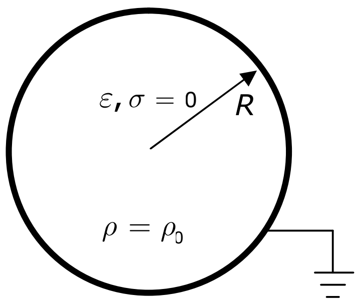
6.641 Electromagnetic Fields, Forces, and Motion
Spring 2009

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641, Electromagnetic Fields, Forces, and Motion
Mid-Term Exam
March 17, 2009

6.641 Formula Sheets appear at the end of this exam. In addition, an 8½" x 11" formula sheet (both sides) that you have prepared is allowed.

1. (25 points)



A perfectly conducting hollow sphere of radius R is perfectly insulating ($\sigma = 0$) and filled with a uniform distribution of volume charge:

$$\rho = \rho_0 \qquad 0 < r < R$$

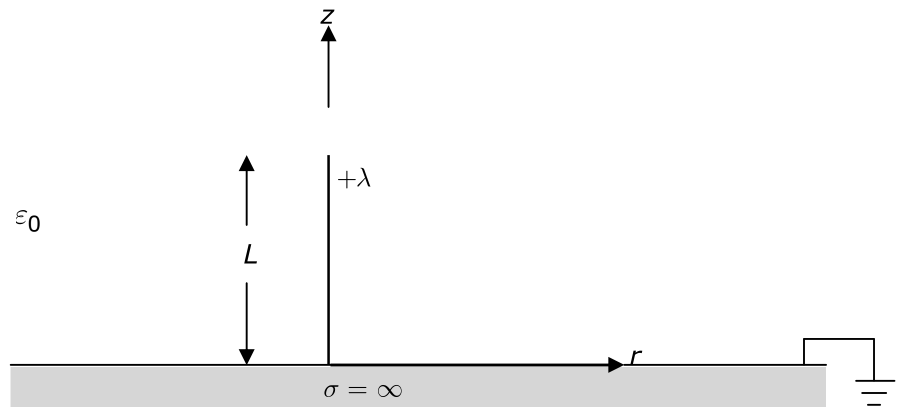
within a medium with permittivity ε. The sphere is grounded at $r = R$ so that the scalar electric potential at $r = R$ is zero, $\Phi(r = R) = 0$. There is no point charge at $r = 0$ so that $E_r(r = 0)$ must be finite.

a) What is the EQS electric field $\bar{E}(r)$ for $0 < r < R$?

b) What is the scalar electric potential $\Phi(r)$ where $\bar{E}(r) = -\nabla\Phi(r)$?

c) What is the free surface charge density $\sigma_s(r = R)$ on the inside surface of the sphere at $r = R$?

2. (25 points)



A uniform line charge λ coulombs/meter of length L stands perpendicularly on a perfectly conducting ground plane of infinite extent in free space with dielectric permittivity ε_0.

a) Find the electric field at the ground plane surface $\bar{E}(r, z = 0_+)$ where r is the cylindrical radial coordinate shown above. See integrals in hint below.

b) Find the surface charge density on the ground plane surface, $\sigma_s(r, z = 0_+)$.

c) Prove that the total charge $q_t(z = 0_+)$ on the ground plane is $-\lambda L$.

Hint for parts (a) and (c): one or more of the following indefinite integrals may be useful.

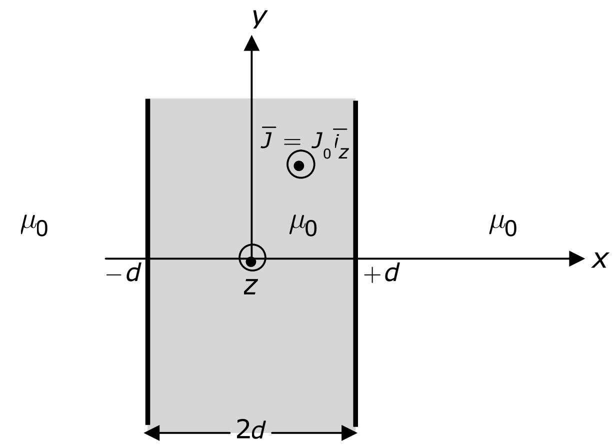
i) $$\int \frac{x\,dx}{[x^2 + L^2]^{1/2}} = \sqrt{x^2 + L^2}$$

ii) $$\int \frac{dx}{[x^2 + L^2]^{1/2}} = \ell n[x + \sqrt{x^2 + L^2}]$$

iii) $$\int \frac{dx}{[x^2 + L^2]^{3/2}} = \frac{x}{L^2[x^2 + L^2]^{1/2}}$$

iv) $$\int \frac{x\,dx}{[x^2 + L^2]^{3/2}} = -\frac{1}{[x^2 + L^2]^{1/2}}$$

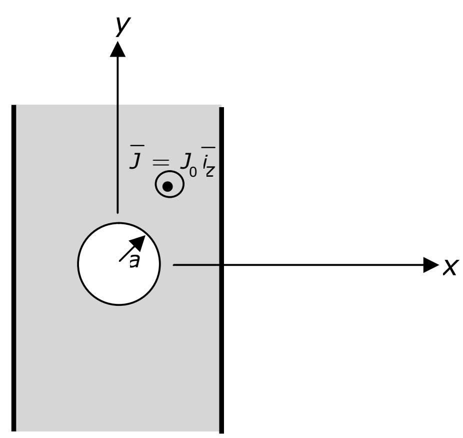
3. (25 points)



An infinite slab in the y and z directions carries a uniform current density $\bar{J} = J_0\bar{i}_z$ for $-d < x < d$. The current carrying slab has magnetic permeability of free space μ_0 and is surrounded by free space for $x > d$ and $x < -d$. There are no surface currents on the $x = \pm d$ surfaces, $\bar{K}(x = d) = \bar{K}(x = -d) = 0$ and the magnetic field only depends on the x coordinate.

a) Find the magnetic field $\bar{H}(x)$ everywhere and plot versus x.

b)



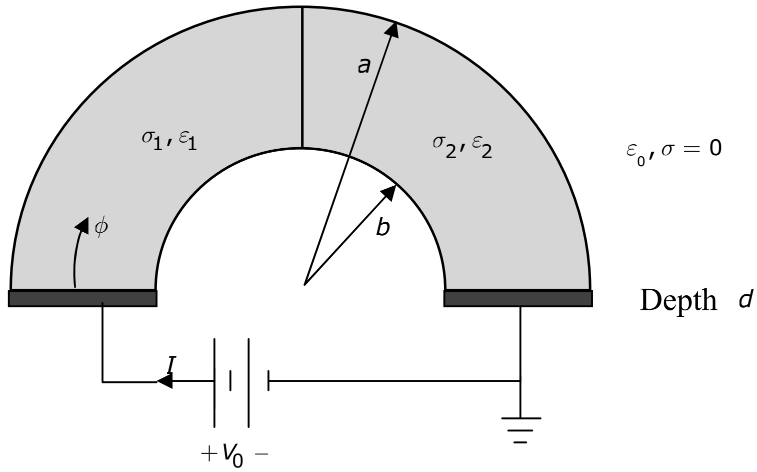
A small cylindrical hole of radius a and of infinite extent in the y and z directions is drilled into the current carrying slab of part (a) and is centered within the slab. The magnetic permeability of all regions is μ_0. Within the hole for $r < a$ the current density is zero, $\bar{J} = 0$. What is the total magnetic field $\bar{H}$ in the hole?

Hint 1: Use superposition replacing the cylindrical hole by two oppositely directed currents.

Hint 2: $r\bar{i}_\phi = r(-\sin\phi\bar{i}_x + \cos\phi\bar{i}_y) = (-y\bar{i}_x + x\bar{i}_y)$ where $r = \sqrt{x^2 + y^2}$.

c) Verify that your solution of part (b) satisfies the MQS Ampere's law within the hole where $\bar{J} = 0$.

4. (25 points)



A resistor is formed in the shape of a circular cylindrical half-shell of inner radius b and outer radius a and is composed of two materials with ohmic conductivities and permittivities $(\sigma_1, \varepsilon_1)$ for $0 < \phi < \frac{\pi}{2}$ and $(\sigma_2, \varepsilon_2)$ for $\frac{\pi}{2} < \phi < \pi$. A dc voltage V_0 is applied to the electrode at $\phi = 0$ while the electrode at $\phi = \pi$ is grounded. The EQS scalar potential is thus imposed as $\Phi(\phi = 0) = V_0$, $\Phi(\phi = \pi) = 0$. The cylindrical system has a depth d.

a) The solution for the EQS scalar potential in each conducting material can be written in the form

$$\Phi_1 = A_1\phi + B_1 \qquad 0 < \phi < \frac{\pi}{2}$$

$$\Phi_2 = A_2\phi + B_2 \qquad \frac{\pi}{2} < \phi < \pi$$

In the dc steady state what are the boundary conditions that allow calculation of A_1, A_2, B_1, and B_2? Find A_1, A_2, B_1, and B_2.

b) What is the electric field in each region of the resistor?

c) What are the free surface charge densities on the interfaces at $\phi = 0$, $\phi = \frac{\pi}{2}$, and $\phi = \pi$?

d) What is the dc terminal current I that flows from the battery?

e) What is the resistance between the electrodes at $\phi = 0$ and $\phi = \pi$?