symmetric solutions

① <u>absolute orientations</u>



(assume correspondence problem is solved)

D.O.F : 3 + 3 = 6

3 constraints from each measurement.

↪ 2 points should suffice ...

No! There is a constraint in 3D between the two points so we have 5 constraints with 2 points. With 3 points, we get 3 constraints.

2D Version :

(x_{ℓ_i}, y_{ℓ_i}) "left"

(x_{r_i}, y_{r_i}) "right"

$$\begin{vmatrix} x_{r_i} \\ y_{r_i} \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} x_{\ell_i} \\ y_{\ell_i} \end{vmatrix} + \begin{vmatrix} x_0 \\ y_0 \end{vmatrix}$$

$$\min_{x_0, y_0, \theta} \sum_{i=1}^{n} (x_{r_i} - \cos\theta\, x_{\ell_i} - \sin\theta\, y_{\ell_i} - x_0)^2 + (y_{r_i} + \sin\theta\, x_{\ell_i} - \cos\theta\, y_{\ell_i} - y_0)^2$$

$\frac{d}{dx_0}() = 0$

$\frac{d}{dy_0}() = 0$

$$\left\{ \begin{array}{l} \overline{x_r} = \cos\theta\, \overline{x_\ell} + \sin\theta\, \overline{y_\ell} + x_0 \\ \overline{y_r} = -\sin\theta\, \overline{x_\ell} + \cos\theta\, \overline{y_\ell} + y_0 \end{array} \right\} \quad (1)$$

↪ separate rotation from translation

then $\left\{ \begin{array}{l} x'_{\ell_i} = x_{\ell_i} - \overline{x} \\ y'_{\ell_i} = y_{\ell_i} - \overline{y} \end{array} \right.$

$$\left. \begin{array}{l} -2 \sum x'_{r_i} (\cos\theta\, x'_{\ell_i} + \sin\theta\, y'_{\ell_i}) \\ -2 \sum y'_{r_i} (-\sin\theta\, x'_{\ell_i} + \cos\theta\, y'_{\ell_i}) \end{array} \right\} \max$$

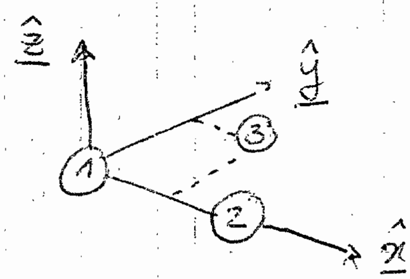
$$\sin\vartheta \sum (x'_{r_i} y'_{l_i} - y'_{r_i} x'_{l_i}) + \cos\vartheta \sum (x'_{r_i} x'_{l_i} + y'_{r_i} y'_{l_i})$$

And thus: $S\sin\vartheta + C\cos\vartheta$ max for $S\cos\vartheta - C\sin\vartheta = 0$

$\rightarrow \vartheta$

Finally we can easily get translation from (1).

3-D



Bla bla bla ... ugly computation and it's not the right method.

Let us do it right:

$\{ \underline{r}_{l,i} \}$ $\{ \underline{r}_{r,i} \}$ $\underline{r}_r = R(\underline{r}_l) + \underline{r}_0$

Minimize the error:

$$\min_{R()\, r_0} \sum \| \underline{r}_{r_i} - R(\underline{r}_{l,i}) - r_0 \|^2$$

$$\frac{d}{d\underline{r}_0}() = 0 \quad : \quad \frac{1}{n} \sum_{i=1}^{n} \underline{r}_{r,i} = \frac{1}{n} \sum_{i=1}^{n} (R(r_{l_i}) + \underline{r}_0)$$

$$\left\{ \bar{r}_r = R(\bar{\underline{r}}_l) + \underline{r}_0 \right\}$$

$\rightarrow$ the centroid goes to the centroid and then we just have to solve for rotation

$$\max \quad \mathring{q}^T \left(\sum \bar{\mathbb{R}}_{\ell_i}^T \mathbb{R}_{r_i} \right) \mathring{q} \quad \text{subject to } \mathring{q}\mathring{q} = 1$$

Raleigh quotient : $\dfrac{1}{\mathring{q}\mathring{q}} \cdot \mathring{q} \left(\sum \ldots \right) \mathring{q}$

where $\mathbb{R}\mathring{q} = \mathring{r}\mathring{q}$

We are then looking for the largest eigenvector :

* Ferrari's formula
* brute force computation

$\Big\}$ in both cases, there is a <u>closed-form</u> solution

$$\underset{3\times 3}{M} = \sum \underline{r}_{\ell_i}' \underline{r}_{r_i}'^T \qquad S_{xx} = \sum x_{\ell_i}' x_{r_i}'$$

$$M = \begin{vmatrix} S_{xx} + S_{yy} + S_{zz} & (S_{yz} - S_{zy})(S_{zx} - S_{xz}) \\ & \ldots \end{vmatrix}$$

and finally rotation $\rightarrow$ translation :

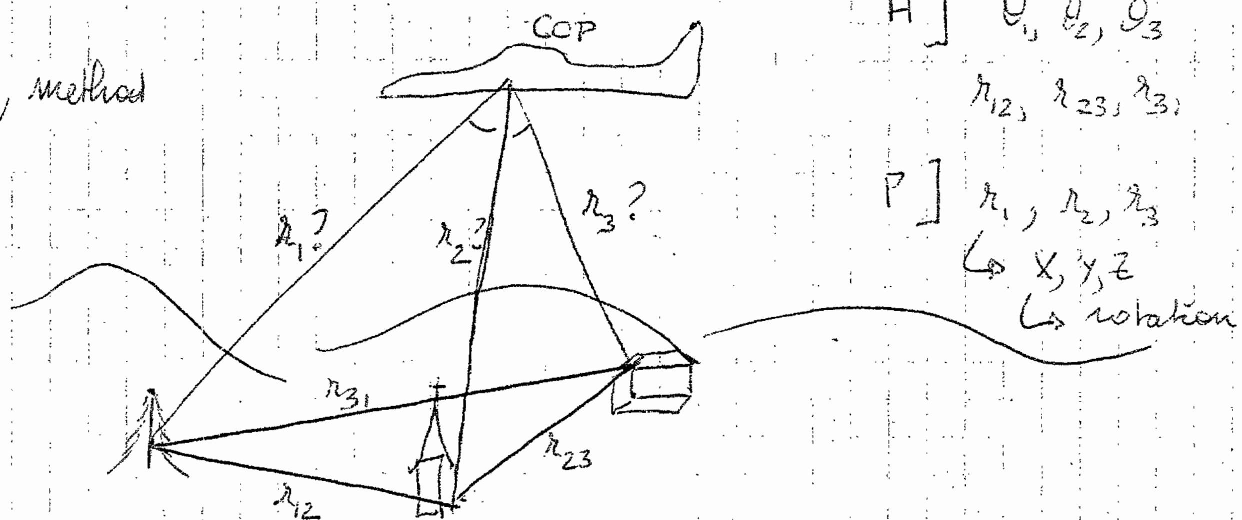
$$\underline{r}_0 = \bar{\underline{r}}_r - R(\underline{r}_\ell)$$

Special Cases

* planar dataset
* quartic binomes
* quadratic in X^2

Problem #2: Exterior orientation

Church method



H] $\theta_1, \theta_2, \theta_3$

r_{12}, r_{23}, r_{31}

P] r_1, r_2, r_3

↳ X, Y, Z

↳ rotation

<u>Step 1</u>

$$\begin{cases} r_{12}^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta_{12} \\ r_{23}^2 = r_2^2 + r_3^2 - 2r_2 r_3 \cos\theta_{23} \\ r_{31}^2 = r_3^2 + r_1^2 - 2r_3 r_1 \cos\theta_{31} \end{cases}$$

Bezout $\leq$ 8 solutions

boils down to 2 linear + 1 quadratic → 2 solutions

↳ r_1, r_2, r_3

<u>Step 2</u> intersect 3 spheres $\underline{r}_0 = (X, Y, Z)$

Step 3: camera orientation

$$\begin{cases} r_{w_1} = R\, r_{c_1} \\ r_{w_2} = R\, r_{c_2} \\ r_{w_3} = R\, r_{c_3} \end{cases}$$

ray in the world

ray in the camera ref. frame

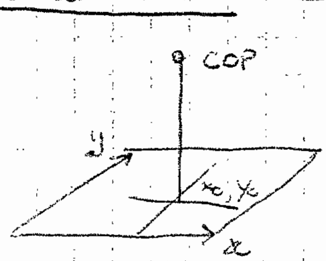
$$| r_{w_1} \;\; r_{w_2} \;\; r_3 | = R \; | r_{c_1} \;\; r_2 \;\; r_{c_3} |$$

$$R = | r_{w_1} \;\; r_{w_2} \;\; r_{w_3} | \, | r_{c_1} \;\; r_{c_2} \;\; r_{c_3} |^{-1}$$

stability? depends on c_1, c_2, c_3

is R orthonormal? Hard to prove, but yes.

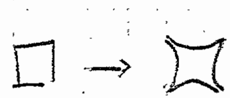
Problem #3: Interior Orientation (camera calibration)



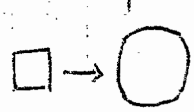
(x_0, y_0, f) + distorsion

$$\delta x = (k_1 r^2 + k_2 r^4 + \dots)\, x$$

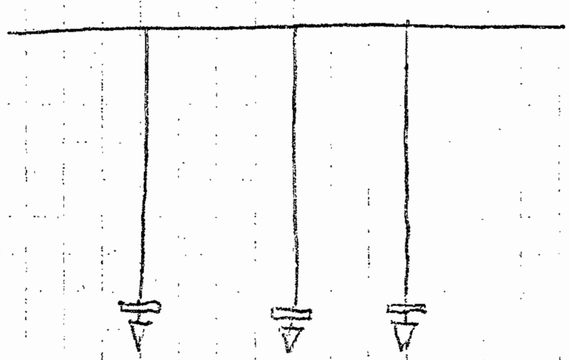
$$\delta y = (k_1 r^2 + k_2 r^4 + \dots)\, y$$



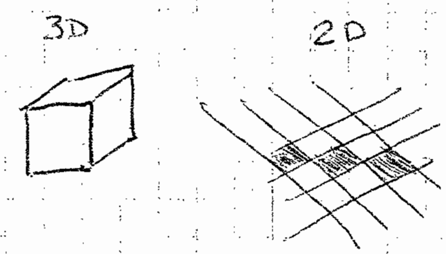
or

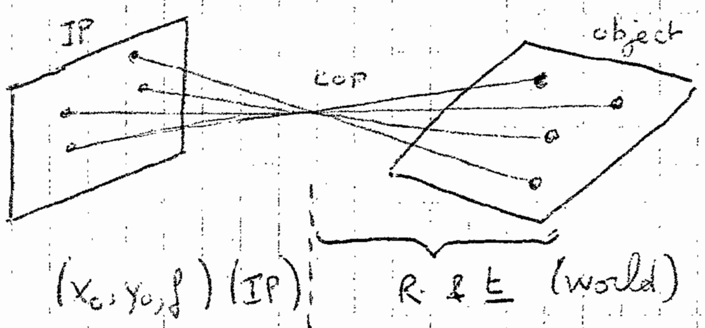


Plumb line method



Calibration object:





3 DOF + 6 DOF = 9
I.O. E.O.

Tsai's method : minimize error in the image plane

Get good guesses.

P_j : $\dfrac{x_I - x_0}{f} = \dfrac{x_c}{z_c}$ $\dfrac{y_I - y_0}{f} = \dfrac{y_c}{z_c}$ $\longrightarrow$ x_0, y_0, f ?

$\underline{r}_c = R(\underline{r}_s) + \underline{t}$ $\longrightarrow$ $R, \underline{t}$?