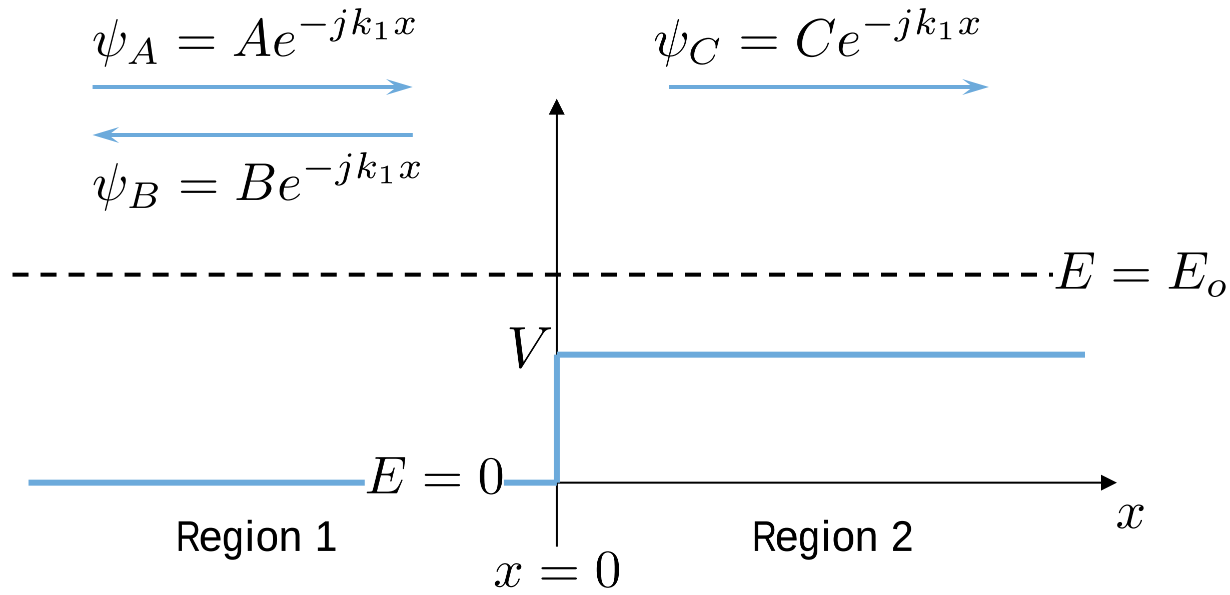
Tunneling

Outline

- Review: Barrier Reflection
- Barrier Penetration (Tunneling)
- Flash Memory

A Simple Potential Step

CASE I : $E_o > V$



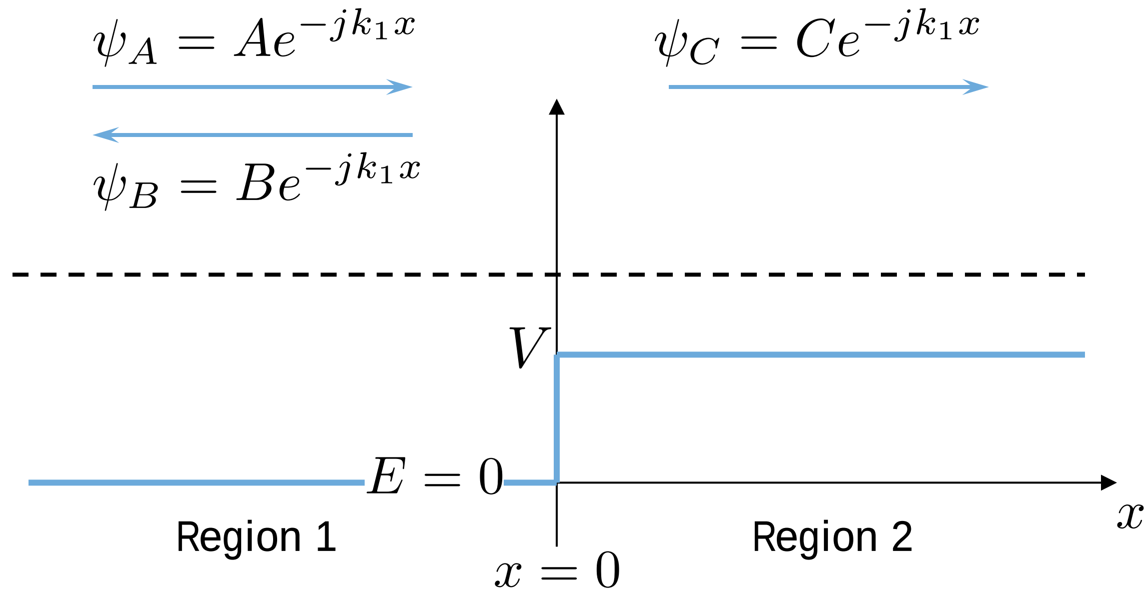
In Region 1:

$$E_o\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \quad \Rightarrow \quad k_1^2 = \frac{2mE_o}{\hbar^2}$$

In Region 2:

$$(E_o - V)\,\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \quad \Rightarrow \quad k_2^2 = \frac{2m\,(E_o - V)}{\hbar^2}$$

A Simple Potential Step



CASE I : $E_o > V$

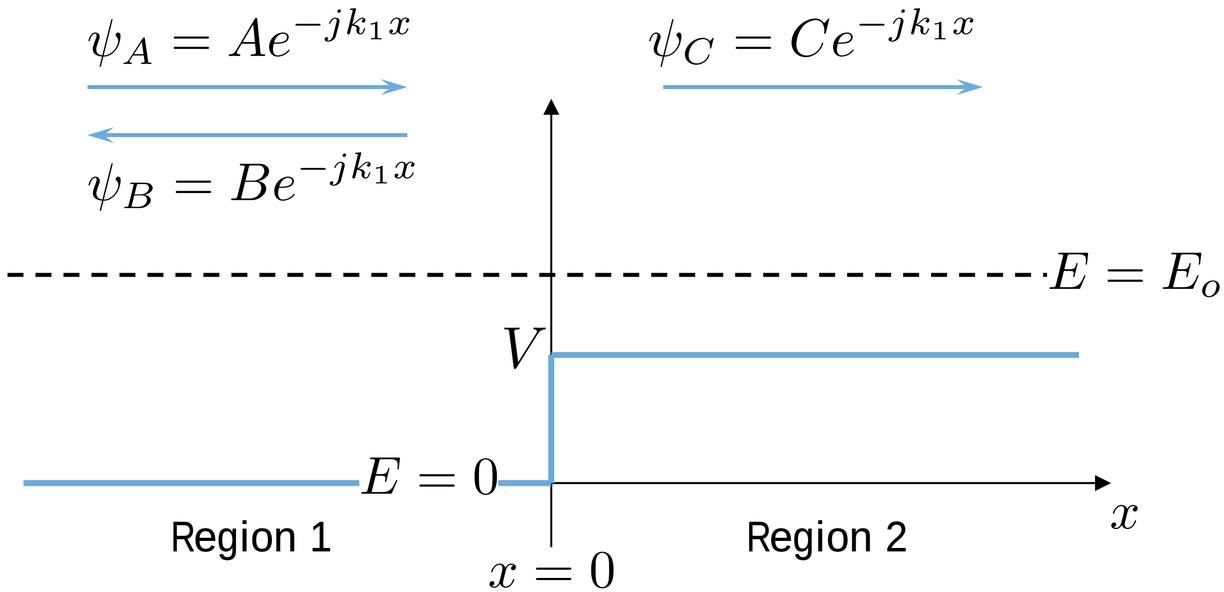
$$\psi_1 = Ae^{-jk_1x} + Be^{jk_1x} \qquad \psi_2 = Ce^{-jk_2x}$$

ψ is continuous: $\psi_1(0) = \psi_2(0) \Rightarrow A + B = C$

$\frac{\partial \psi}{\partial x}$ is continuous: $\frac{\partial}{\partial x}\psi(0) = \frac{\partial}{\partial x}\psi_2(0) \Rightarrow A - B = \frac{k_2}{k_1}C$

A Simple Potential Step

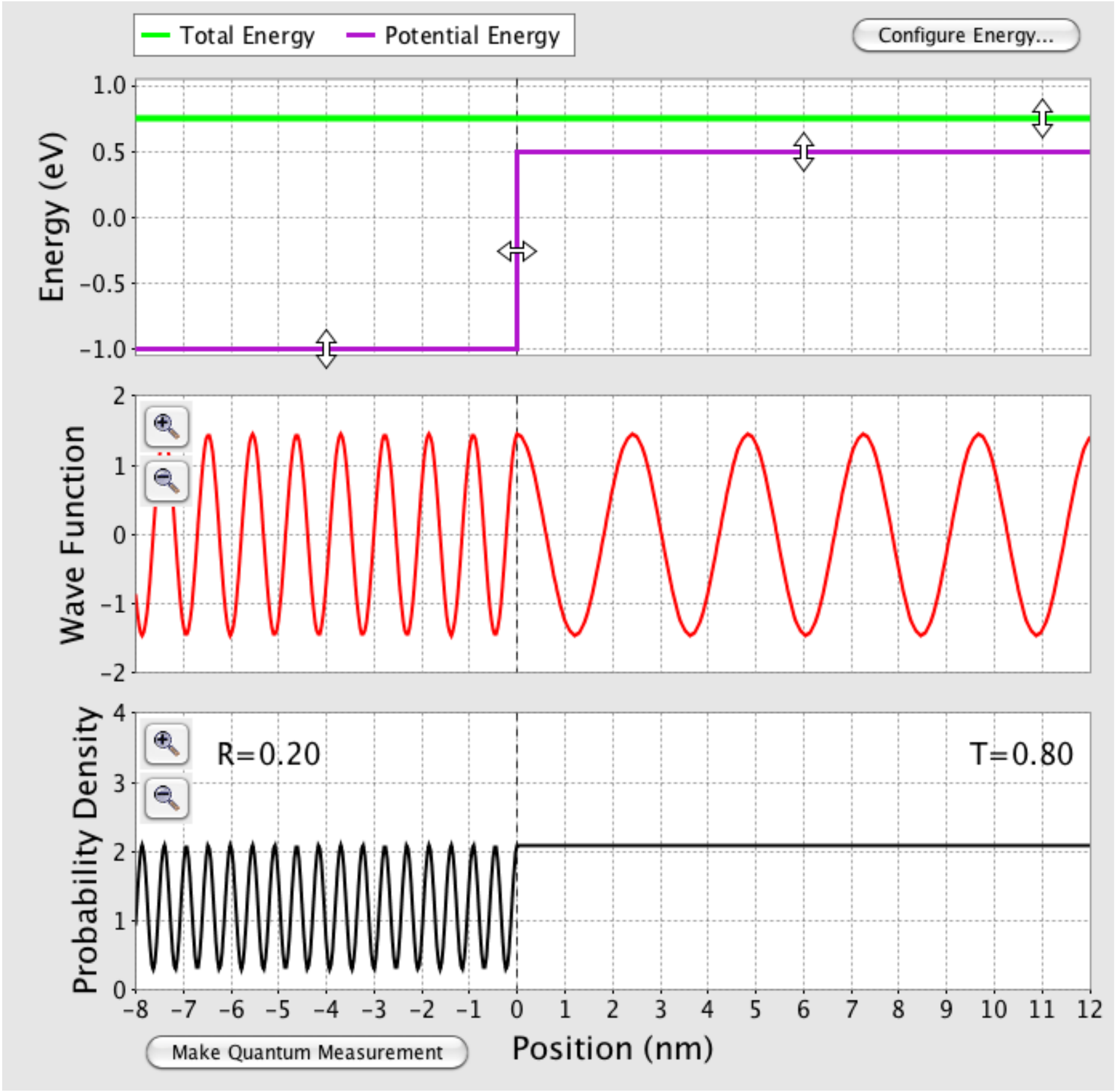
CASE I : $E_o > V$



$$\begin{cases} A + B = C \\ A - B = \dfrac{k_2}{k_1} C \end{cases}$$

$$\frac{B}{A} = \frac{1 - k_2/k_1}{1 + k_2/k_1} = \frac{k_1 - k_2}{k_1 + k_2}$$

$$\frac{C}{A} = \frac{2}{1 + k_2/k_1} = \frac{2k_1}{k_1 + k_2}$$



Example from: http://phet.colorado.edu/en/get-phet/one-at-a-time

Quantum Electron Currents

Given an electron of mass m

that is located in space with charge density $\rho = q\left|\psi(x)\right|^2$

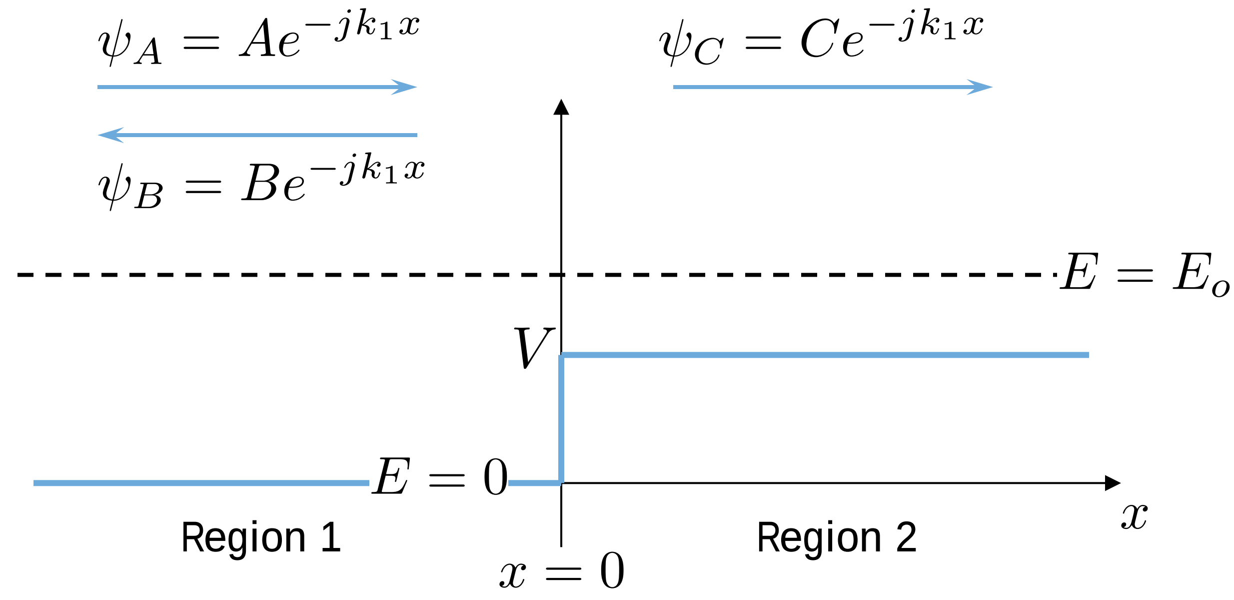
and moving with momentum $< p >$ corresponding to $< v >= \hbar k/m$

... then the current density for a *single electron* is given by

$$\boxed{J = \rho v = q\left|\psi\right|^2(\hbar k/m)}$$

A Simple Potential Step

CASE I : $E_o > V$



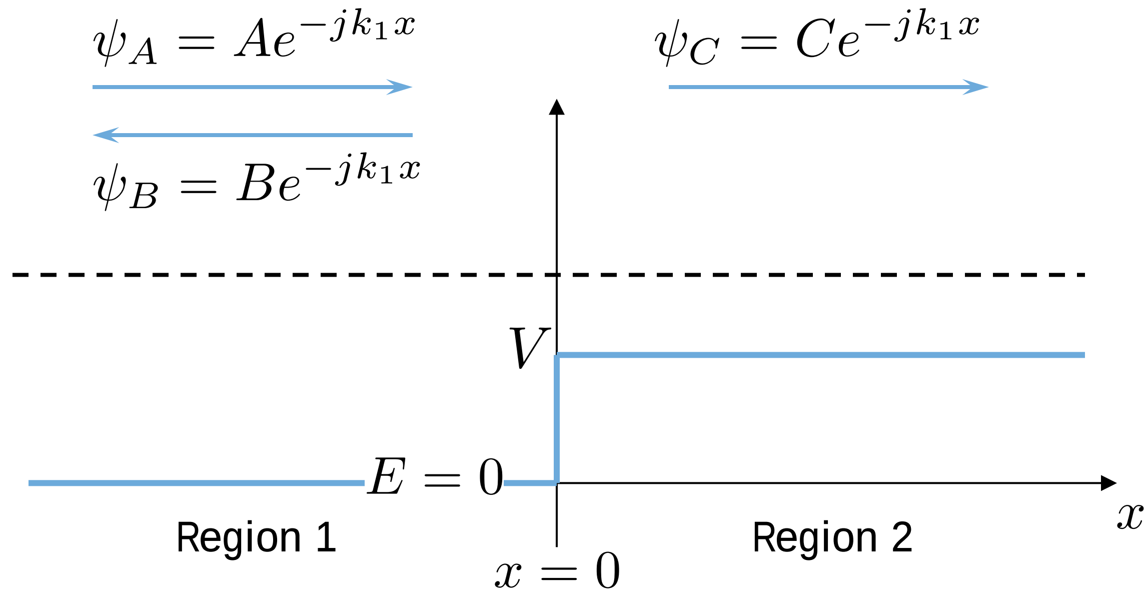
$$\text{Reflection} = R = \frac{J_{reflected}}{J_{incident}} = \frac{J_B}{J_A} = \frac{|\psi_B|^2(\hbar k_1/m)}{|\psi_A|^2(\hbar k_1/m)} = \left|\frac{B}{A}\right|^2$$

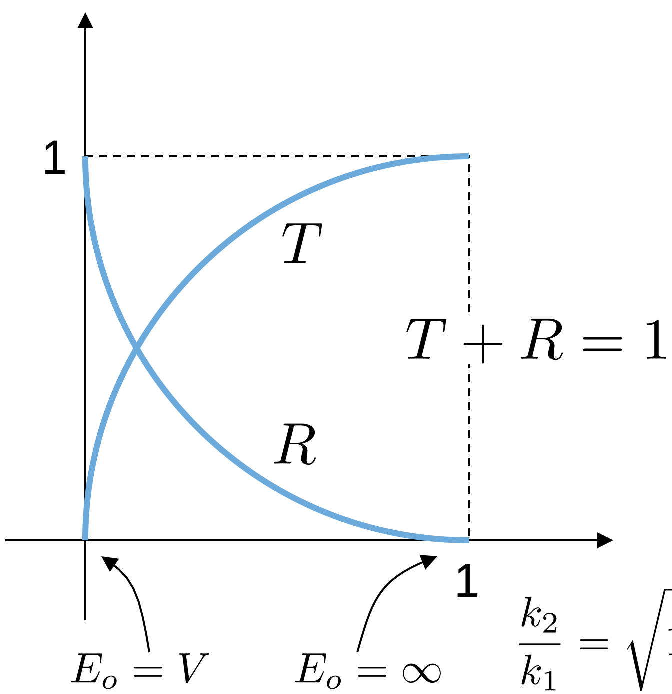
$$\text{Transmission} = T = \frac{J_{transmitted}}{J_{incident}} = \frac{J_C}{J_A} = \frac{|\psi_C|^2(\hbar k_2/m)}{|\psi_A|^2(\hbar k_1/m)} = \left|\frac{C}{A}\right|^2 \frac{k_2}{k_1}$$

$$\frac{B}{A} = \frac{1 - k_2/k_1}{1 + k_2/k_1} \qquad \frac{C}{A} = \frac{2}{1 + k_2/k_1}$$

A Simple Potential Step

CASE I : $E_o > V$



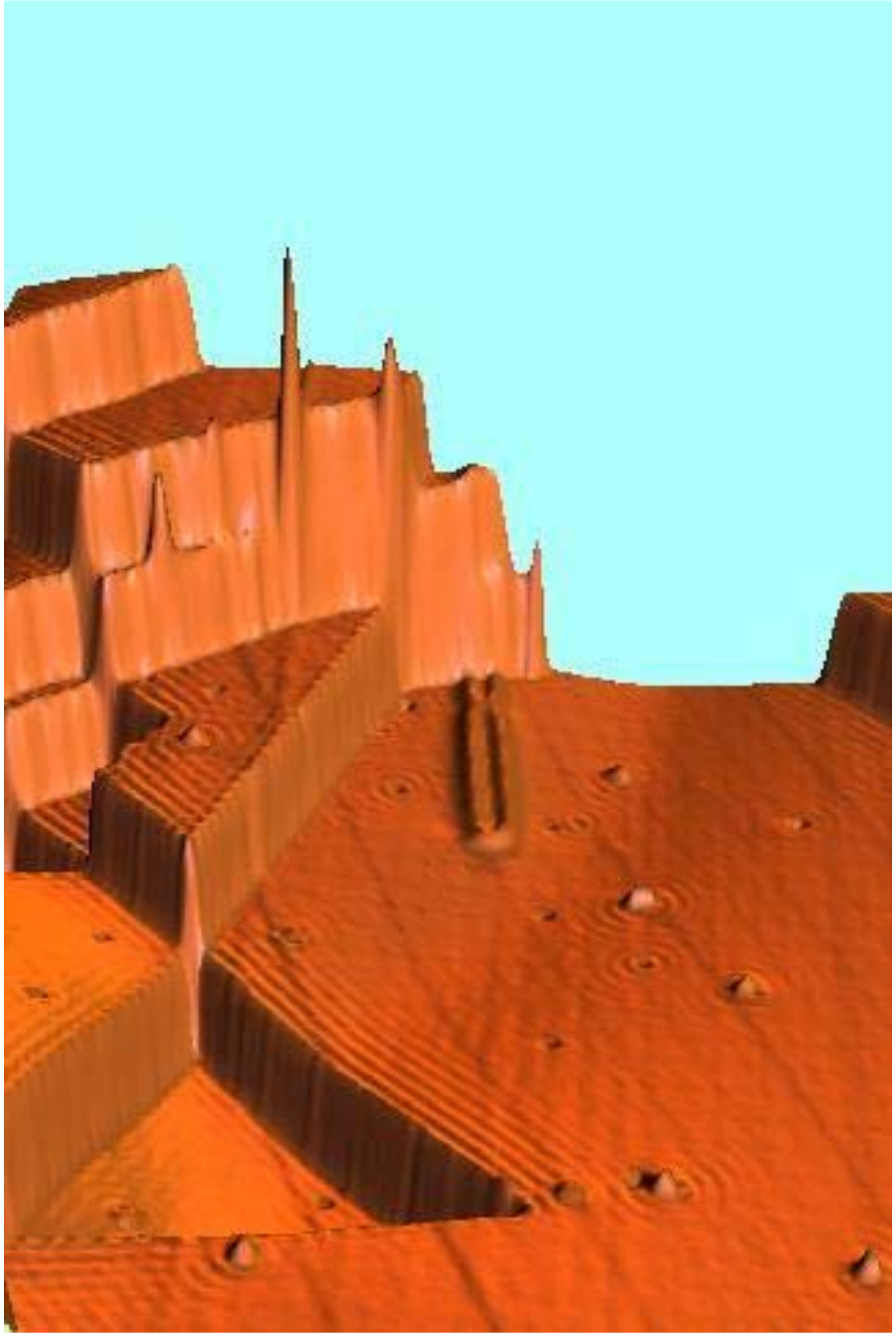


$$\frac{k_2}{k_1} = \sqrt{1 - \frac{V}{E_o}}$$

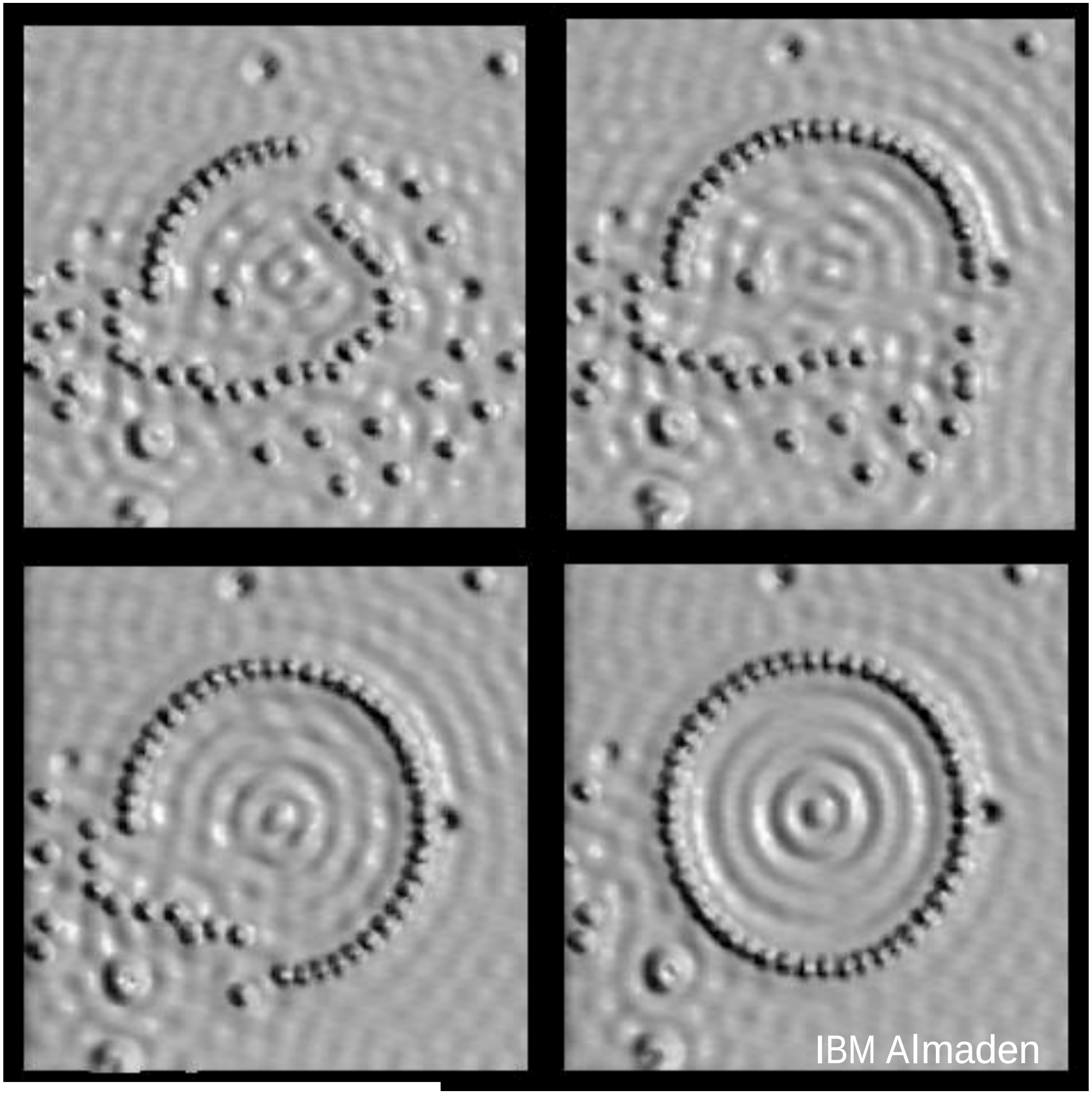
$$\text{Reflection} = R = \left|\frac{B}{A}\right|^2 = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2$$

$$\text{Transmission} = T = 1 - R$$

$$= \frac{4k_1k_2}{|k_1 + k_2|^2}$$




IBM Almaden STM of Copper
Image originally created by the IBM Corporation.



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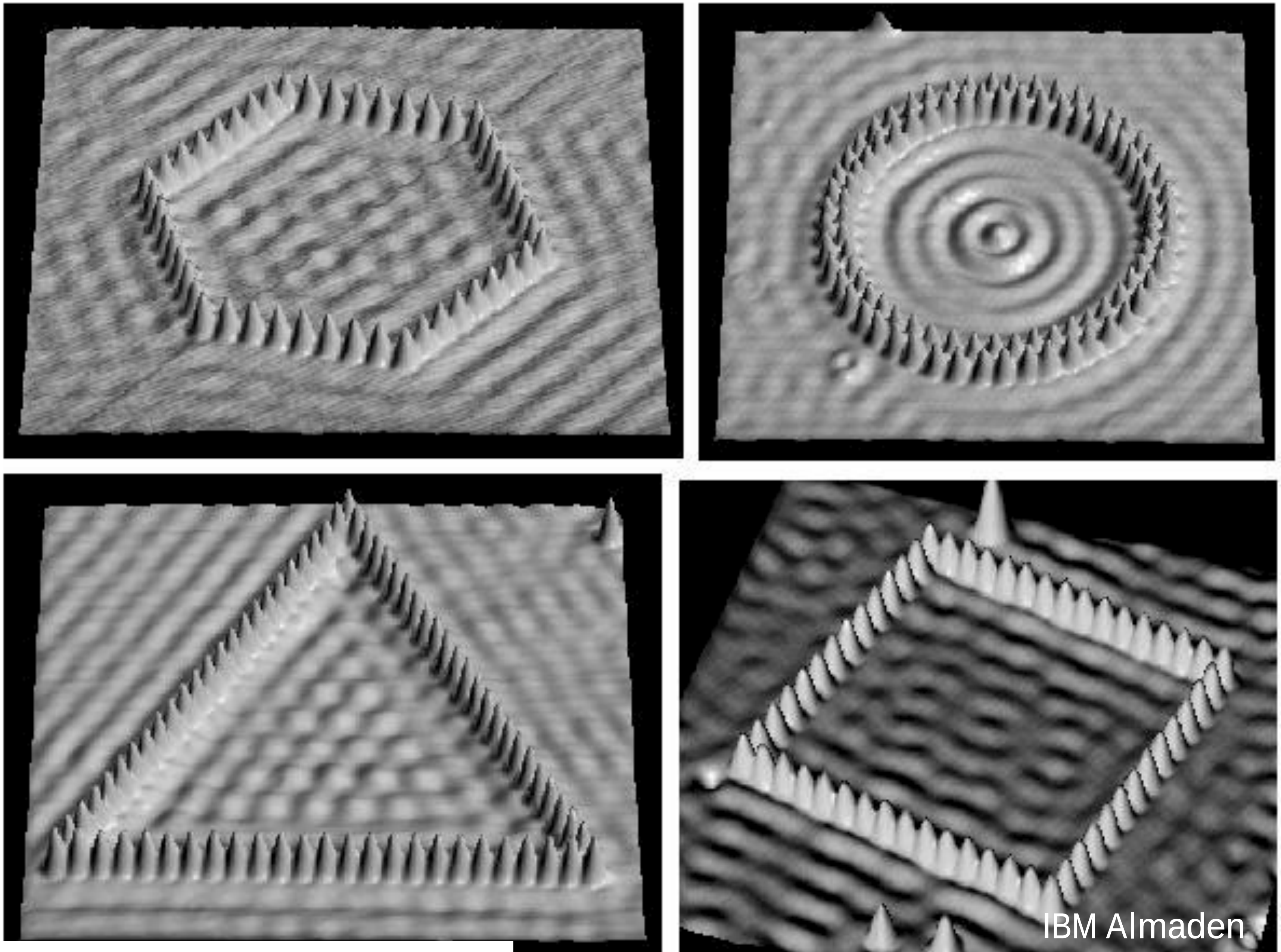

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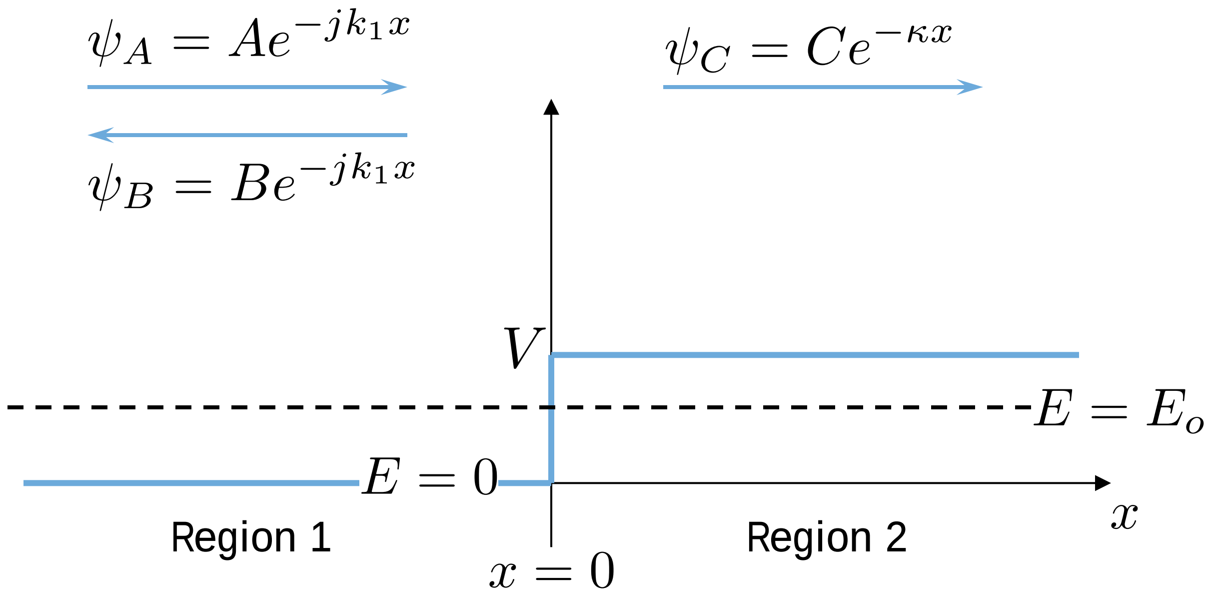
A Simple Potential Step

CASE II : $E_o < V$

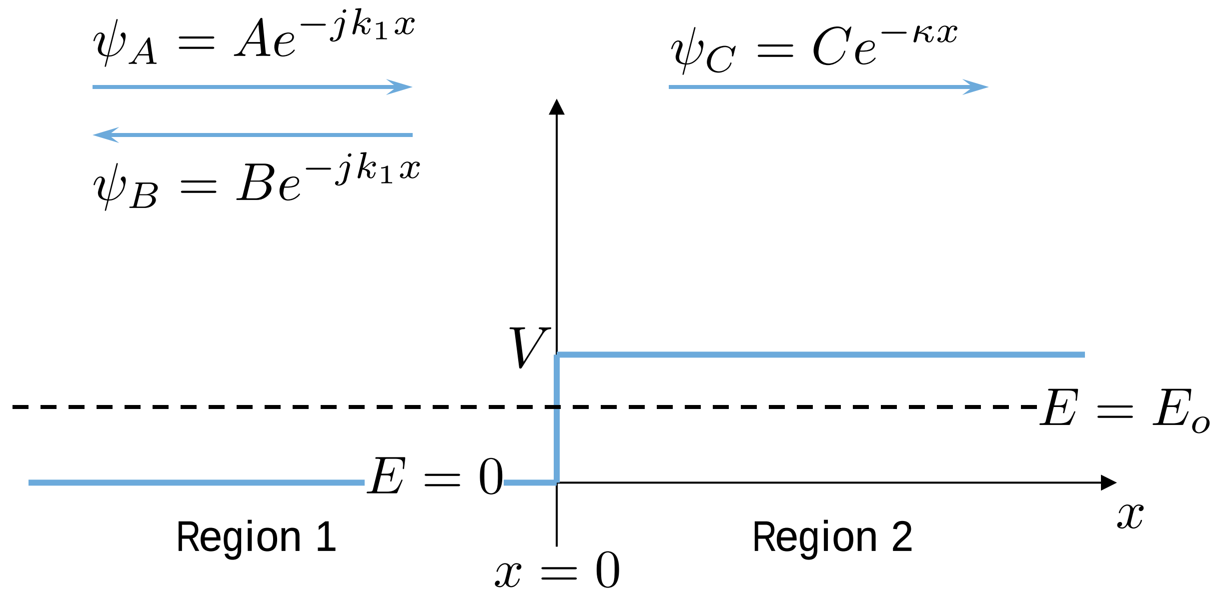


In Region 1: $$E_o\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \Rightarrow k_1^2 = \frac{2mE_o}{\hbar^2}$$

In Region 2: $$\left(E_o - V\right)\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \Rightarrow \kappa^2 = \frac{2m\left(E_o - V\right)}{\hbar^2}$$

A Simple Potential Step

CASE II : $E_o < V$



$$\psi_1 = Ae^{-jk_1x} + Be^{jk_1x} \qquad \psi_2 = Ce^{-\kappa x}$$

ψ is continuous: $\quad \psi_1(0) = \psi_2(0) \quad \Rightarrow \quad A + B = C$

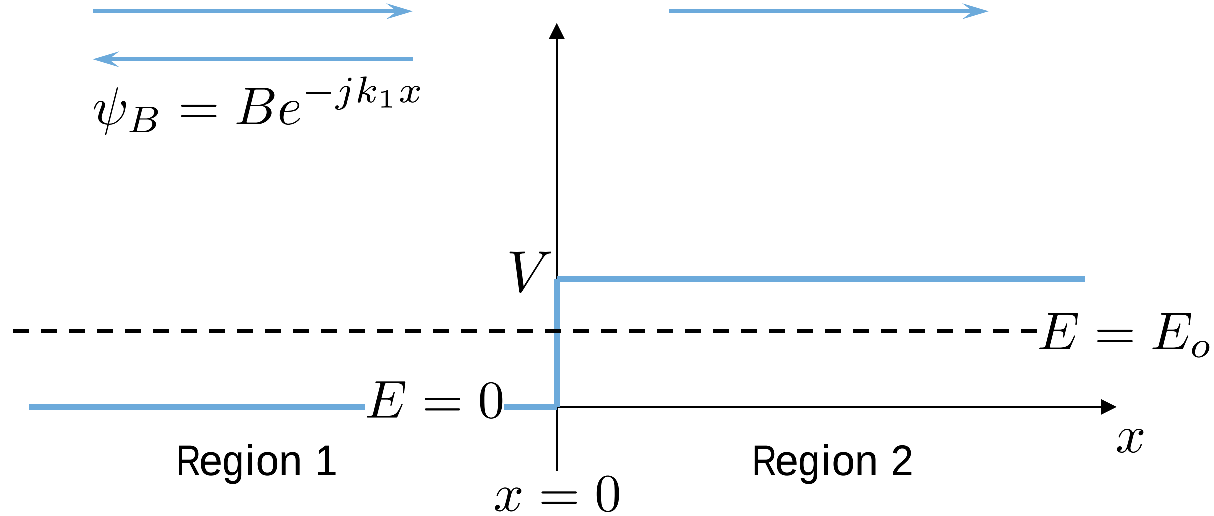
$\dfrac{\partial \psi}{\partial x}$ is continuous: $\quad \dfrac{\partial}{\partial x}\psi(0) = \dfrac{\partial}{\partial x}\psi_2(0) \quad \Rightarrow \quad A - B = -j\dfrac{\kappa}{k_1}C$

A Simple Potential Step

$$\psi_A = Ae^{-jk_1 x} \qquad \psi_C = Ce^{-\kappa x}$$

$$\psi_B = Be^{-jk_1 x}$$

CASE II : $E_o < V$



$$\begin{cases} A + B = C \\ A - B = -j\dfrac{\kappa}{k_1}C \end{cases}$$

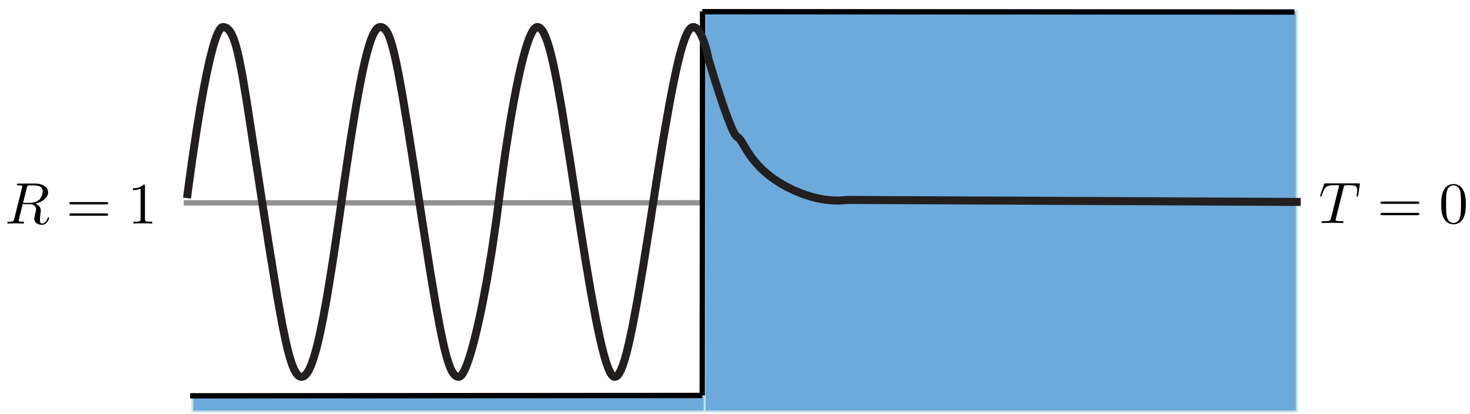
$$\frac{B}{A} = \frac{1 + j\kappa/k_1}{1 - j\kappa/k_1} \qquad \frac{C}{A} = \frac{2}{1 - j\kappa/k_1}$$

$$\boxed{R = \left|\frac{B}{A}\right|^2 = 1 \qquad T = 0}$$

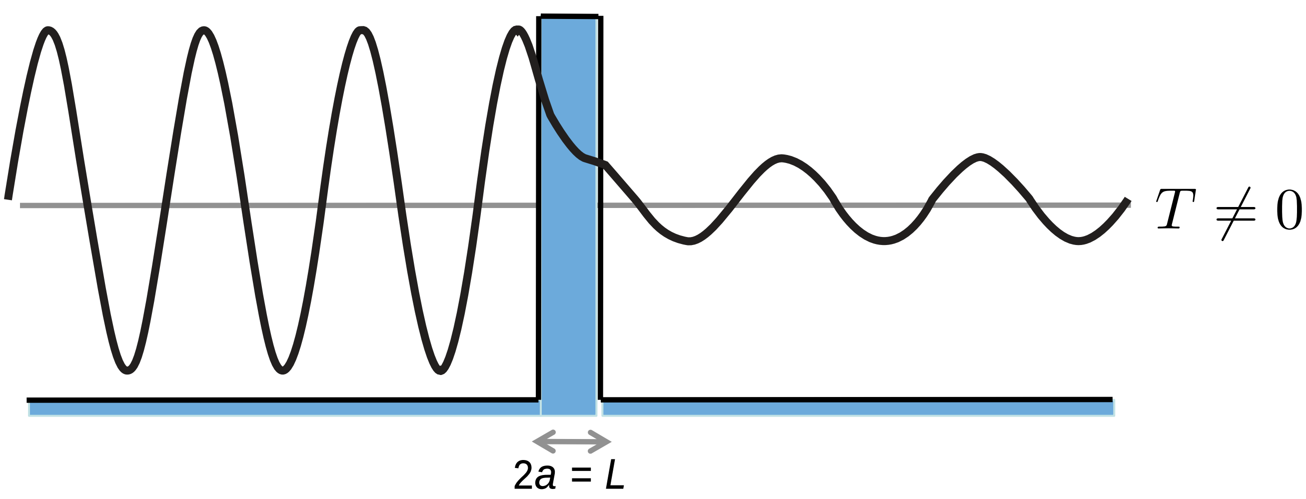
Total reflection → Transmission must be zero

Quantum Tunneling Through a Thin Potential Barrier

Total Reflection at Boundary

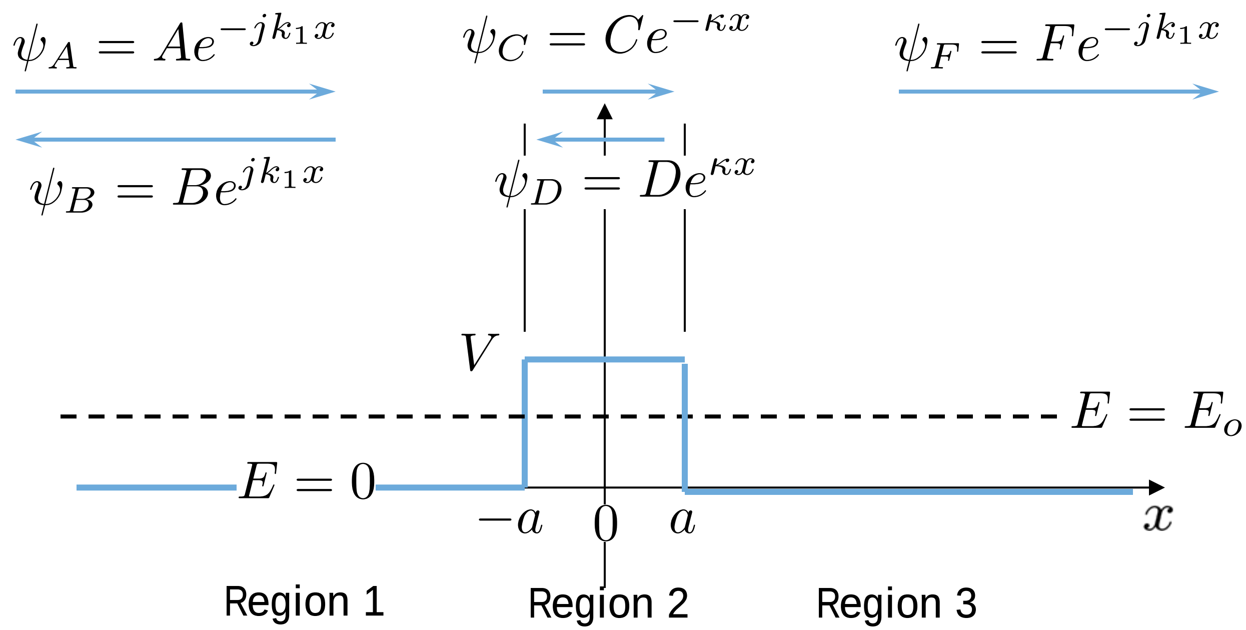


Frustrated Total Reflection (Tunneling)



A Rectangular Potential Step

CASE II : $E_o < V$



In Regions 1 and 3: $$E_o\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \Rightarrow k_1^2 = \frac{2mE_o}{\hbar^2}$$

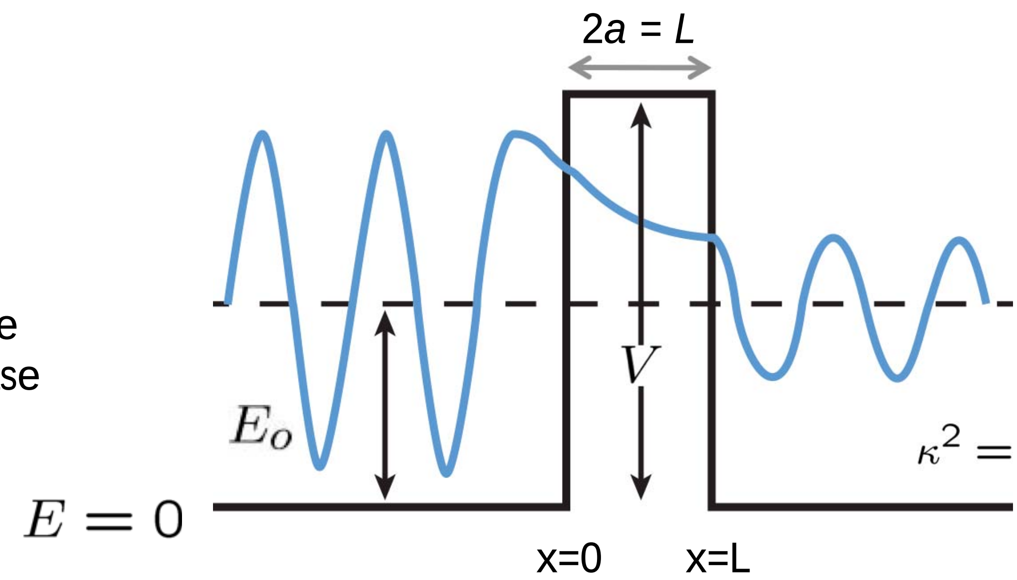
In Region 2: $$(E_o - V)\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \Rightarrow \kappa^2 = \frac{2m(V - E_o)}{\hbar^2}$$

for $E_o < V$:

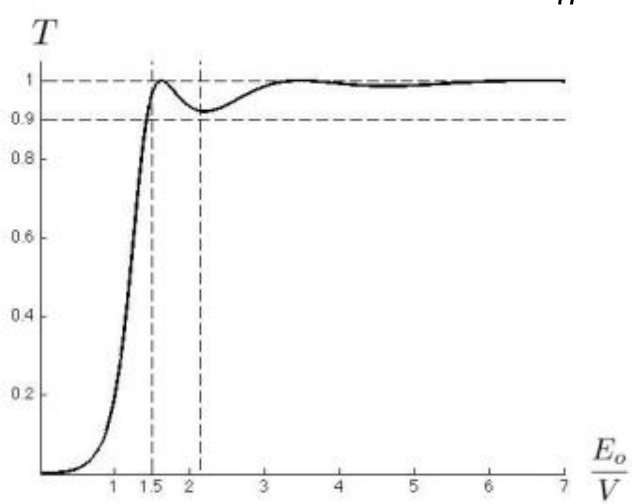
$$\boxed{T = \left|\frac{F}{A}\right|^2 = \frac{1}{1 + \frac{1}{4}\frac{V^2}{E_o(V-E_o)}\sinh^2(2\kappa a)}}$$

A Rectangular Potential Step

Real part of Ψ for $E_o < V$, shows hyperbolic (exponential) decay in the barrier domain and decrease in amplitude of the transmitted wave.



$$\kappa^2 = \frac{2m\,(V - E_o)}{\hbar^2}$$



Transmission Coefficient versus E_o/V for barrier with $2m(2a)^2V/\hbar = 16$

for $E_o < V$:

$$\boxed{T = \left|\frac{F}{A}\right|^2 = \frac{1}{1 + \frac{1}{4}\frac{V^2}{E_o(V-E_o)}\sinh^2(2\kappa a)}}$$

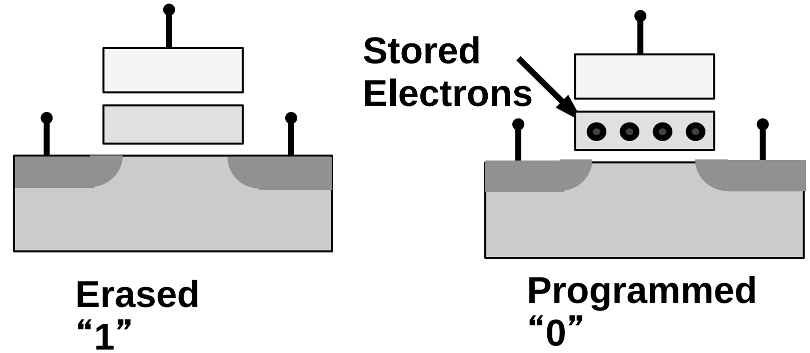
$$\sinh^2(2\kappa a) = \left[e^{2\kappa a} - e^{-2\kappa a}\right]^2 \approx e^{-4\kappa a}$$

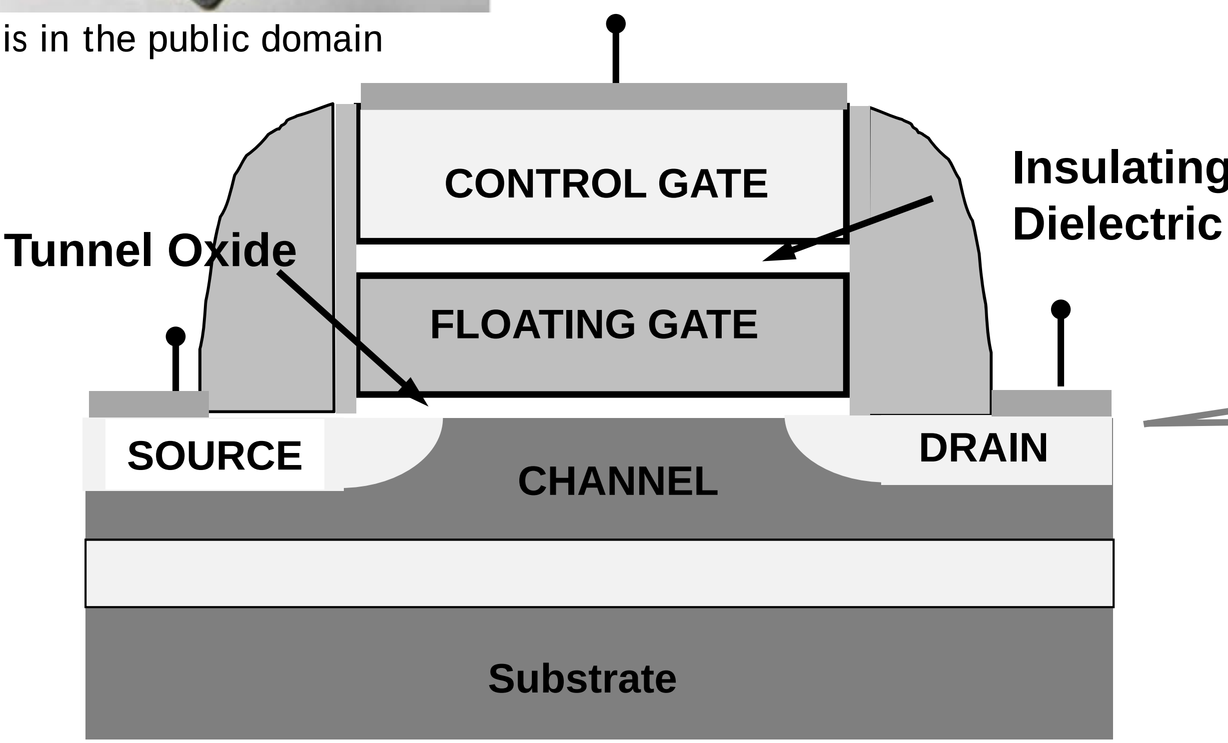
$$\boxed{T = \left|\frac{F}{A}\right|^2 \approx \frac{1}{1 + \frac{1}{4}\frac{V^2}{E_o(V-E_o)}} e^{-4\kappa a}}$$

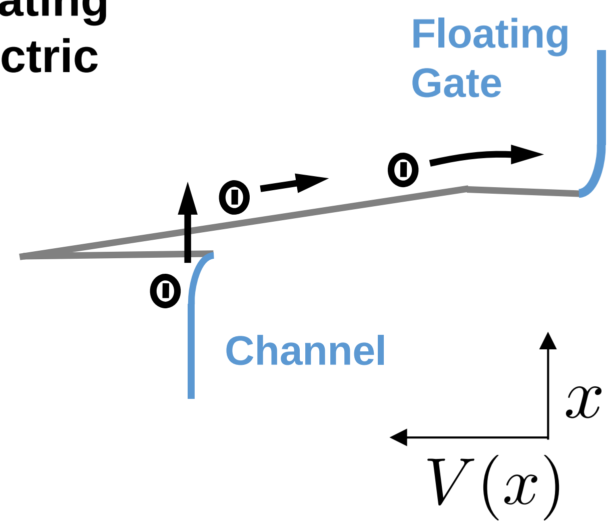
Tunneling Applet: **http://www.colorado.edu/physics/phet/dev/quantum-tunneling/1.07.00/**

Flash Memory

Image is in the public domain







Electrons tunnel preferentially when a voltage is applied

MOSFET: Transistor in a Nutshell

Conduction electron flow

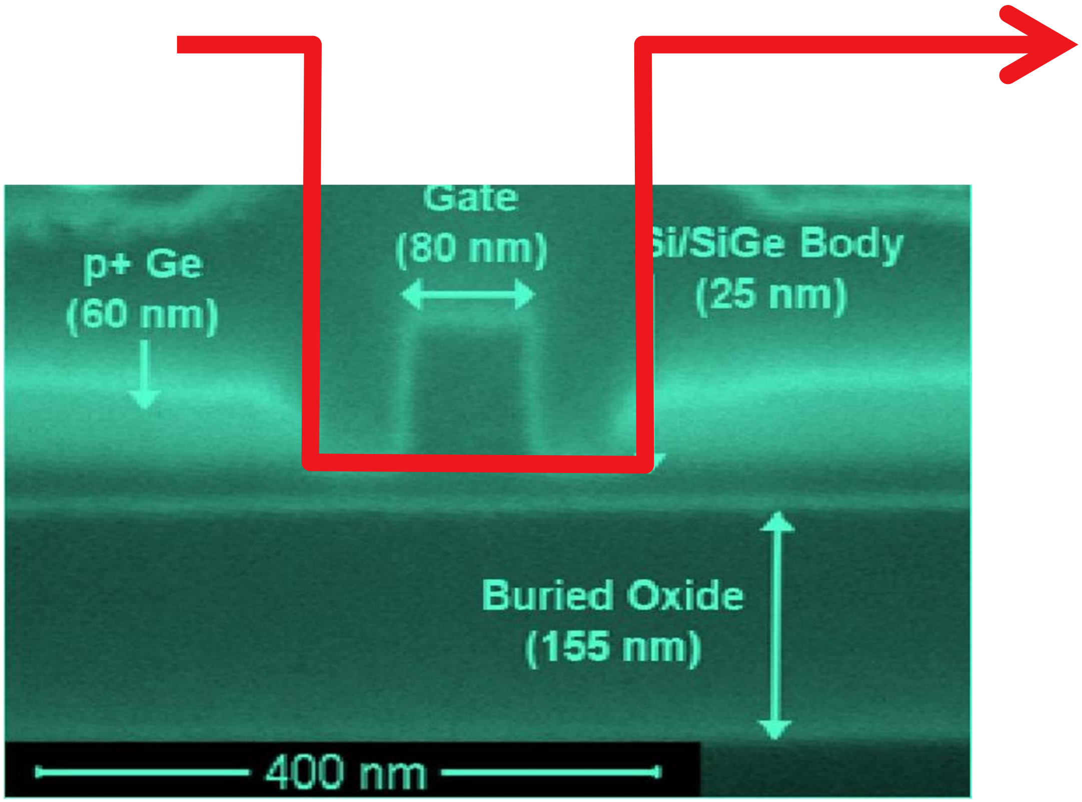


Image courtesy of J. Hoyt Group, EECS, MIT. Photo by L. Gomez

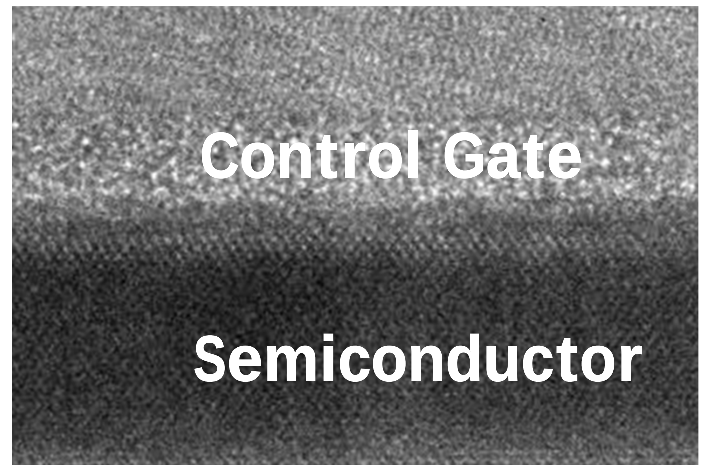


Image courtesy of J. Hoyt Group, EECS, MIT. Photo by L. Gomez

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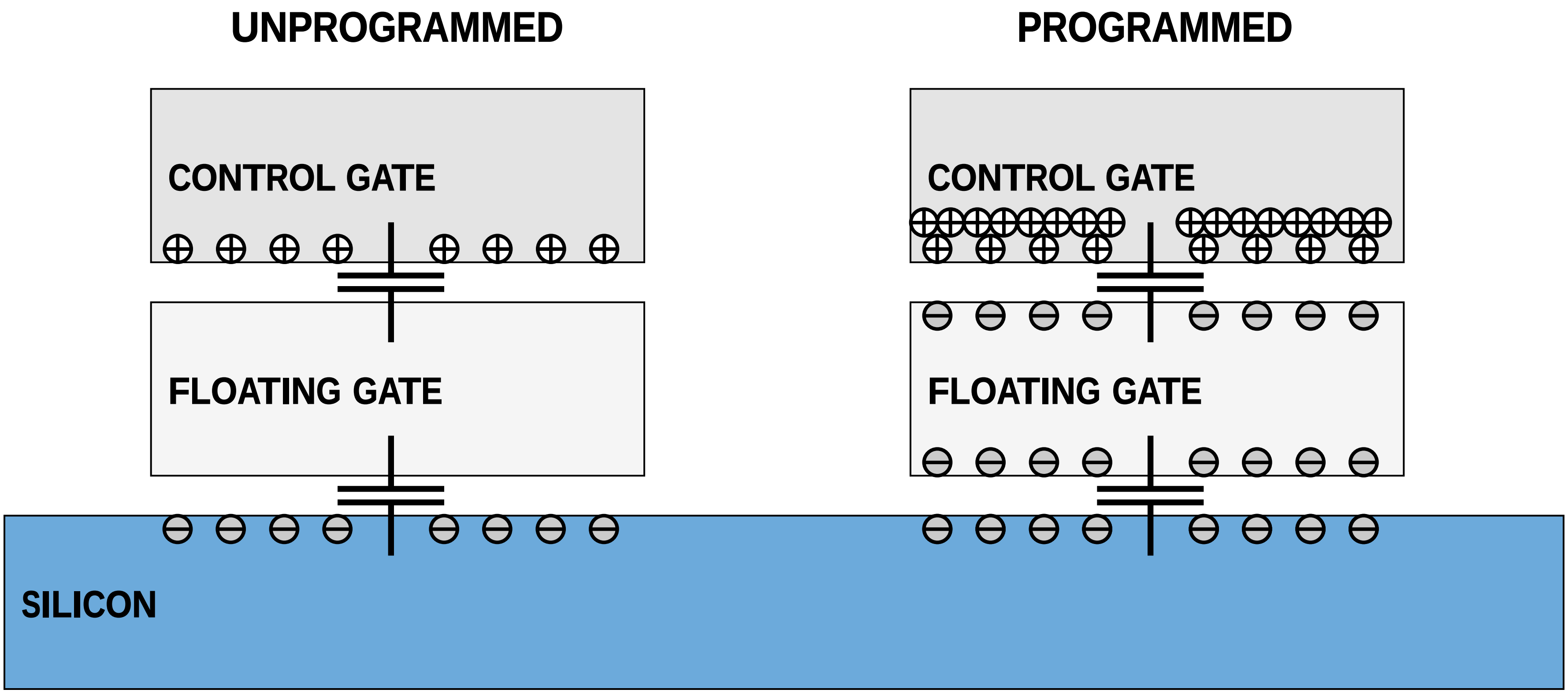
Tunneling causes thin insulating layers to become leaky !

Reading Flash Memory



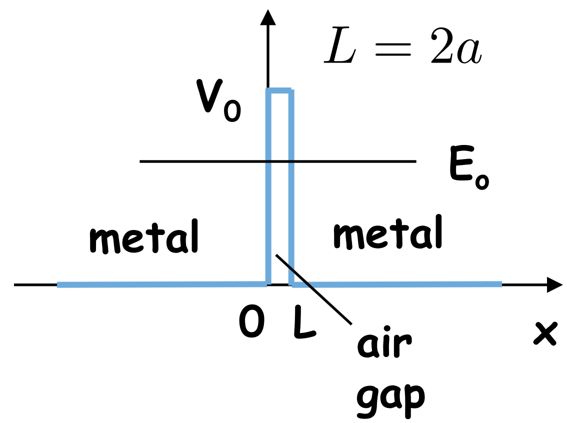
To obtain the same channel charge, the programmed gate needs a higher control-gate voltage than the unprogrammed gate

How do we WRITE Flash Memory ?

Example: Barrier Tunneling

- Let's consider a tunneling problem:

An electron with a total energy of $E_o = 6$ eV approaches a potential barrier with a height of $V_0 = 12$ eV. If the width of the barrier is $L = 0.18$ nm, what is the probability that the electron will tunnel through the barrier?



$$T = \left|\frac{F}{A}\right|^2 \approx \frac{16E_o(V - E_o)}{V^2} e^{-2\kappa L}$$

$$\kappa = \sqrt{\frac{2m_e}{\hbar^2}(V - E_o)} = 2\pi\sqrt{\frac{2m_e}{h^2}(V - E_o)} = 2\pi\sqrt{\frac{6\text{eV}}{1.505\text{eV-nm}^2}} \approx 12.6 \text{ nm}^{-1}$$

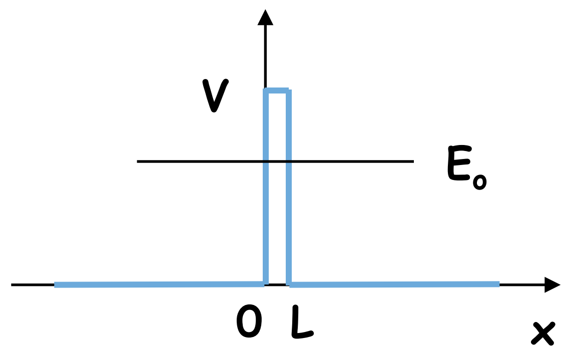
$$T = 4e^{-2(12.6 \text{ nm}^{-1})(0.18 \text{ nm})} = 4(0.011) = \boxed{4.4\%}$$

Question: What will T be if we double the width of the gap?

Multiple Choice Questions

Consider a particle tunneling through a barrier:

1. Which of the following will increase the likelihood of tunneling?



a. decrease the height of the barrier
b. decrease the width of the barrier
c. decrease the mass of the particle

2. What is the energy of the particles that have successfully "escaped"?

a. < initial energy
b. = initial energy
c. > initial energy

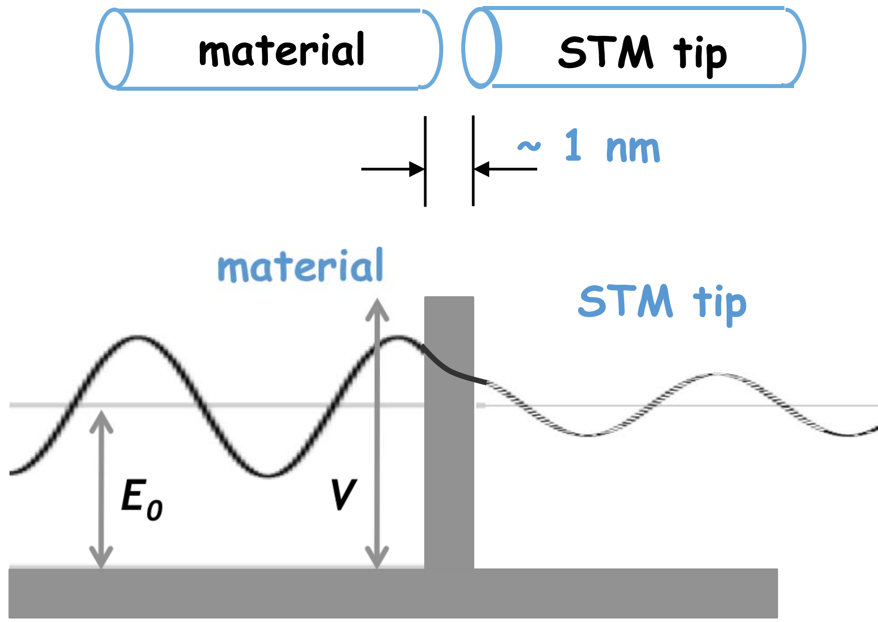
Although the *amplitude* of the wave is smaller after the barrier, no energy is lost in the tunneling process

Application of Tunneling:

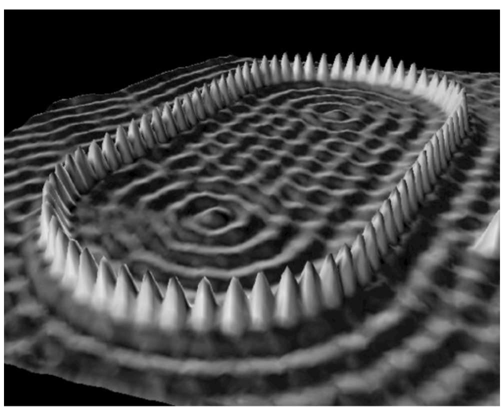
Scanning Tunneling Microscopy (STM)

Due to the quantum effect of "barrier penetration," the electron density of a material extends beyond its surface:

One can exploit this to measure the electron density on a material's surface:



Sodium atoms on metal:



STM images

Image originally created by IBM Corporation

Single walled carbon nanotube:

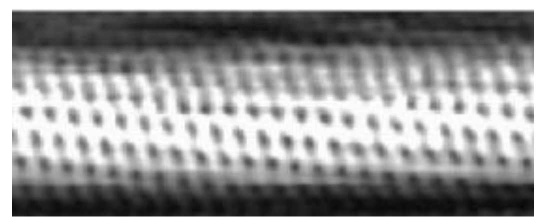


Image is in the public domain

Reflection of EM Waves and QM Waves

$$P = \hbar\omega \times \frac{\text{photons}}{\text{s cm}^2}$$

$$P = \frac{|E|^2}{\eta}$$

$$R = \frac{P_{reflected}}{P_{incident}} = \left|\frac{E_o^r}{E_o^i}\right|^2$$

Then for optical material when $\mu=\mu_0$:

$$R = \left|\frac{B}{A}\right|^2 = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2$$

$$= \left|\frac{n_1 + n_2}{n_1 + n_2}\right|^2$$

= probability of a particular photon being reflected

$$J = q \times \frac{\text{electrons}}{\text{s cm}^2}$$

$$J = \rho v = q\,|\psi|^2\,(\hbar k/m)$$

$$R = \frac{J_{reflected}}{J_{incident}} = \frac{|\psi_B|^2}{|\psi_A|^2}$$

$$R = \left|\frac{B}{A}\right|^2 = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2$$

= probability of a particular electron being reflected