### 6.013 - Electromagnetics and Applications

## Problem Set 6 (five problems)

Suggested Reading: $\quad$ Course notes, Sections 7.1.1-7.1.2, 7.2.1-7.2.2, 8.1, 8.3.1

## Problem 6.1

Stripline can be approximated as an ideal parallel-plate TEM line if its width W is much greater than the separation d between top and bottom plates (i.e., if fringing fields can be neglected). Consider the illustrated infinitely long stripline for which $\mathrm{d}=1$ micron and the medium between the plates has $\mu=\mu_{0}$ and $\varepsilon=4 \varepsilon_{0}$.

(a) For what width W is the impedance of this line $50 \Omega$ ? Is the ideal parallel plate model valid for these dimensions? Discuss briefly.
(b) For a 1 -volt DC signal, what is the intensity I (time-average Poynting vector magnitude [W]) of the TEM electromagnetic field propagating between the plates?
(c) Evaluate the time average electric and magnetic energy densities per meter, i.e., $\mathrm{W}_{\mathrm{e}}$ and $\mathrm{W}_{\mathrm{m}}[\mathrm{J} / \mathrm{m}]$, on this line for case (b) (neglect fringing fields).
(d) Show that the average power on the line, $\mathrm{c}_{\mathrm{line}}\left(\mathrm{W}_{\mathrm{e}}+\mathrm{W}_{\mathrm{m}}\right)$, equals the I found in (b).
(e) Show that if two arbitrary signals flowing in opposite directions are superimposed so that $\mathrm{v}(\mathrm{z}, \mathrm{t})=\mathrm{f}_{+}(\mathrm{t}-\mathrm{z} / \mathrm{v})+\mathrm{f}_{-}(\mathrm{t}+\mathrm{z} / \mathrm{v})$, then the total power flowing down this line in the +z direction at any ( $\mathrm{t}, \mathrm{z}$ ) equals the power flowing in the +z direction minus the power flowing in the -z direction. Show whether or not such superposition of powers also applies when the two signals flow in the same direction on TEM lines.

## Problem 6.2

A 30-centimeter-long, air-filled $100 \Omega$ TEM line is excited at one end by a matched voltage source $V(t)$, where $V(t)$ is a step function $2 u(t)$ volts. Sketch and quantitatively dimension $\mathrm{V}(\mathrm{z})$ and $\mathrm{I}(\mathrm{z})$ on the line at $\mathrm{t}=15 \times 10^{-10}$ sec for the case where the load is:
(a) a $300 \Omega$ resister
(b) a capacitor $\mathrm{C}=2 \times 10^{-12} \mathrm{~F}$
(c) a diode back-biased with a 1 -volt battery, as shown.


## Problem 6.3

A line driver at one end of a $2-\mathrm{cm}$ long 200-ohm TEM transmission line triggers a flipflop at the other end with a step function, as illustrated. The dielectric in the line has $\varepsilon=$ $4 \varepsilon_{0}$ and $\mu=\mu_{0}$.

(a) Sketch and dimension $\mathrm{v}(\mathrm{t}, \mathrm{z})$ on the line at $\mathrm{t}=0.1 \mathrm{~ns}\left(10^{-10} \mathrm{sec}\right)$.
(b) Repeat (a) for $t=0.2 \mathrm{~ns}$.
(c) Sketch quantitatively the load voltage $\mathrm{v}_{\mathrm{L}}(\mathrm{t})$ until the flip-flop is triggered; its trigger voltage is 4 volts. Note that triggering is excessively delayed.
(d) What is the asymptotic value of the load voltage $\mathrm{v}_{\mathrm{L}}(\mathrm{t})$ as $\mathrm{t} \rightarrow \infty$ ?
(e) If the line impedance were matched at 50 ohms, would there still be excessive delay?
(f) Write a simple equation for $\mathrm{v}(\mathrm{z}, \mathrm{t})$ valid for $0<\mathrm{t}<0.1 \mathrm{~ns}$, then extend it to 0.2 ns .

## Problem 6.4

A unit-step current source $I_{0}(t)=u(t)$ directly drives the illustrated circuit. Sketch and dimension the voltage $\mathrm{v}(\mathrm{z})$ on all lines at:
(a) $t=D / 2 c$
(b) $t=3 D / 2 c$
(c) $t=5 D / 2 c$.


## Problem 6.5

A current source $I_{0}$ drives a delicate transistor that has an input impedance of $4 \mathrm{Z}_{0}$ through a TEM line of impedance $Z_{0}$, as illustrated.

(a) At $t=0$ the switch at $\mathrm{z}=\mathrm{D} / 2$ opens for $\mathrm{D} / 10 \mathrm{c}$ seconds and then recloses. Sketch the voltage $\mathrm{v}(\mathrm{z})$ on the line at $\mathrm{t}=\mathrm{D} / 5 \mathrm{c}$.
(b) Will $\mathrm{v}_{\mathrm{L}}(\mathrm{t})$ across the transistor load ever exceed its breakdown limit of $7 \mathrm{Z}_{0} \mathrm{I}_{0}$ volts? Briefly explain.

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Spring 2009

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