6.01 Midterm 1 Spring 2011

Name:	**Section:**

Enter all answers in the boxes provided.

During the exam you may:

- read any paper that you want to
- use a calculator

You may not

- use a computer, phone or music player

For staff use:

1.	/16
2.	/24
3.	/16
4.	/28
5.	/16
total:	/100

1 Difference Equations (16 points)

System 1: Consider the system represented by the following difference equation

$$y[n] = x[n] + \frac{1}{2}\Big(5y[n-1] + 3y[n-2]\Big)$$

where $x[n]$ and $y[n]$ represent the n^{th} samples of the input and output signals, respectively.

1.1 Poles (4 points)

Determine the pole(s) of this system.

number of poles:

list of pole(s):

1.2 Behavior (4 points)

Does the unit-sample response of the system converge or diverge as $n \to \infty$?

converge or **diverge:**

Briefly explain.

System 2: Consider a different system that can be described by a difference equation of the form

$$y[n] = x[n] + Ay[n-1] + By[n-2]$$

where A and B are real-valued constants. The system is known to have two poles, given by $\frac{1}{2} \pm j\frac{1}{3}$.

1.3 Coefficients (4 points)

Determine A and B.

A = [ ] B = [ ]

1.4 Behavior (4 points)

Does the unit-sample response of the system converge or diverge as $n \to \infty$?

converge or **diverge:** [ ]

Briefly explain.

2 Geometry OOPs (24 points)

We will develop some classes and methods to represent polygons. They will build on the following class for representing points.

```
class Point:
    def __init__(self, x, y):
        self.x = x
        self.y = y
    def distanceTo(self, p):
        return math.sqrt((self.x - p.x)**2 + (self.y - p.y)**2)
    def __str__(self):
        return 'Point'+str((self.x, self.y))
    __repr__ = __str__
```

2.1 Polygon class (6 points)

Define a class for a Polygon, which is defined by a list of `Point` instances (its *vertices*). You should define the following methods:

- `__init__`: takes a list of the points of the polygon, in a counter-clockwise order around the polygon, as input
- `perimeter`: takes no arguments and returns the perimeter of the polygon

```
>>> p = Polygon([Point(0,0),Point(1,0),Point(1,1),Point(0,1)])
>>> p.perimeter()
4.0
```

```
class Polygon:
```

2.2 Rectangles (6 points)

Define a `Rectangle` class, which is a subclass of the `Polygon` class, for an axis-aligned rectangle which is defined by a center point, a width (measured along x axis), and a height (measured along y axis).

```
>>> s = Rectangle(Point(0.5, 1.0), 1, 2)
```

This has a result that is equivalent to

```
>>> s = Polygon([Point(0, 0), Point(1, 0), Point(1, 2), Point(0, 2)])
```

Define the `Rectangle` class; write as little new code as possible.

2.3 Edges (6 points)

Computing the perimeter, and other algorithms, can be conveniently organized by iterating over the *edges* of a polygon. So, we can describe the polygon in terms of edges, as defined in the following class:

```
class Edge:
    def __init__(self, p1, p2):
        self.p1 = p1
        self.p2 = p2
    def length(self):
        return self.p1.distanceTo(self.p2)
    def determinant(self):
        return self.p1.x * self.p2.y - self.p1.y * self.p2.x
    def __str__(self):
        return 'Edge'+str((self.p1, self.p2))
    __repr__ = __str__
```

Assume that the `__init__` method for the `Polygon` class initializes the attribute `edges` to be a list of `Edge` instances for the polygon, as well as initializing the points.

Define a new method, `sumForEdges`, for the `Polygon` class that takes a procedure as an argument, which applies the procedure to each edge and returns the sum of the results. The example below simply returns the number of edges in the polygon.

```
>>> p = Polygon([Point(0,0),Point(2,0),Point(2,1),Point(0,1)])
>>> p.sumForEdges(lambda e: 1)
4
```

2.4 Area (6 points)

A very cool algorithm for computing the area of an arbitrary polygon hinges on the fact that:

The area of a planar non-self-intersection polygon with vertices $(x_0, y_0), \ldots, (x_n, y_n)$ is

$$A = \frac{1}{2}\left(\begin{vmatrix} x_0 & x_1 \\ y_0 & y_1 \end{vmatrix} + \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \cdots + \begin{vmatrix} x_n & x_0 \\ y_n & y_0 \end{vmatrix}\right)$$

where $|M|$ denotes the determinant of a matrix, defined in the two by two case as:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$$

Note that the `determinant` method has already been implemented in the `Edge` class.

Use the `sumForEdges` method and any other methods in the `Edge` class to implement an `area` method for the `Polygon` class.

3 Signals and Systems (16 points)

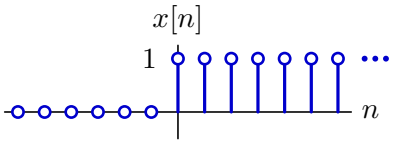
Consider the system described by the following difference equation:

$$y[n] = x[n] + y[n-1] + 2y[n-2]\,.$$

3.1 Unit-Step Response (4 points)

Assume that the system starts at rest and that the input $x[n]$ is the **unit-step** signal $u[n]$.

$$x[n] = u[n] \equiv \begin{cases} 1 & n \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$



Find $y[4]$ and enter its value in the box below.

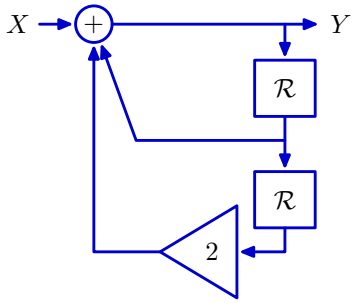
$y[4] =$ ____

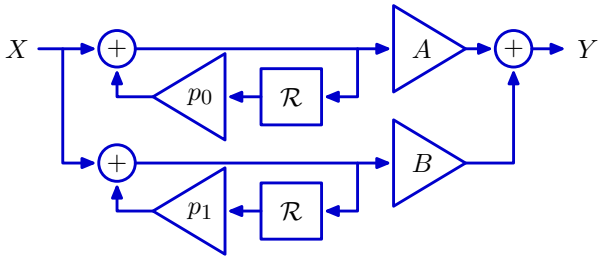
3.2 Block Diagrams (12 points)

The system that is represented by the following difference equation

$$y[n] = x[n] + y[n-1] + 2y[n-2]$$

can also be represented by the block diagram below (left).





It is possible to choose coefficients for the block diagram on the right so that the systems represented by the left and right block diagrams are "equivalent".[1]

Enter values of p_0, p_1, A, and B that will make the systems equivalent in the boxes below

$p_0 =$ $\quad$ A = $\quad$ $p_1 =$ $\quad$ B =

[1] Two systems are "equivalent" if identical inputs generate identical outputs when each system is started from "rest" (i.e., all delay outputs are initially zero).

4 Robot SM (28 points)

There is a copy of this page at the back of the exam that you can tear off for reference.

In Design Lab 2, we developed a state machine for getting the robot to follow boundaries. Here, we will develop a systematic approach for specifying such state machines.

We start by defining a procedure called `inpClassifier`, which takes an input of type `io.SensorInput` and classifies it as one of a small number of input types that can then be used to make decisions about what the robot should do next.

Recall that instances of the class `io.SensorInput` have two attributes: `sonars`, which is a list of 8 sonar readings and `odometry` which is an instance of `util.Pose`, which has attributes `x`, `y` and `theta`.

Here is a simple example:

```
def inpClassifier(inp):
    if inp.sonars[3] < 0.5: return 'frontWall'
    elif inp.sonars[7] < 0.5: return 'rightWall'
    else: return 'noWall'
```

Next, we create a class for defining "rules." Each rule specifies the next state, forward velocity and rotational velocity that should result when the robot is in a specified current state and receives a particular type of input. Here is the definition of the class `Rule`:

```
class Rule:
    def __init__(self, currentState, inpType, nextState, outFvel, outRvel):
        self.currentState = currentState
        self.inpType = inpType
        self.nextState = nextState
        self.outFvel = outFvel
        self.outRvel = outRvel
```

Thus, an instance of a `Rule` would look like this:

```
  Rule('NotFollowing', 'frontWall', 'Following', 0.0, 0.0)
```

which says that if we are in state `'NotFollowing'` and we get an input of type `'frontWall'`, we should transition to state `'Following'` and output zero forward and rotational velocities.

Finally, we will specify the new state machine class called `Robot`, which takes a start state, a list of `Rule` instances, and a procedure to classify input types. The following statement creates an instance of the `Robot` class that we can use to control the robot in a `Soar` brain.

```
r = Robot('NotFollowing',
          [Rule('NotFollowing', 'noWall', 'NotFollowing', 0.1, 0.0),
           Rule('NotFollowing', 'frontWall', 'Following', 0.0, 0.0),
           Rule('Following', 'frontWall', 'Following', 0.0, 0.1)],
          inpClassifier)
```

Assume it is an error if a combination of state and input type occurs that is not covered in the rules. In that case, the state will not change and the output will be an action with zero velocities.

4.1 Simulate (3 points)

For the input classifier (`inpClassifier`) and `Robot` instance (`r`) shown above, give the outputs for the given inputs.

step	input		next state	output	
	`sonars[3]`	`sonars[7]`		forward vel	rotational vel
1	1.0	5.0			
2	0.4	5.0			
3	0.4	5.0			

4.2 Charge and Retreat (4 points)

We'd like the robot to start at the origin (i.e., x=0, y=0, theta=0), then move forward (along x axis) until it gets within 0.5 meters of a wall, then move backward until it is close to the origin (within 0.02 m), and then repeat this cycle of forward and backward moves indefinitely. Assume that the robot never moves more than 0.01 m per time step.

Write an input classifier for this behavior.

```
def chargeAndRetreatClassifier(inp):
```

4.3 Robot instance (7 points)

Write an instance of the `Robot` class that implements the charge-and-retreat behavior described above. Make sure that you cover all the cases.

4.4 Matching (6 points)

Write a procedure `match(rules, inpType, state)` that takes a list of rules, an input type classification, and a state, and returns the rule in `rules` that matches `inpType` and `state` if there is one, and otherwise returns `None`.

4.5 The Machine (8 points)

Complete the definition of the `Robot` class below; use the `match` procedure you defined above.

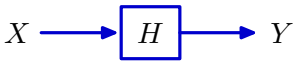
Recall that, at each step, the output must be an instance of the `io.Action` class; to initialize an instance of `io.Action`, you must provide a forward and a rotational velocity.

If a combination of state and input type occurs that is not covered in the rules, remain in the same state and output an action with zero velocity.

```
class Robot(sm.SM):
    def __init__(self, start, rules, inpClassifier):
        self.startState = start
        self.rules = rules
        self.inpClassifier = inpClassifier
```

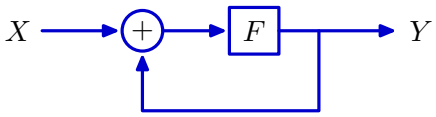
5 Feedback (16 points)

Let H represent a system with input X and output Y as shown below.



System 1

Assume that the system function for H can be written as a ratio of polynomials in $\mathcal{R}$ with constant, real-valued, coefficients. In this problem, we investigate when the system H is equivalent to the following feedback system



System 2

where F is also a ratio of polynomials in $\mathcal{R}$ with constant, real-valued coefficients.

Example 1: Systems 1 and 2 are equivalent when $H = H_1 = \dfrac{\mathcal{R}}{1-\mathcal{R}}$ and $F = F_1 = \mathcal{R}$.

Example 2: Systems 1 and 2 are equivalent when $H = H_2 = \dfrac{\mathcal{R}^2}{1-\mathcal{R}^2}$ and $F = F_2 = \mathcal{R}^2$.

5.1 Generalization (4 points)

Which of the following expressions for F guarantees equivalence of Systems 1 and 2?

$$F_A = \frac{1}{1+H} \qquad F_B = \frac{1}{1-H} \qquad F_C = \frac{H}{1+H} \qquad F_D = \frac{H}{1-H}$$

Enter F_A or F_B or F_C or F_D or **None**:

5.2 Find the Poles (6 points)

Let $H_3 = \frac{9}{2+\mathcal{R}}$. Determine the pole(s) of H_3 and the pole(s) of $\frac{1}{1-H_3}$.

Pole(s) of H_3: | Pole(s) of $\frac{1}{1-H_3}$:

5.3 SystemFunction (6 points)

Write a procedure `insideOut(H)`:

- the input H is a `sf.SystemFunction` that represents the system H, and
- the output is a `sf.SystemFunction` that represents $\frac{H}{1-H}$.

You may use the `SystemFunction` class and other procedures in `sf`:

Attributes and methods of `SystemFunction` class:

```
__init__(self, numPoly, denomPoly)
poles(self)
poleMagnitudes(self)
dominantPole(self)
numerator
denominator
```

Procedures in `sf`

```
sf.Cascade(sf1, sf2)
sf.FeedbackSubtract(sf1, sf2)
sf.FeedbackAdd(sf1, sf2)
sf.Gain(c)
```

```
def insideOut(H):
```

Worksheet (intentionally blank)

Worksheet (intentionally blank)

Robot SM: Reference Sheet

This is the same as the first page of problem 4.

In Design Lab 2, we developed a state machine for getting the robot to follow boundaries. Here, we will develop a systematic approach for specifying such state machines.

We start by defining a procedure called `inpClassifier`, which takes an input of type `io.SensorInput` and classifies it as one of a small number of input types that can then be used to make decisions about what the robot should do next.

Recall that instances of the class `io.SensorInput` have two attributes: `sonars`, which is a list of 8 sonar readings and `odometry` which is an instance of `util.Pose`, which has attributes `x`, `y` and `theta`.

Here is a simple example:

```
def inpClassifier(inp):
    if inp.sonars[3] < 0.5: return 'frontWall'
    elif inp.sonars[7] < 0.5: return 'rightWall'
    else: return 'noWall'
```

Next, we create a class for defining "rules." Each rule specifies the next state, forward velocity and rotational velocity that should result when the robot is in a specified current state and receives a particular type of input. Here is the definition of the class `Rule`:

```
class Rule:
    def __init__(self, currentState, inpType, nextState, outFvel, outRvel):
        self.currentState = currentState
        self.inpType = inpType
        self.nextState = nextState
        self.outFvel = outFvel
        self.outRvel = outRvel
```

Thus, an instance of a `Rule` would look like this:

```
  Rule('NotFollowing', 'frontWall', 'Following', 0.0, 0.0)
```

which says that if we are in state `'NotFollowing'` and we get an input of type `'frontWall'`, we should transition to state `'Following'` and output zero forward and rotational velocities.

Finally, we will specify the new state machine class called `Robot`, which takes a start state, a list of `Rule` instances, and a procedure to classify input types. The following statement creates an instance of the `Robot` class that we can use to control the robot in a `Soar` brain.

```
r = Robot('NotFollowing',
          [Rule('NotFollowing', 'noWall', 'NotFollowing', 0.1, 0.0),
           Rule('NotFollowing', 'frontWall', 'Following', 0.0, 0.0),
           Rule('Following', 'frontWall', 'Following', 0.0, 0.1)],
          inpClassifier)
```

Assume it is an error if a combination of state and input type occurs that is not covered in the rules. In that case, the state will not change and the output will be an action with zero velocities.

6.01SC Introduction to Electrical Engineering and Computer Science

Spring 2011