Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

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1. **Example 9.1, page 463 in textbook**
   Romeo and Juliet start dating, but Juliet will be late on any date by a random amount X, uniformly distributed over the interval $[0, \theta]$. The parameter θ is unknown. Assuming that Juliet was late by an amount x on their first date, find the ML estimate of θ based on the observation $X = x$.

2. **Example 9.4, page 464 in textbook**
   Estimate the mean μ and variance v of a normal distribution using n independent observations $X_1, \ldots, X_n$.

3. **Example 9.8, page 474 of textbook**
   We would like to estimate the fraction of voters supporting a particular candidate for office. We collect n independent sample voter responses $X_1, \ldots, X_n$, where X_i is viewed as a Bernoulli random variable, with $X_i = 1$ if the ith voter supports the candidate. We conducted a poll of 1200 people in North Carolina, and found that 684 were supporting the candidate. We would like to construct a 95% confidence interval for θ, the proportion of people who support the candidate. As we saw in lecture, using the central limit theorem, an (approximate) 95% confidence interval can be defined as

   $$\hat{\Theta}^- = \hat{\Theta}_n - 1.96\sqrt{\frac{v}{n}}, \qquad \hat{\Theta}^+ = \hat{\Theta}_n + 1.96\sqrt{\frac{v}{n}}$$

   where $v = \text{Var}(X_i)$, and $\hat{\Theta}_n = (X_1 + \ldots + X_n)/n$. Unfortunately, we don't know the value for v. Construct confidence intervals for θ using the following three ways of estimating or bounding the value for v (in each case simply assume that v is equal to the given estimate; note that this is a further approximation in cases (a) and (b)).

   (a)

   $$\hat{S}_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \hat{\Theta}_n)^2$$

   (b)

   $$\hat{\Theta}_n(1 - \hat{\Theta}_n)$$

   (c) The most conservative upper bound for the variance.