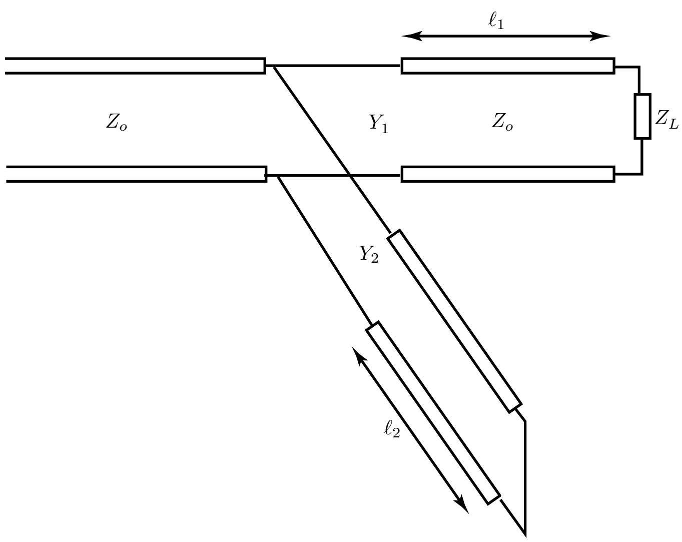
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 9 | 6.630 Electromagnetics | Issued: Week 10
Fall Term 2006 | | Due: Week 11

Reading assignment: Section 2.6; J. A. Kong, "*Electromagnetic Wave Theory*".

Problem P9.1



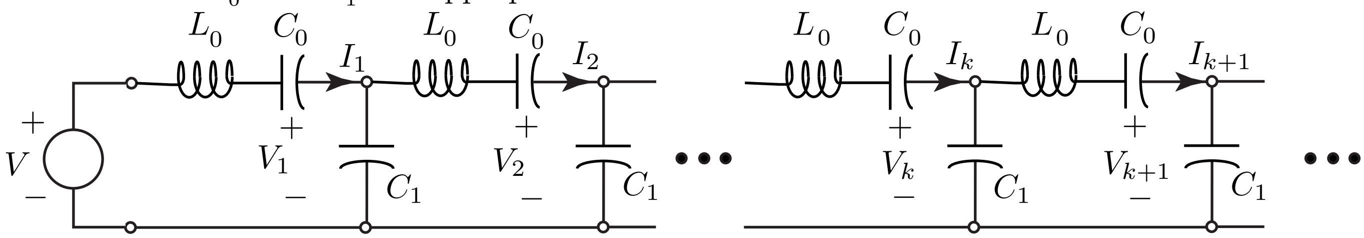
Use the Smith chart to match the following load impedence, Z_L to the line impedance, $Z_o = 50\Omega$ by finding a suitable ℓ_1 and ℓ_2.
(a) $Z_L = (100 + 25j)\Omega$
(b) $Z_L = (100 - 25j)\Omega$
(c) $Z_L = (25 + 50j)\Omega$
(d) $Z_L = (25 - 50j)\Omega$

Problem P9.2

(a) Determine the propagation constant $\theta(\omega)$, for the lumped transmission line shown in following figure . Show that the result can be placed in the form

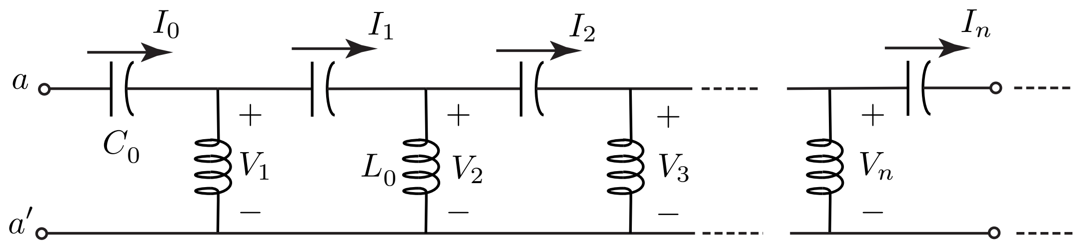
$$\sin^2 \frac{\theta}{2} = \frac{\omega^2 - {\omega_0}^2}{\omega_1^2}$$

where ω_0^2 and ω_1^2 are appropriate constants.



(b) For what range of ω will the line support a propagating wave, i.e., admit real θ solutions?

Problem P9.3



(a) Determine the dispersion relation for the backward wave line shown in following figure. That is, assume $V_n = Ae^{-jn\theta}$, $I_n = Be^{-jn\theta}$ and determine $\theta(\omega)$. Sketch $\theta(\omega)$ for $\omega > \omega_0 = \frac{1}{2\sqrt{L_0 C_0}}$.

(b) For a given $\omega > \omega_0$, the result in part (a) yields two real real values of θ (excluding values which differ by $2n\pi$). Determine the impedance, $Z = \frac{V_n}{I_n}$, for each of these modes and show that the time-averaged power flow is in the direction opposite the phase velocity.

(c) A voltage source $v_s(t) = V_s \sin \omega_0 t$ is connected to terminal pair $a - a'$. Determine the steady-state $v_n(t)$.

(d) Suppose the source connected to $a - a'$ is given by

$$v_s(t) = V_s \frac{\sin \omega_1 t}{\omega_1 t} \sin \omega_0 t$$

where $\omega_1 \ll \omega_0$. Determine $v_n(t)$.