April 18, 2001 - Quiz #2

Name: SOLUTIONS

Recitation: ______________________

problem	grade
1	
2	
3	
4	
5	
total	

General guidelines (please read carefully before starting):

- Make sure to write your name on the space designated above.
- **Open book**: you can use any material you wish.
- All answers should be given in the space provided. Please do not turn in any extra material. If you need more space, use the back page.
- You have **120 minutes** to complete your quiz.
- Make reasonable approximations and *state them*, i.e. quasi-neutrality, depletion approximation, etc.
- Partial credit will be given for setting up problems without calculations. **NO** credit will be given for answers without reasons.
- Use the symbols utilized in class for the various physical parameters, i.e. μ_n, I_D, E, etc.
- Every numerical answer must have the proper units next to it. Points will be subtracted for answers without units or with wrong units.
- Use $\phi = 0$ at $n_o = p_o = n_i$ as potential reference.
- Use the following fundamental constants and physical parameters for silicon and silicon dioxide at room temperature:

$$n_i = 1 \times 10^{10}\ cm^{-3}$$
$$kT/q = 0.025\ V$$
$$q = 1.60 \times 10^{-19}\ C$$
$$\epsilon_s = 1.05 \times 10^{-12}\ F/cm$$
$$\epsilon_{ox} = 3.45 \times 10^{-13}\ F/cm$$

(1b) *(5 points)* We want this inverter to have an average propagation delay $t_p = 1\ ns$ when driving a $C_L = 1\ pF$ capacitive load. Calculate W_n and W_p.

The average propagation delay is:

$$t_p = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}\left[\frac{C_L V_{DD}}{\frac{W_n}{L_n}\mu_n C_{ox}(V_{DD}-V_{Tn})^2} + \frac{C_L V_{DD}}{\frac{W_p}{L_p}\mu_p C_{ox}(V_{DD}+V_{Tp})^2}\right] =$$

$$= \frac{C_L V_{DD} L_n}{2 C_{ox} W_p}\left[\frac{W_p}{W_n}\cdot\frac{1}{\mu_n(V_{DD}-V_{Tn})^2} + \frac{1}{\mu_p(V_{DD}+V_{Tp})^2}\right] =$$

Solving for W_p:

$$W_p = \frac{C_L V_{DD} L_n}{2 C_{ox} t_p}\left[\frac{W_p}{W_n}\cdot\frac{1}{\mu_n(V_{DD}-V_{Tn})^2} + \frac{1}{\mu_p(V_{DD}+V_{Tp})^2}\right] =$$

$$= \frac{10^{-12}\times 5\times 1}{2\times 3.5\times 10^{-7}\times 10^{-9}}\left[3.6\,\frac{1}{400\,(5-0.5)^2} + \frac{1}{200\,(5-1)^2}\right] = 5.4\,\mu m$$

where we have used

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45\times 10^{-13}}{10\times 10^{-7}} \simeq 3.5\times 10^{-7}\ F/cm^2$$

Hence

$$W_n = \frac{5.4}{3.6} = 1.5\ \mu m$$

1. *(25 points)* You are given a CMOS inverter with the following parameters:

$V_{Tn} = 0.5\ V$	$t_{ox} = 10\ nm$	$\mu_n = 400\ cm^2/V \cdot s$
$V_{Tp} = -1\ V$	$\lambda_n = \lambda_p = 0.1\ V^{-1}$	$\mu_p = 200\ cm^2/V \cdot s$
$V_{DD} = 5\ V$	$L_n = L_p = 1\ \mu m$	

(1a) *(5 points)* Calculate the ratio W_p/W_n so that $V_M = 2.5\ V$.

V_M is given by:

$$V_M = \frac{V_{Tn} + \sqrt{\frac{k_p}{k_n}}(V_{DD} + V_{Tp})}{1 + \sqrt{\frac{k_p}{k_n}}}$$

let us solve for k_p/k_n:

$$\frac{k_p}{k_n} = \left(\frac{V_M - V_{Tn}}{V_{DD} - V_M + V_{Tp}}\right)^2 = \left(\frac{2.5 - 0.5}{5 - 2.5 - 1}\right)^2 = 1.8$$

This ratio is equal to

$$\frac{k_p}{k_n} = \frac{\frac{W_p}{\cancel{L_p}} \mu_p \cancel{C_{ox}}}{\frac{W_n}{\cancel{L_n}} \mu_n \cancel{C_{ox}}} = \frac{W_p \mu_p}{W_n \mu_n}$$

Hence

$$\frac{W_p}{W_n} = \frac{k_p}{k_n} \cdot \frac{\mu_n}{\mu_p} = 1.8 \times 2 = 3.6$$

(1c) *(10 points)* Estimate NM_L and NM_H for this inverter.

We have to start by computing the voltage gain at V_M:

$$|A_v| = (g_{mn} + g_{mp})(r_{on} \| r_{op}) = \frac{g_{mn} + g_{mp}}{\frac{1}{r_{on}} + \frac{1}{r_{op}}} = \frac{g_{mn} + g_{mp}}{g_{on} + g_{op}} =$$

$$= \frac{\frac{W_n}{L_n} \mu_n \cancel{C_{ox}} (V_M - V_{Tn}) + \frac{W_p}{L_p} \mu_p \cancel{C_{ox}} (V_M + V_{Tp})}{\frac{W_n}{2L_n} \mu_n \cancel{C_{ox}} (V_M - V_{Tn})^2 \lambda_n + \frac{W_p}{2L_p} \mu_p \cancel{C_{ox}} (V_M + V_{Tp})^2} =$$

$$= \frac{2}{\lambda_n} \frac{(V_M - V_{Tn}) + \frac{W_p}{W_n}\frac{\mu_p}{\mu_n}(V_M + V_{Tp})}{(V_M - V_{Tn})^2 + \frac{W_p}{W_n}\frac{\mu_p}{\mu_n}(V_M + V_{Tp})^2} = \frac{2}{0.1} \cdot \frac{(2.5 - 0.5) + 1.8 \times (2.5 - 1)}{(2.5 - 0.5)^2 + 1.8\,(2.5 - 1)^2} = 11.7$$

We now use the standard formulae for NM_L and NM_H:

$$NM_L = V_M - \frac{V_{DD} - V_M}{|A_v|} = 2.5 - \frac{5 - 2.5}{11.7} = 2.3 \text{ V}$$

$$NM_H = V_{DD} - V_M \left(1 + \frac{1}{|A_v|}\right) = 5 - 2.5\left(1 + \frac{1}{11.7}\right) = 2.3 \text{ V}$$

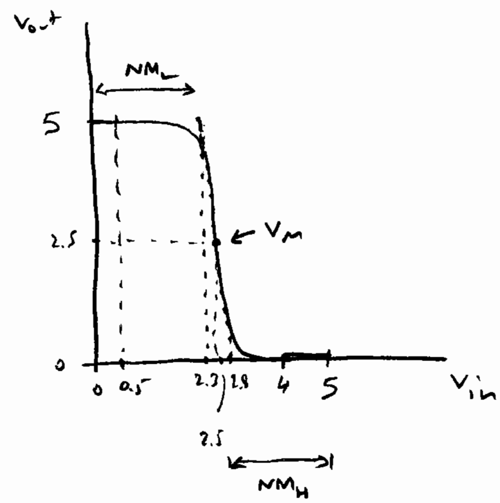
Here is a much faster way to compute the voltage gain:

$$|A_v| = \frac{g_{mn} + g_{mp}}{g_{on} + g_{op}} = \frac{\sqrt{2\frac{W_n}{L_n}\mu_n C_{ox} I_D} + \sqrt{2\frac{W_p}{L_p}\mu_p C_{ox} I_D}}{\lambda_n I_D + \lambda_p I_D} = \frac{\cancel{\sqrt{2}}}{2\lambda} \frac{\sqrt{\frac{W_n}{L_n}\mu_n C_{ox}} + \sqrt{\frac{W_p}{L_p}\mu_p C_{ox}}}{\sqrt{\frac{1}{\cancel{2}}\frac{W_n}{L_n}\mu_n C_{ox}}\,(V_M - V_{Tn})} =$$

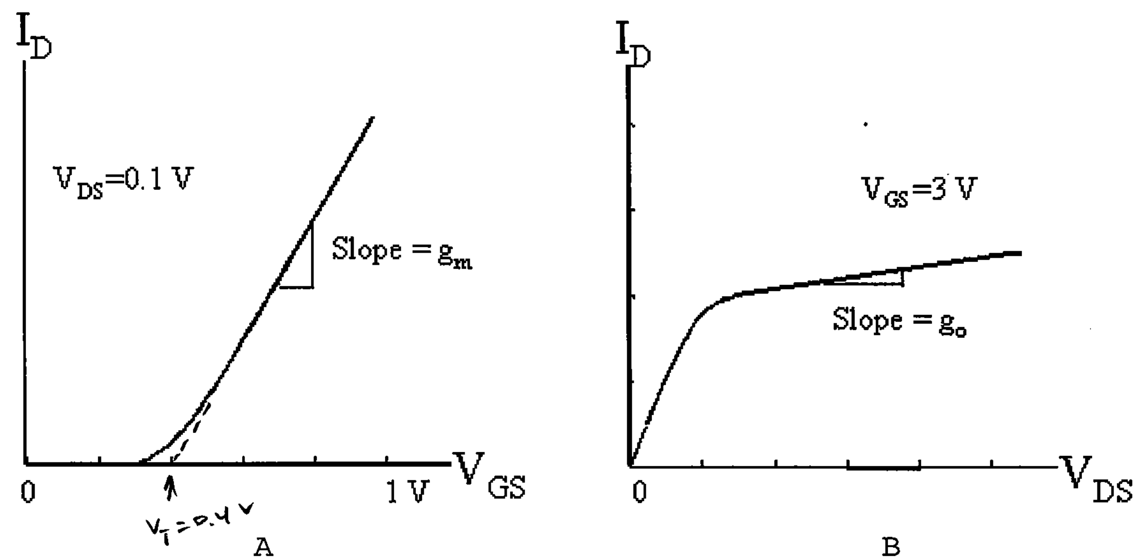
$$= \frac{1}{\lambda} \frac{\sqrt{1 + \frac{k_p}{k_n}}}{V_M - V_{Tn}}$$

where we have used the fact that $I_{Dn} = -I_{Dp} = I_D$ and $\lambda_n = \lambda_p = \lambda$.

(1d) *(5 points)* Sketch and appropriately label the voltage transfer characteristics of this inverter.



2. *(20 points)* You are given the following I-V characteristics for a n-MOSFET with $t_{ox} = 10\ nm$, $W = 10\ \mu m$, and $L = 1\ \mu m$. The gate material is n$^+$-doped polysilicon. The body is tied up to the source.



In (A), $g_m = 1.4 \times 10^{-4}\ A/V$. In (B), $g_o = 5.7 \times 10^{-5}\ A/V$.

(2a) *(5 points)* From (A), estimate the threshold voltage, V_T.

From (A), by simple extrapolation we get:

$V_T \simeq 0.4$ V

(2b) *(5 points)* From (A), estimate the electron mobility, μ_n.

In (A) the transistor is biased in the linear regime (V_{DS} is small).

The transistor current is given by:

$$I = \frac{W}{L} \mu_n C_{ox} \left(V_{GS} - \frac{V_{DS}}{2} - V_T\right) V_{DS} \quad .$$

The transconductance is then:

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{W}{L} \mu_n C_{ox} V_{DS}$$

Solving for μ_n:

$$\mu_n = \frac{g_m L}{W\, C_{ox} V_{DS}} = \frac{1.4 \times 10^{-4} \times 10^{-4}}{10 \times 10^{-4} \times 3.45 \times 10^{-7} \times 0.1} = 406 \quad cm^2/V.s$$

with

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-13}}{10 \times 10^{-7}} = 3.45 \times 10^{-7} \quad F/cm^2$$

(2d) *(5 points)* Estimate the saturation voltage, V_{DSsat}, and the saturation current, I_{Dsat}, corresponding to the characteristics in (B).

In a MOSFET:

$$V_{DSsat} = V_{GS} - V_T$$

Hence, for the data in (B):

$$V_{DSsat} = 3 - 0.4 = 2.6 \text{ V.}$$

I_{Dsat} is simply:

$$I_D = \frac{W}{2L}\mu_n C_{ox} (V_{GS} - V_T)^2 = \frac{10}{2 \times 1} 406 \times 3.45 \times 10^{-7} (3 - 0.4)^2 = 4.7 \text{ mA}$$

(2e) *(5 points)* From (B), estimate the width of the channel pinch-off region, ΔL, at $V_{DS} = 4\ V$.

To the first order we know that

$$g_o = \lambda I_D = \frac{\Delta L}{L} \frac{I_D}{V_{DS} - V_{DSsat}}$$

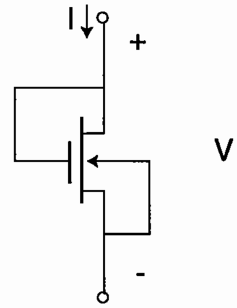
Solving for ΔL:

$$\Delta L = \frac{g_o L (V_{DS} - V_{DSsat})}{I_D} = \frac{5.7 \times 10^{-5} \times 10^{-4} (4 - 2.6)}{4.7 \times 10^{-3}} = 1.7 \times 10^{-6} \text{ cm} =$$

$$= 0.017\ \mu m$$

or 1.7 % of the channel length (pretty small).

3. *(20 points)* An n-channel MOSFET is wired up in the form indicated below. This is an enhancement-mode device ($V_T > 0$). Neglect channel length modulation.



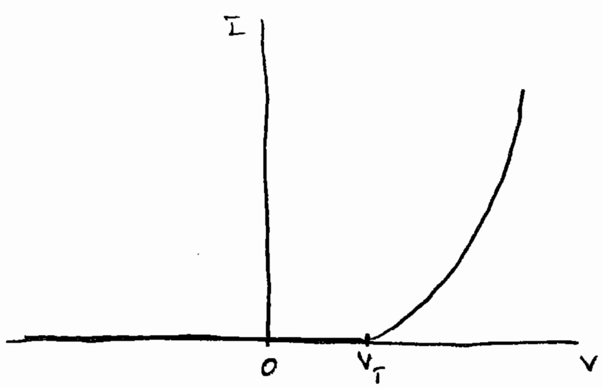
(3a) *(10 points)* In terms of usual MOSFET parameters, derive suitable equations for the I-V characteristics of the resulting two-terminal device. Sketch the I-V characteristics in a linear scale.

For $V > V_T$, the MOSFET is in the saturation regime since $V_{GD} = 0 < V_T$. The I-V characteristics are then:

$$I = \frac{W}{2L} \mu_n C_{ox} (V - V_T)^2$$

For $V < V_T$, the MOSFET is cut-off and $I = 0$

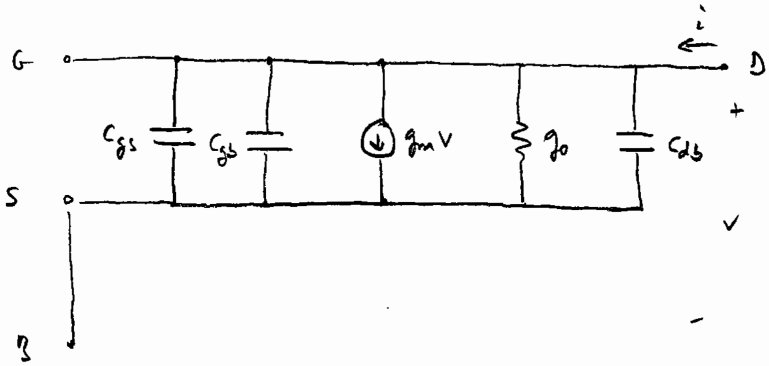
The I-V characteristics then look like:



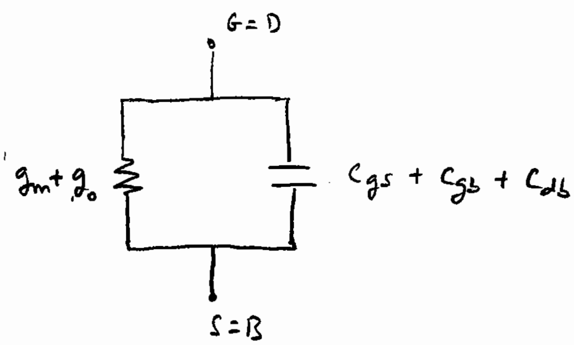
This looks pretty much like a diode except that the forward current branch is quadratic rather than exponential.

(3b) *(10 points)* Sketch a complete high-frequency small-signal equivalent circuit model for this two-terminal device for situations in which $V > V_T$. Express all small-signal elements in terms of those of the MOSFET, *i.e.*: as a function of g_m, g_o, C_{gs}, C_{gd}, C_{sb}, etc.

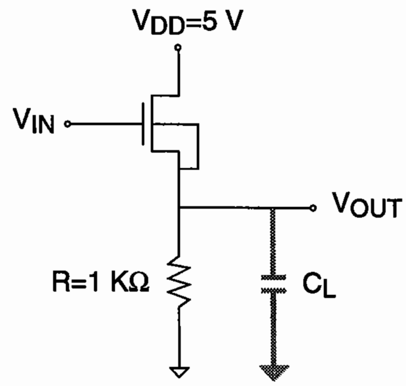
If we look at the small-signal equivalent circuit model of a MOSFET in saturation (appropriate in this case for $v > V_T$), we see that it gets simplified because the body is shorted to the source and the gate is shorted to the drain. That effectively kills C_{sb}, g_{mb} and C_{gd}. We are then left with:



This can be simplified to:



4. *(15 points)* An NMOS inverter with a resistor pull up was miswired and ended up as sketched below.



The parameters of the transistor are: $\mu_n C_{ox} = 50\ \mu A/V^2$, $W/L = 5$, and $V_T = 1\ V$. Neglect channel length modulation in this problem.

(4a) *(5 points)* For $V_{IN} = 0$, in what regime is the transistor biased? How much is V_{OUT}? *(numerical answer expected).*

For $V_{IN} = 0$ the transistor is cut-off, as a result, no current flows through the resistor and $V_{OUT} = 0$.

(4b) *(10 points)* For $V_{IN} = 5\ V$, in what regime is the transistor biased? How much is V_{OUT}? (you can leave the result in the form of an equation where V_{OUT} is the only unknown).

For $V_{IN} = 5\,V$, the transistor is in saturation since $V_{GD} = 0\ V$ ($< V_T$).

The current flowing through the transistor is

$$I_D = \frac{K}{2}(V_{IN} - V_{OUT} - V_T)^2 = \frac{5}{2}\,50\times10^{-6}\,(5 - V_{OUT} - 1)^2 =$$

$$= 1.25\times10^{-4}\,(4 - V_{OUT})^2$$

The current flowing through the resistor is

$$I_R = \frac{V_{OUT}}{R} = 10^{-3}\,V_{OUT}$$

Since $I_R = I_D$, we have

$$1.25\times10^{-4}\,(4 - V_{OUT})^2 = 10^{-3}\,V_{OUT}$$

or

$$1.25\times10^{-4}\,V_{out}^2 - 2\times10^{-3}\,V_{out} + 2\times10^{-3} = 0$$

or

$$V_{out}^2 - 16\,V_{out} + 16 = 0$$

The solution is:

$$V_{out} \simeq 1.1\,V$$

5. *(20 points)* In a certain pn junction diode at room temperature at a particular forward bias voltage, the current supported by hole injection into the n-side of the diode is 100 μA.

The quasi-neutral width of the n-side of the diode is $w_n - x_n = 1\ \mu m$. The hole diffusion coefficient is 10 cm^2/s. The pn junction area is 10 μm^2.

Make and state suitable approximations.

(5a) *(5 points)* Estimate the hole concentration at the space-charge region edge of the n quasi-neutral region. *(numerical answer expected).*

The current is given by:

$$I_p = qAD_p \frac{p(x_n)}{w_n - x_n}$$

Solving for $p(x_n)$:

$$p(x_n) = \frac{I_p (w_n - x_n)}{qAD_p} = \frac{100 \times 10^{-6} \times 10^{-4}}{1.6 \times 10^{-19} \times 10 \times 10^{-8} \times 10} = 6.3 \times 10^{16}\ cm^{-3}$$

(5b) *(5 points)* Estimate the velocity at which holes are injected at the edge of the n quasi-neutral region *(numerical answer expected).*

At the edge of the quasi-neutral region we can write:

$$I_p = qA\, p(x_n)\, v_p(x_n)$$

Solving for $v_p(x_n)$:

$$v_p(x_n) = \frac{I_p}{qA\, p(x_n)} = \frac{100 \times 10^{-6}}{1.6 \times 10^{-19} \times 10^{-7} \times 6.3 \times 10^{16}} = 10^{5}\ cm/s$$

(5c) *(5 points)* Estimate the hole flux arriving at the surface of the n quasi-neutral region. *(numerical answer expected).*

The hole flux arriving at the surface of the n quasi-neutral region is the same one that was injected at its edge with the space-charge region. This is because all holes that are injected reach the surface.

Hence
$$F_p(W_n) = F_p(x_n) = \frac{I_p}{Aq} = \frac{100\times10^{-6}}{10^{-7}\times1.6\times10^{-19}} = 6.3\times10^{21}\ cm^{-2}s^{-1}$$

(5d) *(5 points)* Estimate the diffusion capacitance associated with hole storage in the n quasi-neutral region. *(numerical answer expected).*

The hole charge stored in the n-QNR is:

$$Q_p = \tau_t I_p = \frac{(W_n-x_n)^2}{2D_p} I_p = \frac{(10^{-4})^2}{2\times10} \times 100\times10^{-6} = 5\times10^{-14}\ C$$

The diffusion capacitance is:

$$C_p = \frac{q}{kT} Q_p = \frac{5\times10^{-14}}{0.025} = 2\times10^{-12}\ F$$