Refraction and Snell's Law

Reading - Shen and Kong – Ch. 4

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Willebrord Snellius (1580–1626) was a Dutch astronomer and mathematician

Outline

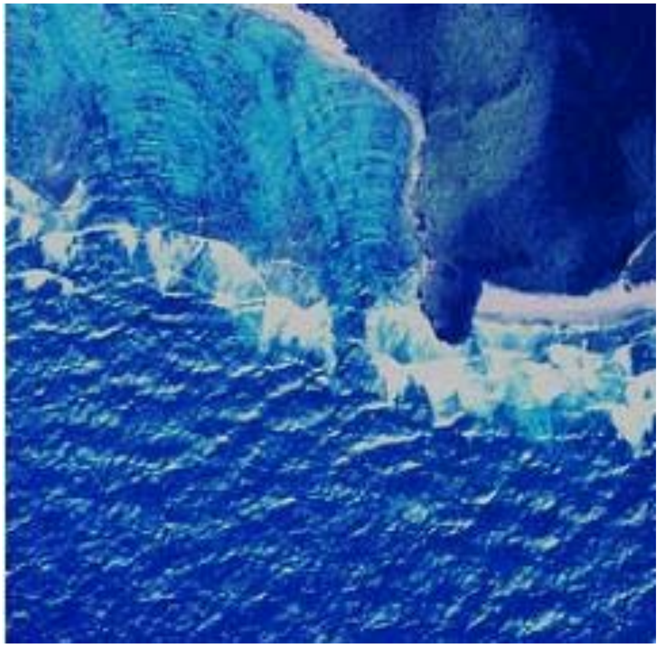
- TE and TM fields
- Refraction and Snell's Law:
  - From TE analysis
  - From Phase Matching
  - From Fermat's Principle of Least Time
- Total Internal Reflection and Fibers
- FIOS

Refraction

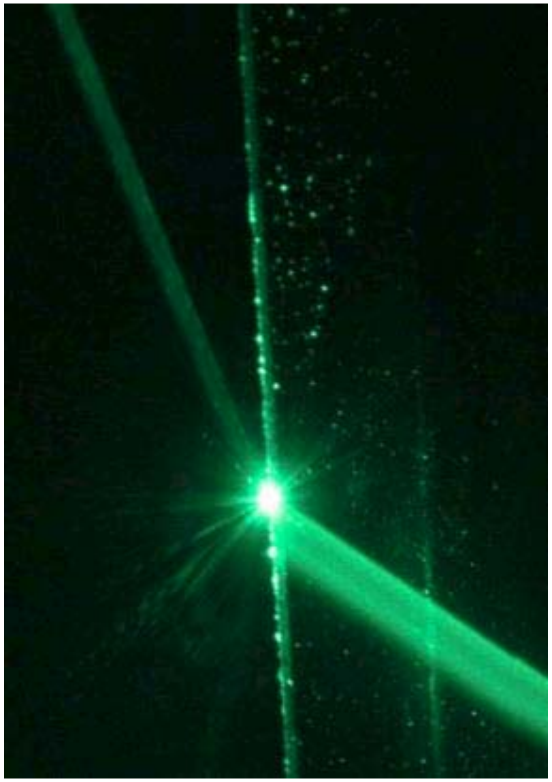
Water Waves

E&M Waves

Image by NOAA Photo library http://www.flickr.com/photos/noaaphotolib/5179052751/ on flickr

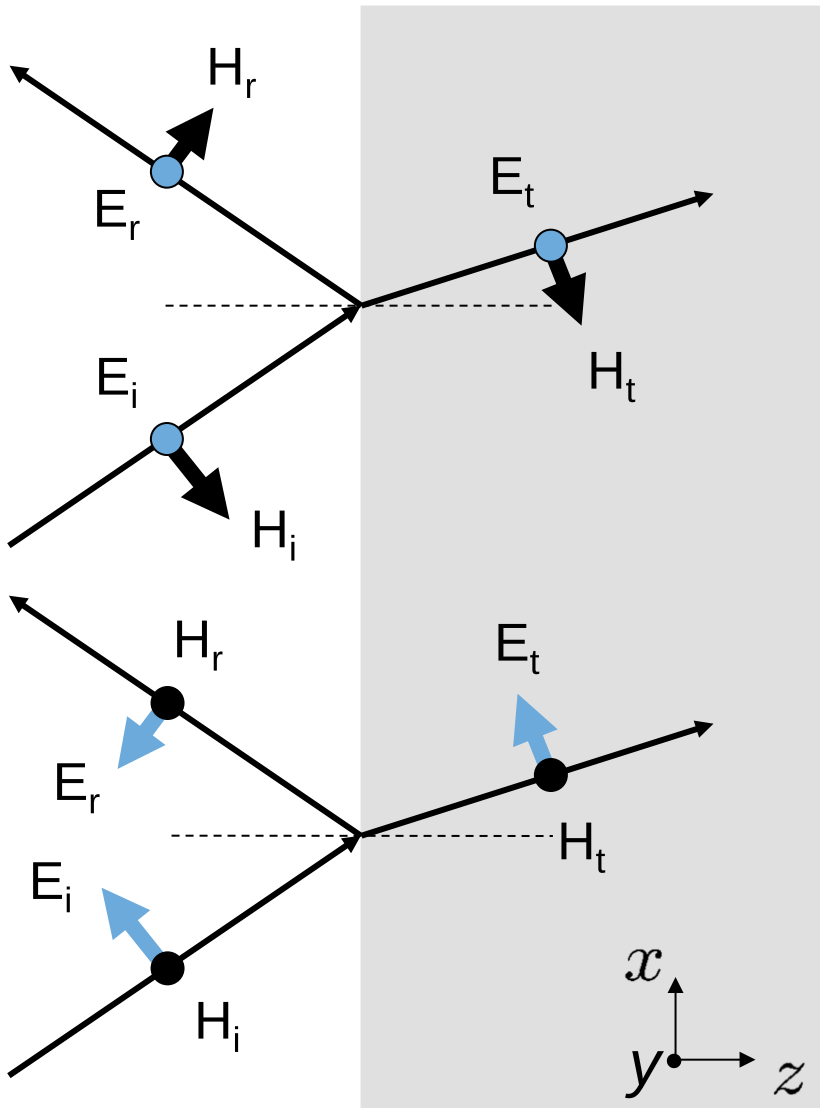


Waves refract at the top where the water is shallower



Refraction involves a change in the direction of wave propagation due to a change in propagation speed. It involves the oblique incidence of waves on media boundaries, and hence wave propagation in at least two dimensions.

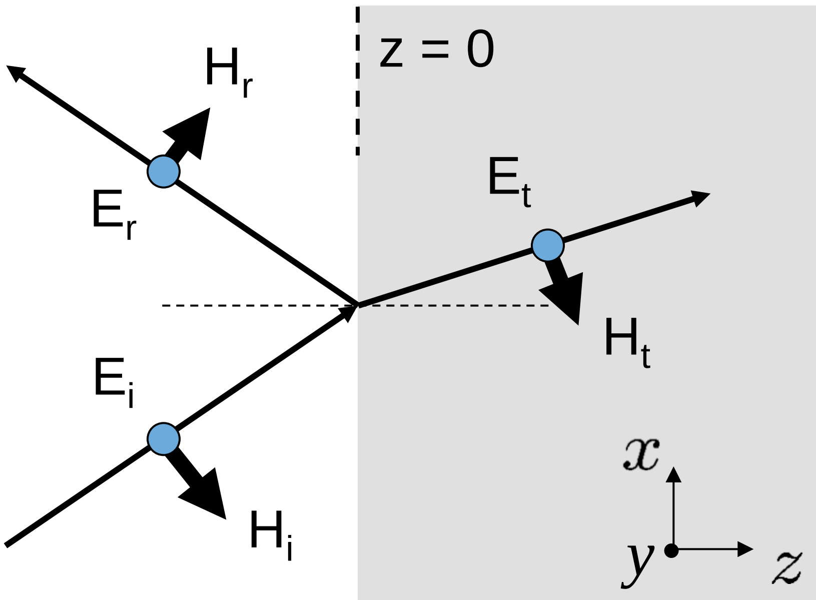
Oblique Incidence at Dielectric Interface



Transverse Electric Field

Transverse Magnetic Field

Partial TE Analysis



$$\vec{E}_i = \hat{y} E_o^i e^{-jk_{ix}x - jk_{iz}z}$$

$$\vec{E}_r = \hat{y} E_o^r e^{-jk_{rx}x + jk_{rz}z}$$

$$\vec{E}_t = \hat{y} E_o^t e^{-jk_{tx}x - jk_{tz}z}$$

$$\omega_i = \omega_r = \omega_t$$

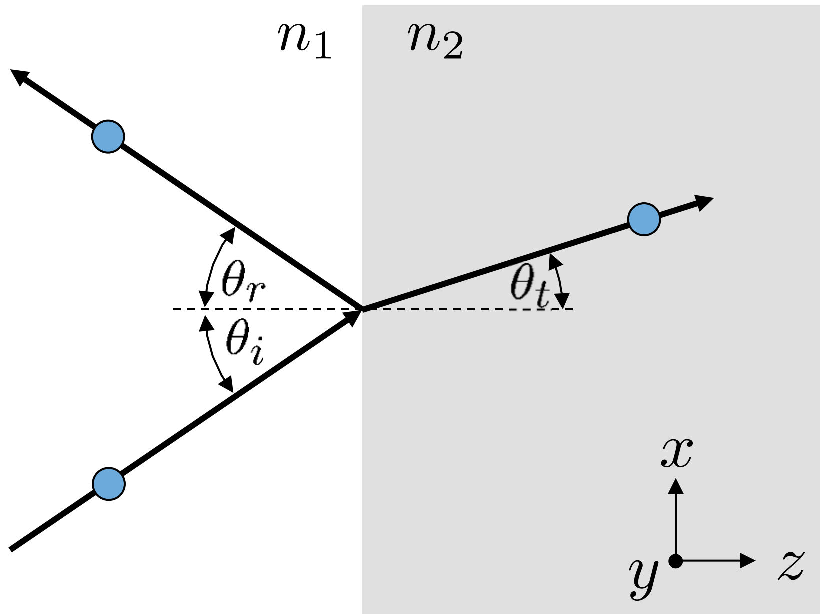
Tangential E must be continuous at the boundary <u>z = 0</u> for all x and for t.

$$E_o^i e^{-jk_{ix}} + E_o^r e^{-jk_{rx}x} = E_o^t e^{-jk_{tx}x}$$

This is possible if and only if $k_{ix} = k_{rx} = k_{tx}$ and $\omega_i = \omega_r = \omega_t$

The former condition is phase matching $k_{ix} = k_{rx} = k_{tx}$

Snell's Law



$$kix = k_{rx}$$

$$kix = k_{tx}$$

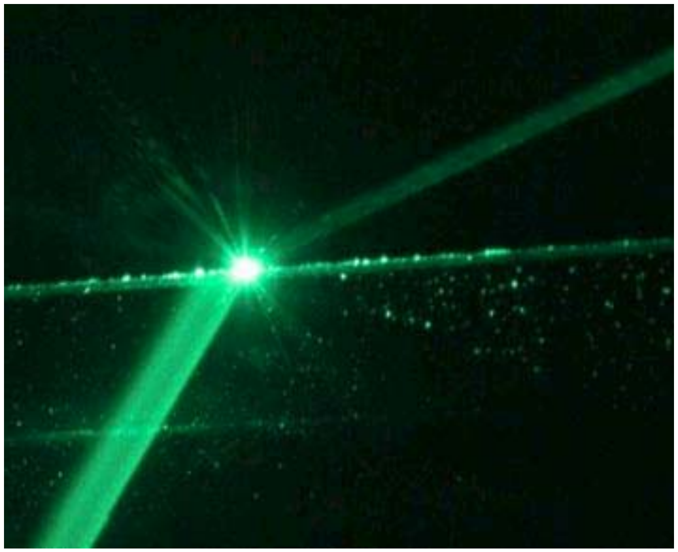
$$n_1 \sin\theta_i = n_1 \sin\theta_r$$

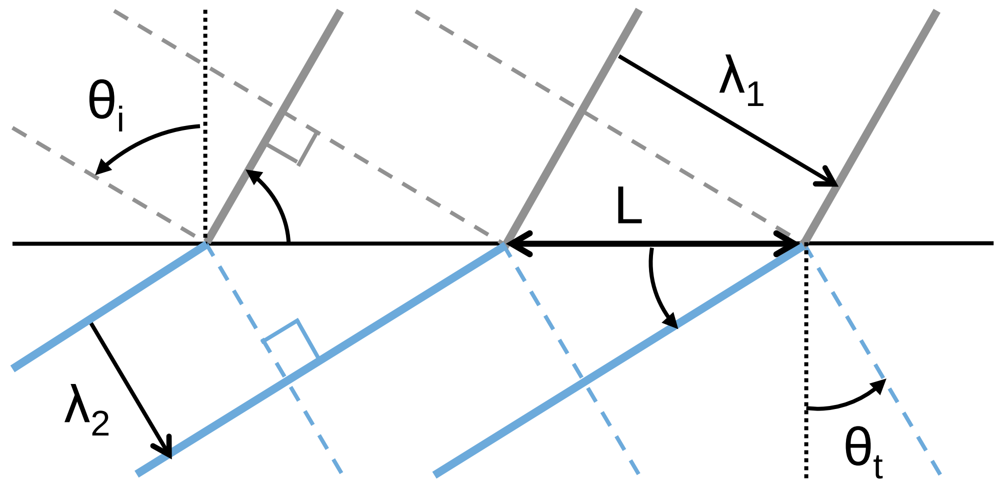
$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$\theta_i = \theta_r$$

SNELL'S LAW

Snells Law via Phase Matching





Following phase continuity, the phase-front separation L is common to both the incident and transmitted, or refracted, waves.

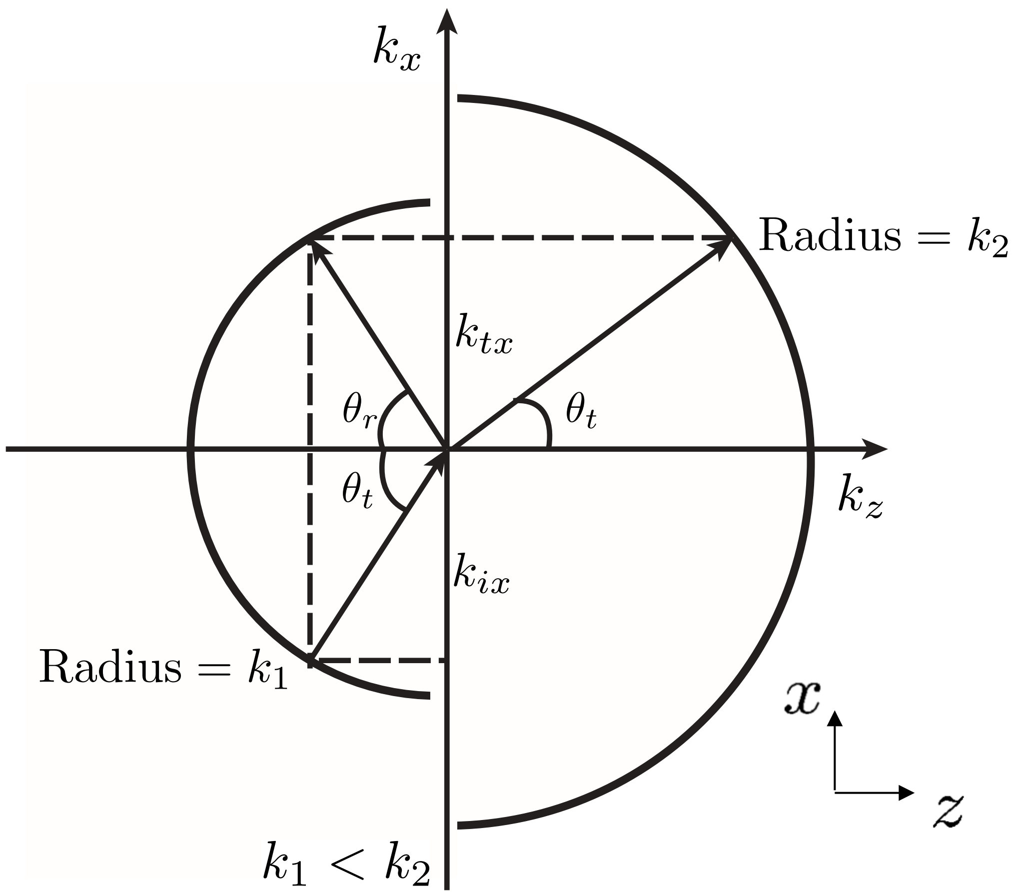
$$L \sin(\theta_i) = \lambda_1 = v_{p1}(2\pi/\omega) \qquad L \sin(\theta_t) = \lambda_2 = v_{p2}(2\pi/\omega)$$

$$\sin(\theta_1)/\sin(\theta_2) = v_{p1}/v_{p2} = n_2/n_1$$

Snell's Law Diagram

Tangential E field
is continuous ...

$$E_o^i e^{-jk_{ix}} + E_o^r e^{-jk_{rx}x} = E_o^t e^{-jk_{tx}x}$$

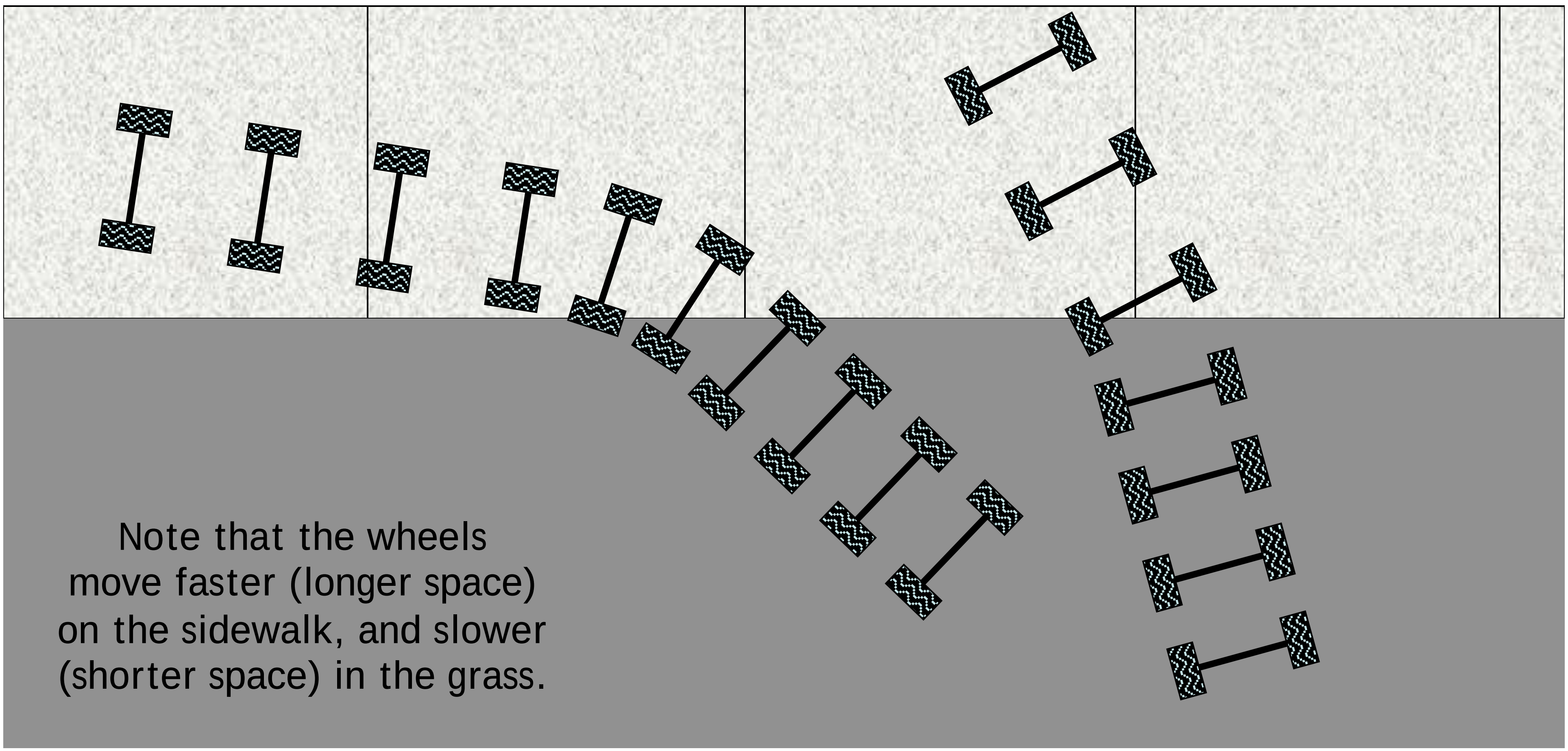


$$k_{ix} = k_{tx}$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

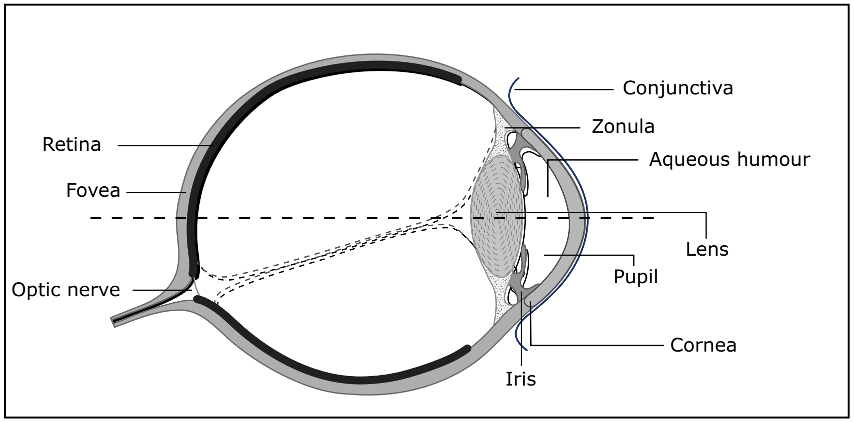
Refraction in Suburbia

Think of refraction as a pair of wheels on an axle going from a sidewalk onto grass. The wheel in the grass moves slower, so the direction of the wheel pair changes.



Note that the wheels move faster (longer space) on the sidewalk, and slower (shorter space) in the grass.

Snell's Law and Lenses



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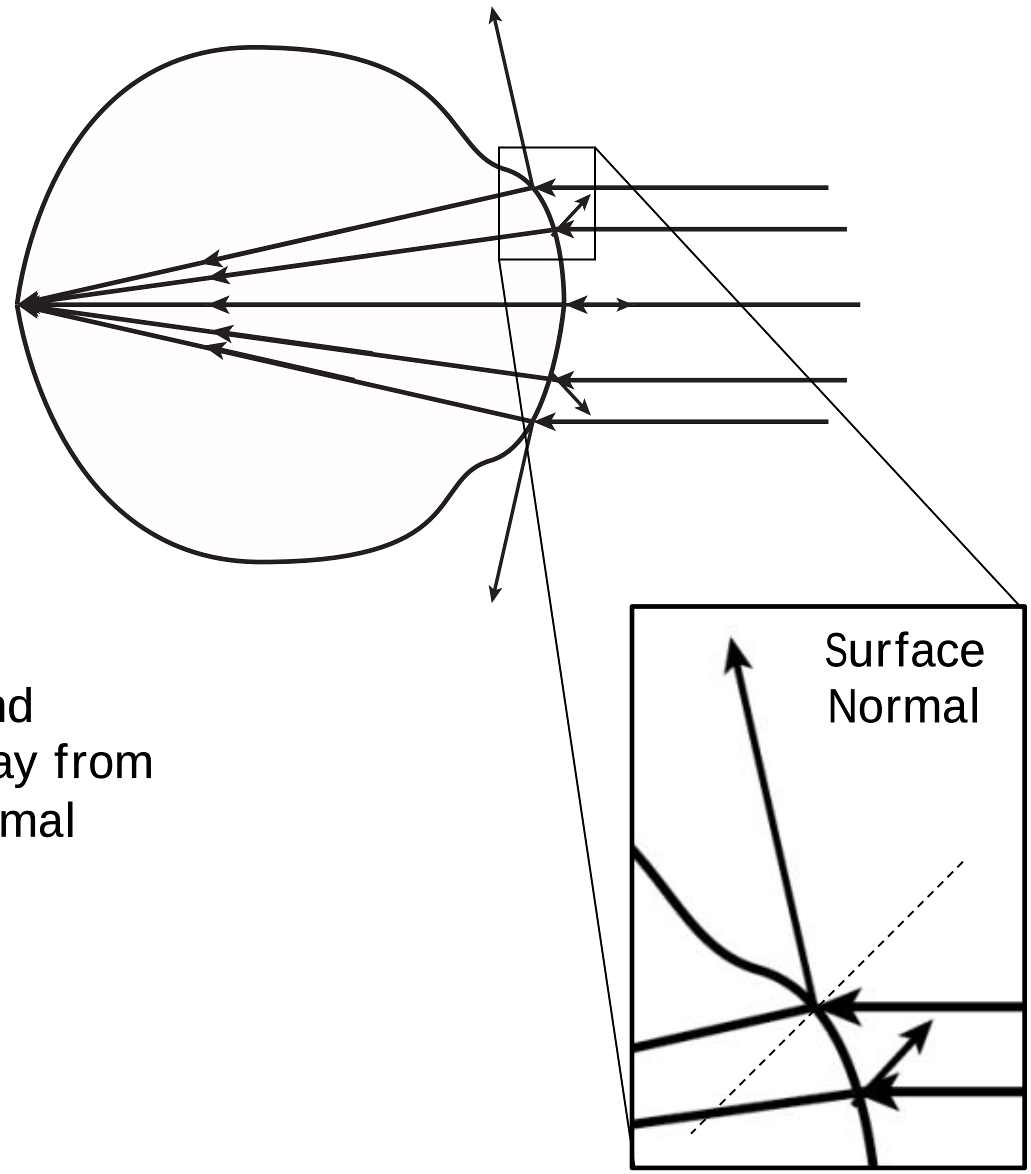
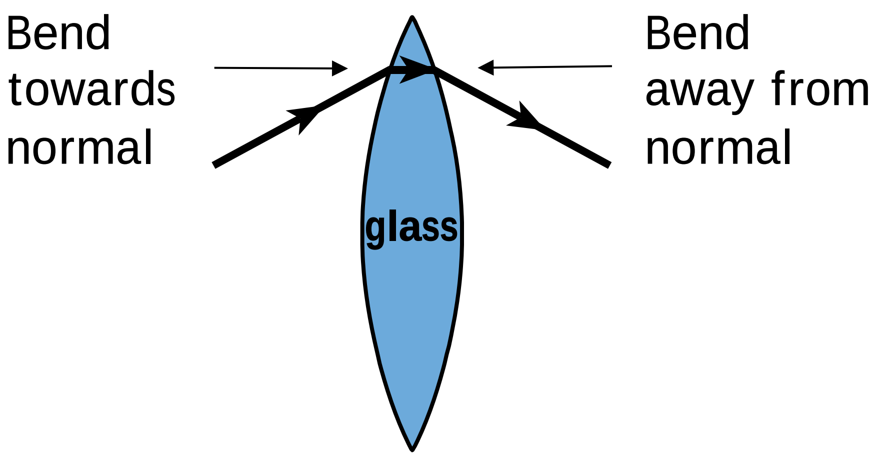
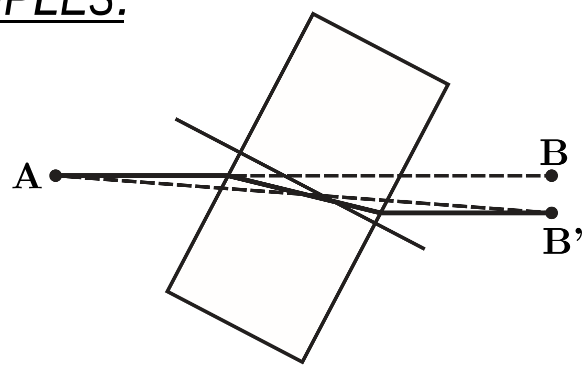




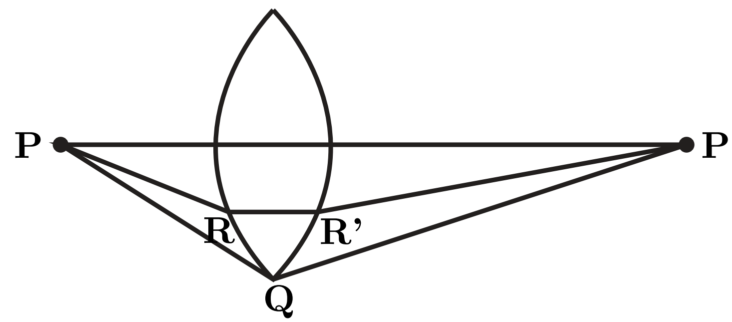
History of Snell's Law

- Snell's Law describing refraction was first recorded by Ptolemy in 140 A.D
- First described by relationship by Snellius in 1621
- First explained in 1650 by Fermat's principle of least time.

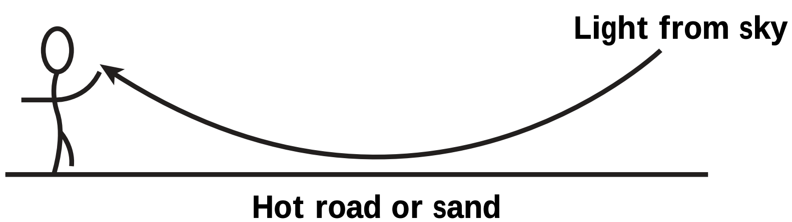
EXAMPLES:



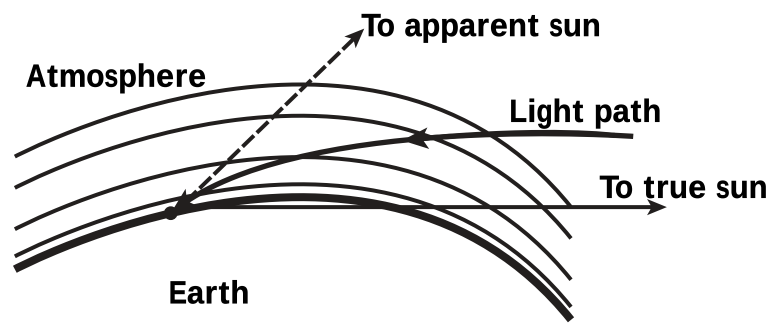
BEAM OF LIGHT IS OFFSET AS IT PASSES THROUGH A TRANSPARENT BLOCK



FOCUSING OPTICAL SYSTEM



MIRAGE

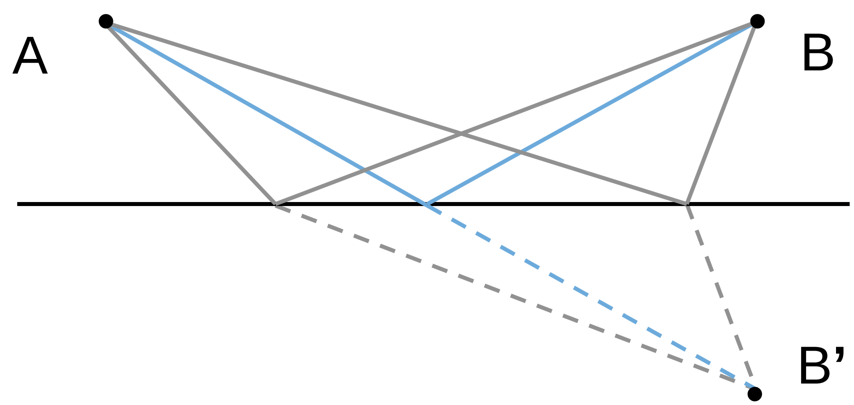


NEAR THE HORIZON, THE APPARENT SUN IS HIGHER THAN THE TRUE SUN BY ABOUT ½ DEGREE

Fermat's Principle of Least Time

Fermat's principle of minimum time argues that light will travel from one point to another along a path that requires the minimum time.

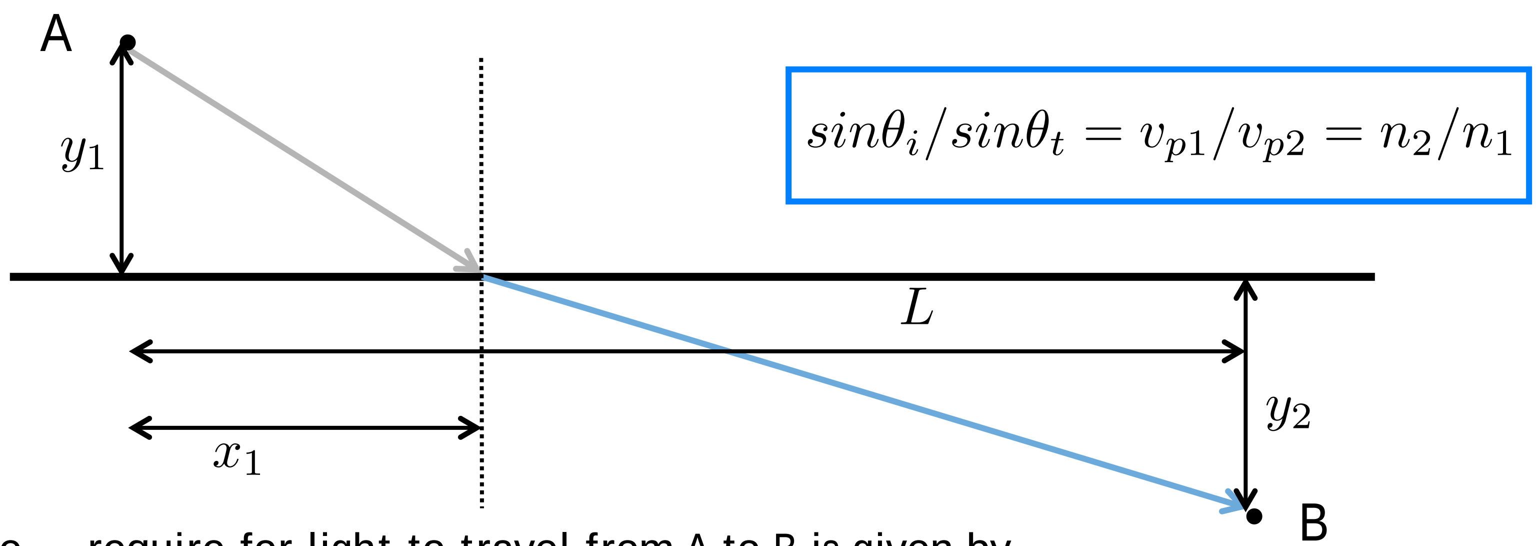
Applied to Reflection



Since it is straight, the blue path is the shortest path from A to B'. So, the blue path is also the shortest reflecting path to B since it images the path to B'. For the blue path, the incidence and reflection angles equal.

Fermat's Principle of Least Time

Refraction



The time t require for light to travel from A to B is given by

$$t = \frac{\sqrt{x_1^2 + y_1^2}}{v_1} + \frac{\sqrt{((L - x_1)^2 + y_2^2)}}{v_2}$$

From $dt/dx_1 = 0$, it follows that

$$\frac{x_1 v_1}{\sqrt{(x_1^2 + y_1^2)}} = \frac{x_2 v_2}{\sqrt{((L - x_1)^2 + y_2^2)}}$$

Total Internal Reflection

Beyond the critical angle, θ_c , a ray within the higher index medium cannot escape at shallower angles

$$n_2 sin\theta_2 = n_1 sin\theta_1 \quad \theta_c = sin^{-1}(n_1/n_2)$$

For glass, the critical internal angle is 42°

For water, it is 49°

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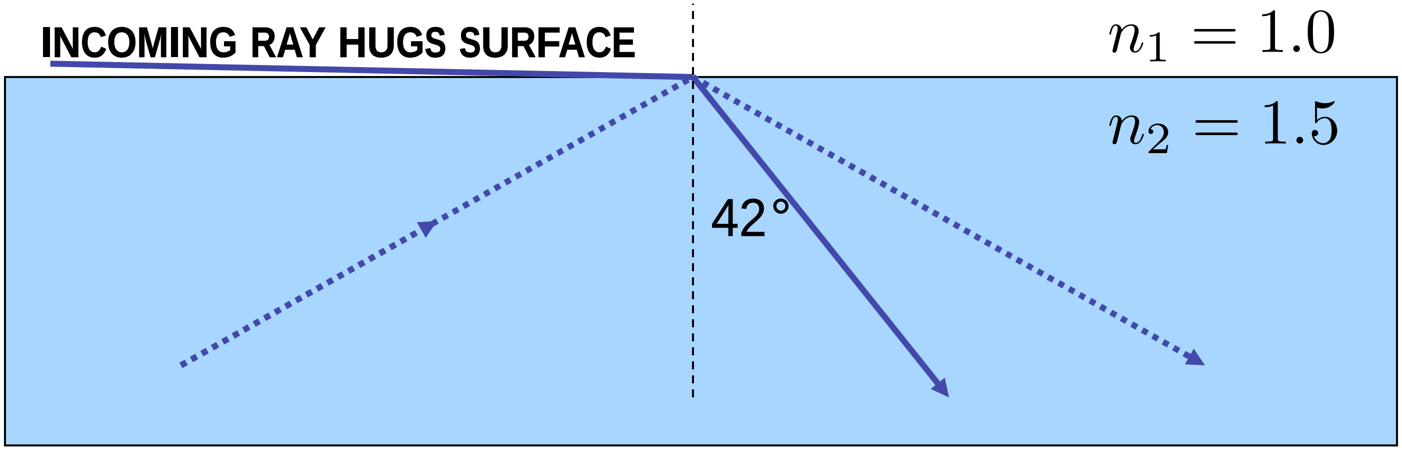
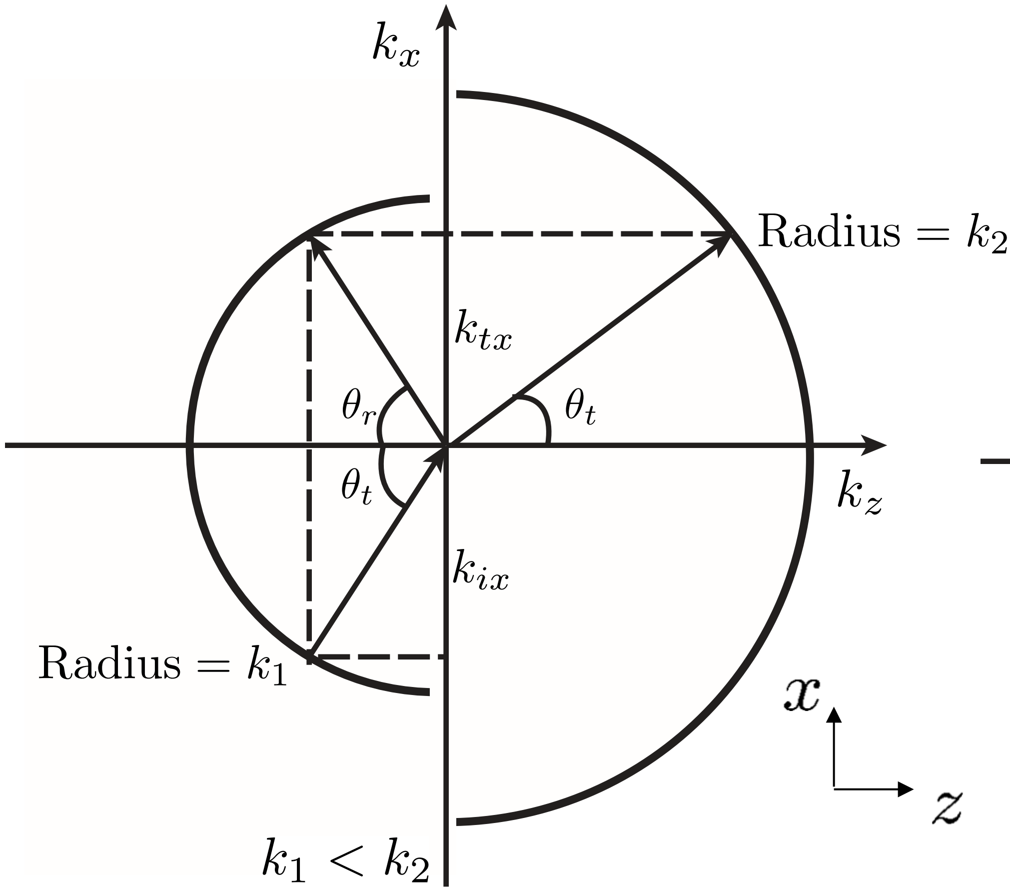


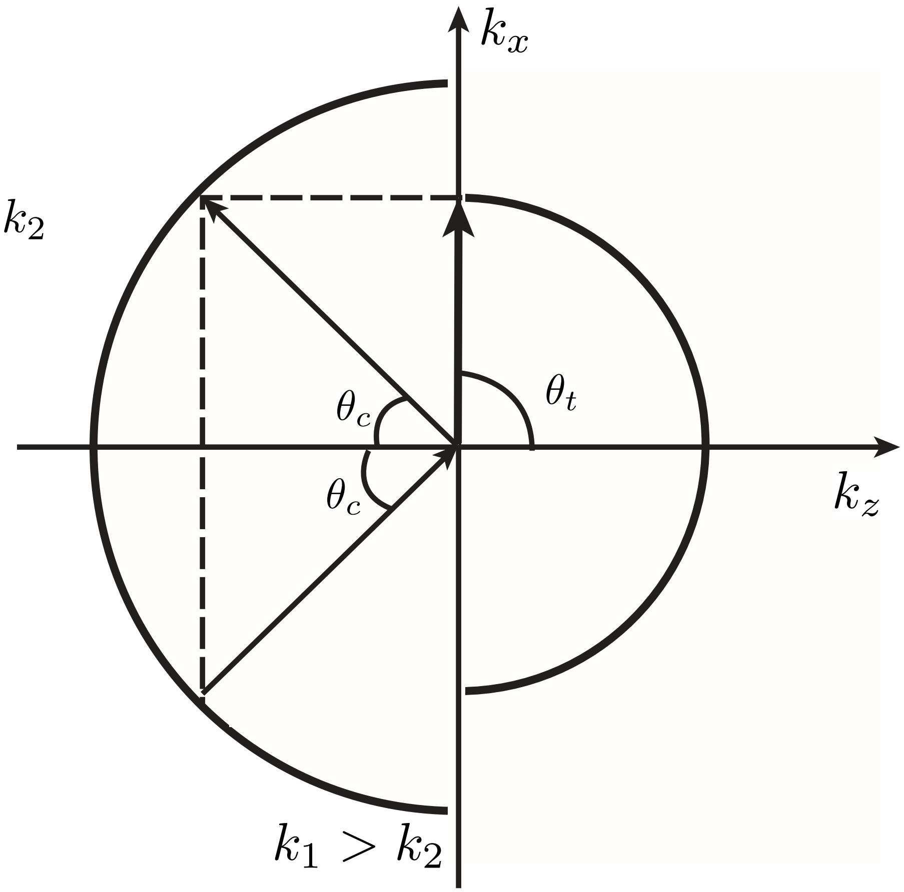
Snell's Law Diagram

Tangential E field is continuous … $k_{ix} = k_{tx}$

Refraction



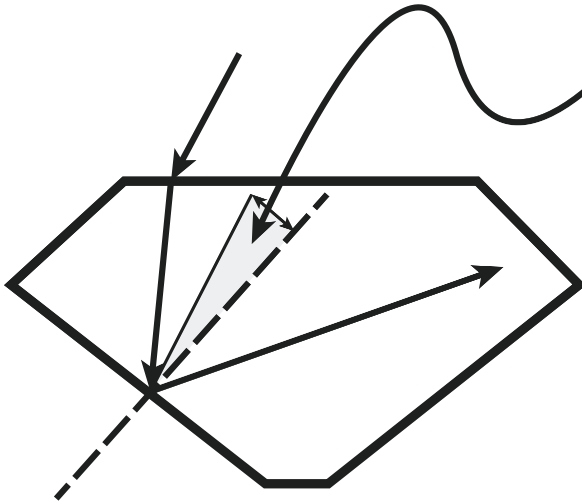
Total Internal Reflection



Applications of Total Internal Reflection

Critical angle (diamond/air interface): $\sin^{-1}(n_2/n_1) = \sin^{-1}(1/2.42) \sim 24°$
Critical angle (glass/air interface) is: $\sim 42°$

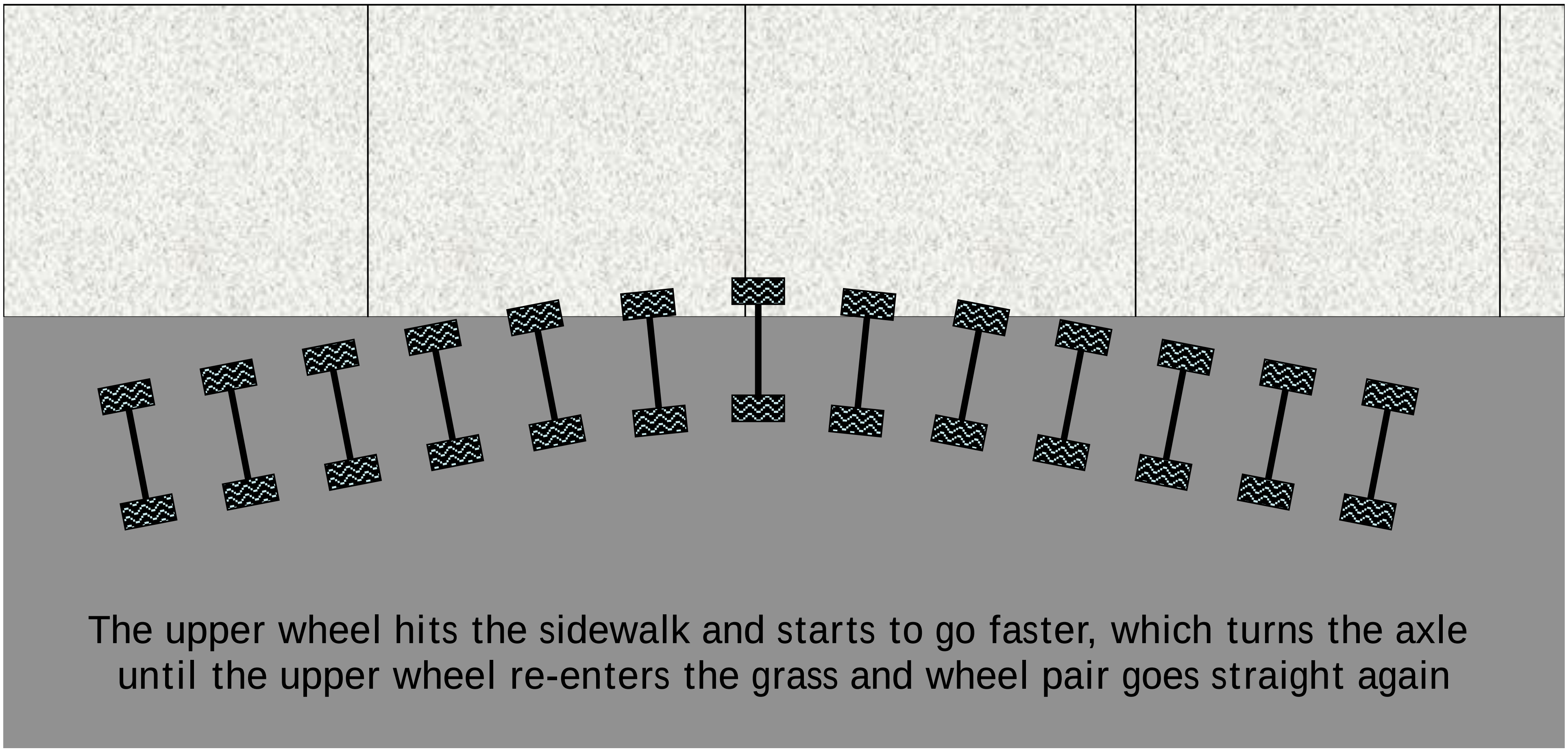
Image by Steve Jurvetson
http://en.wikipedia.org/wiki/File:Apollo_synthetic_diamond.jpg on Wikipedia



Diamonds sparkle as light bounces inside them multiple times due to the high index of refraction

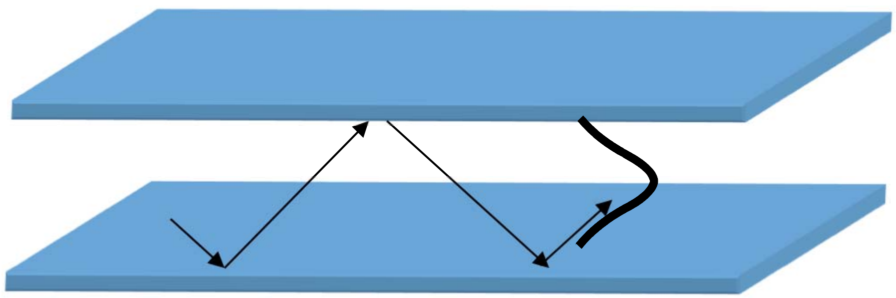
Total Internal Reflection in Suburbia

Moreover, this wheel analogy is mathematically equivalent to the refraction phenomenon.
One can recover Snell's law from it: $n_1 \sin \theta_1 = n_2 \sin \theta_2$.

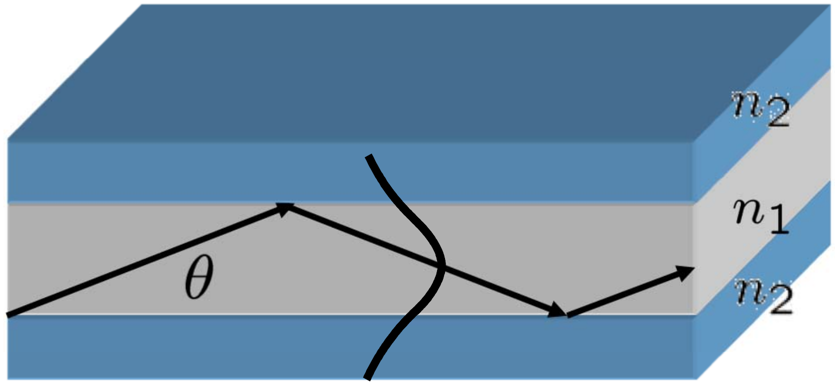


Waveguide Transports Light Between Mirrors

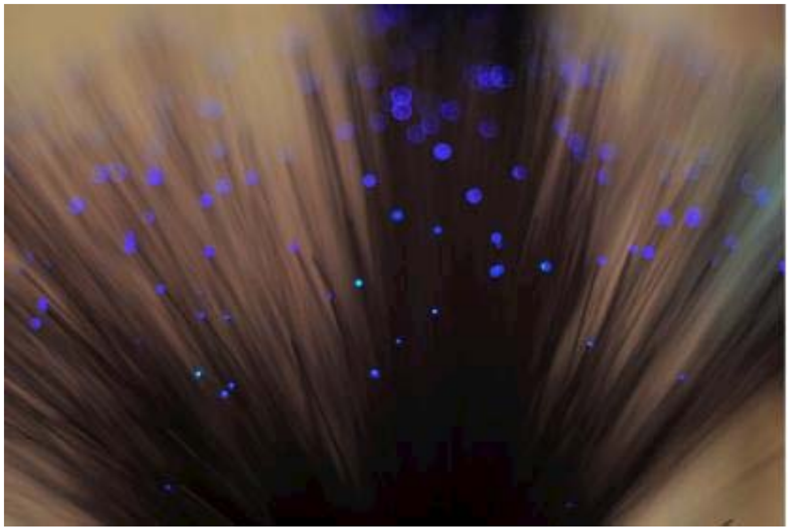
Metal waveguides



Dielectric waveguides



So what kind of waveguide are the optical fibers ?



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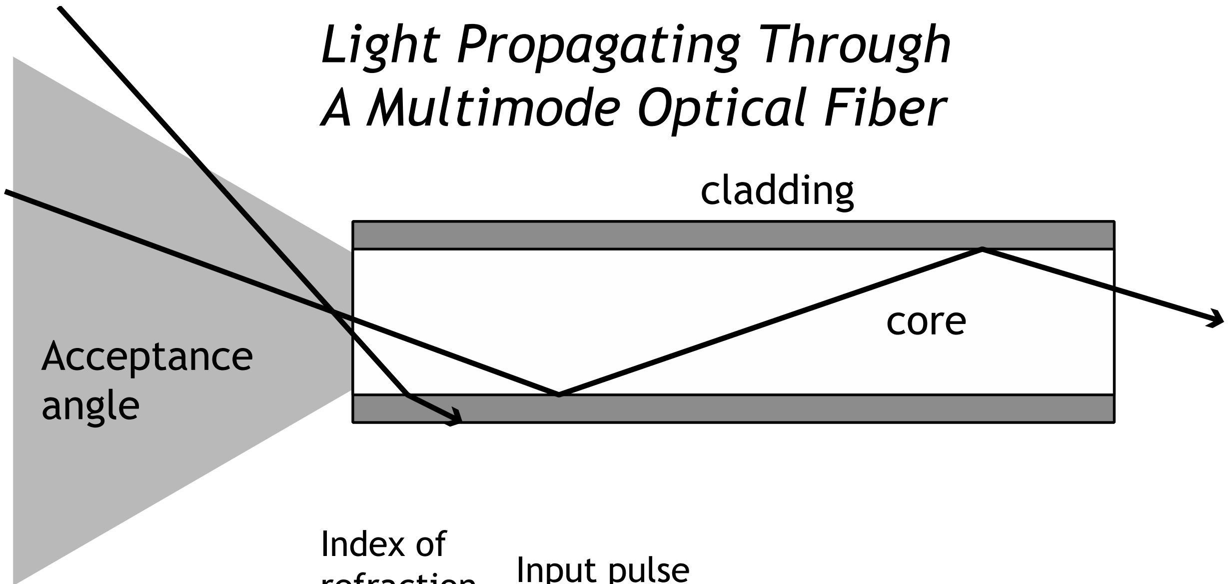
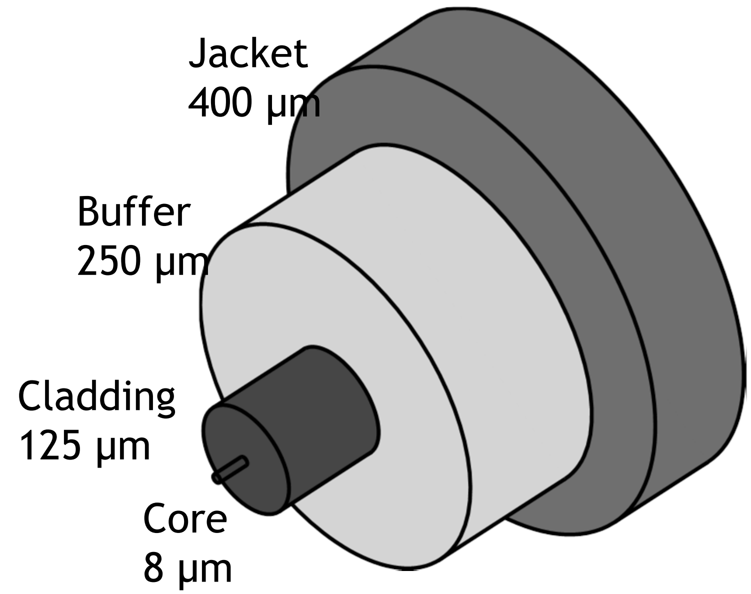
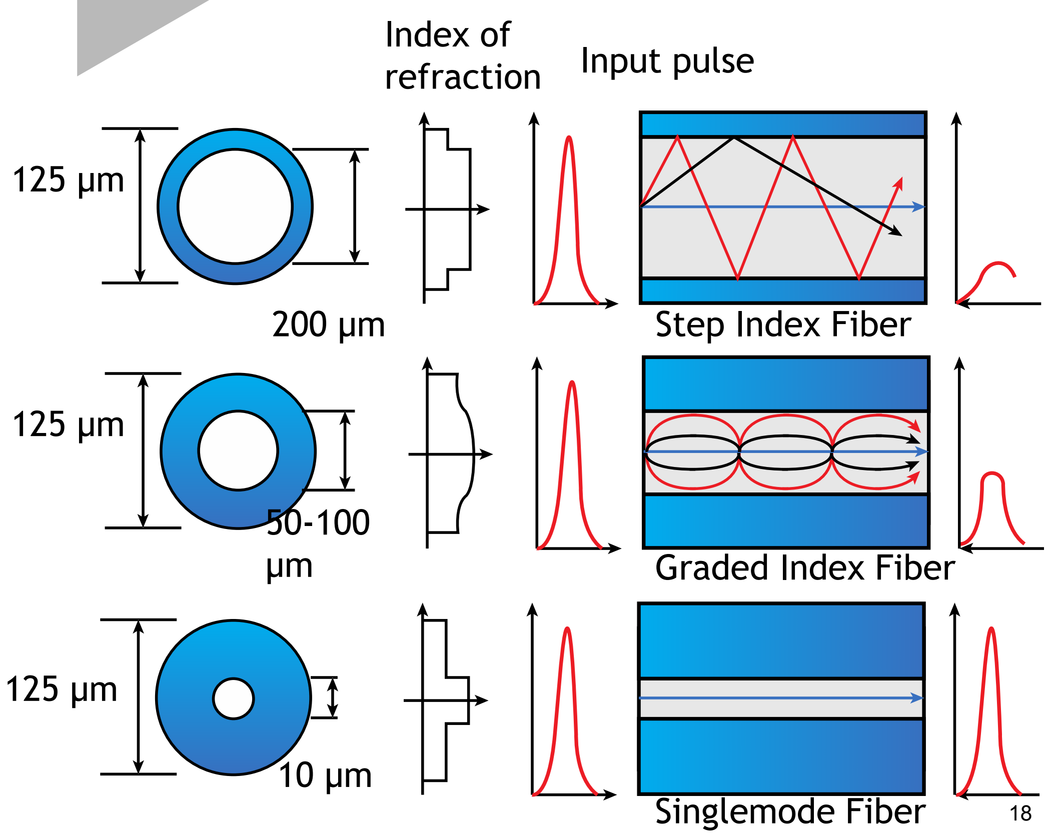
Optical Fibers

Light Propagating Through A Multimode Optical Fiber



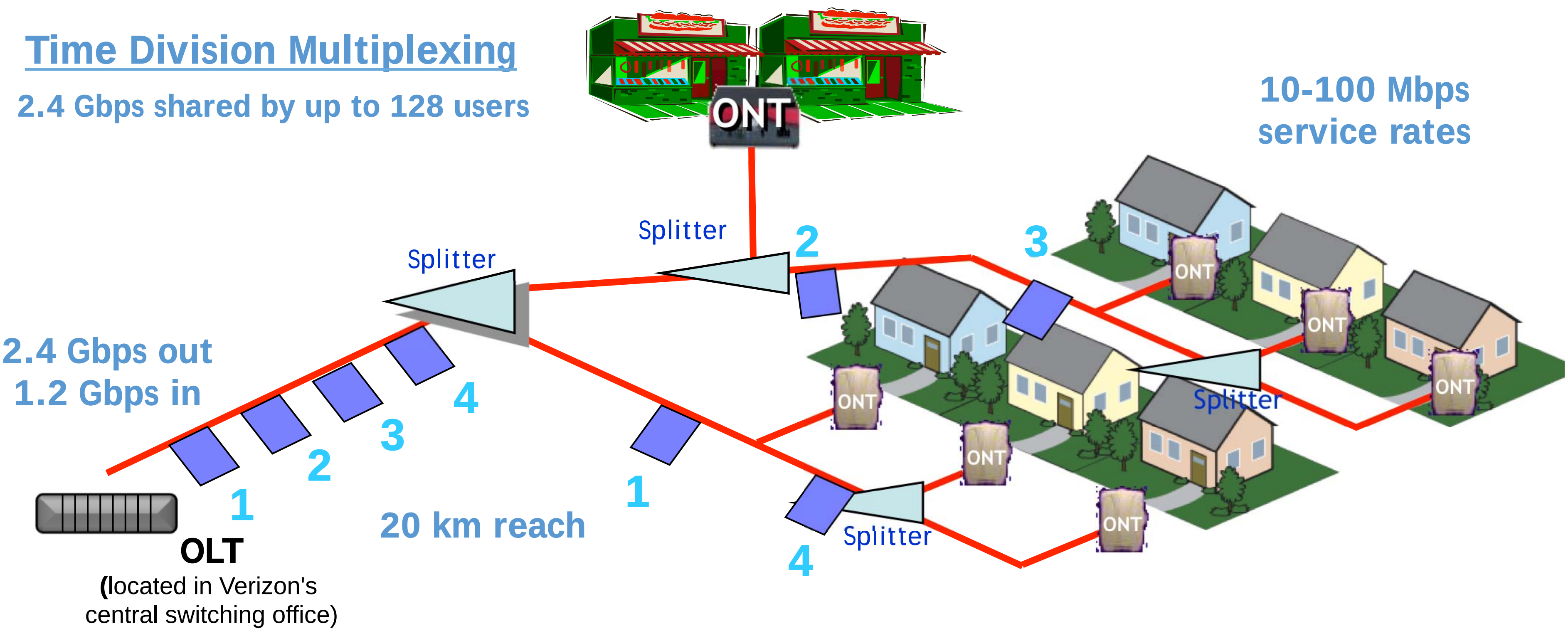
Single Mode Fiber Structure





Modal dispersion is a distortion mechanism occurring in multimode fibers and other waveguides, in which the signal is spread in time because the propagation velocity of the optical signal is not the same for all modes

Fiber to the Home



An ONT (Optical Network Terminal) is a media converter that is installed by Verizon either outside or inside your premises, during FiOS installation. The ONT converts fiber-optic light signals to copper/electric signals. Three wavelengths of light are used between the ONT and the OLT (Optical Line Terminal):

- λ = 1310 nm voice/data transmit
- λ = 1490 nm voice/data receive
- λ = 1550 nm video receive

Each ONT is capable of delivering:
Multiple POTS (plain old telephone service) lines, Internet data, Video

Image by Raj from Chennai, India
http://commons.wikimedia.org/wiki/File:Strings_of_lights.jpg
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Fiber to the Home

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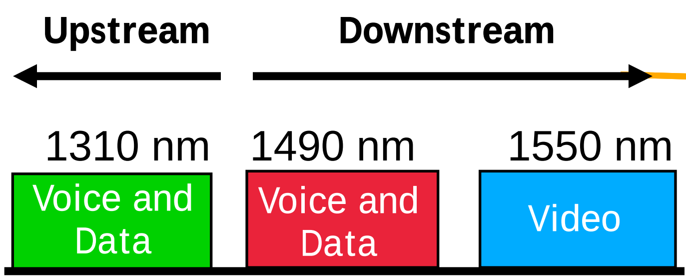
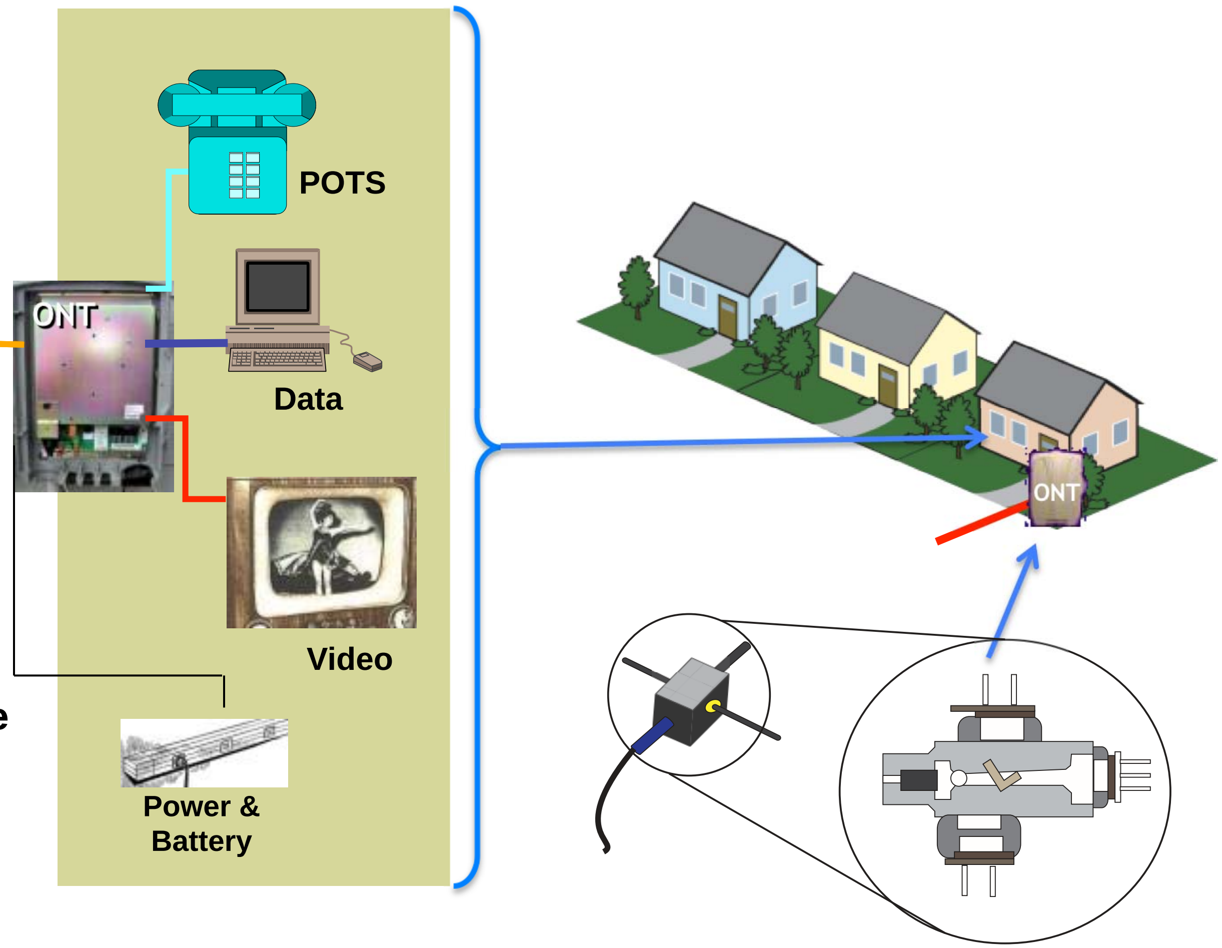
Bandwidths & Services





- **Channels downstream to each home**
  λ = 1490 and λ = 1550 nm
- **Channel upstream from each home**
  λ = 1310 nm

Image of ONT by Josh Bancroft
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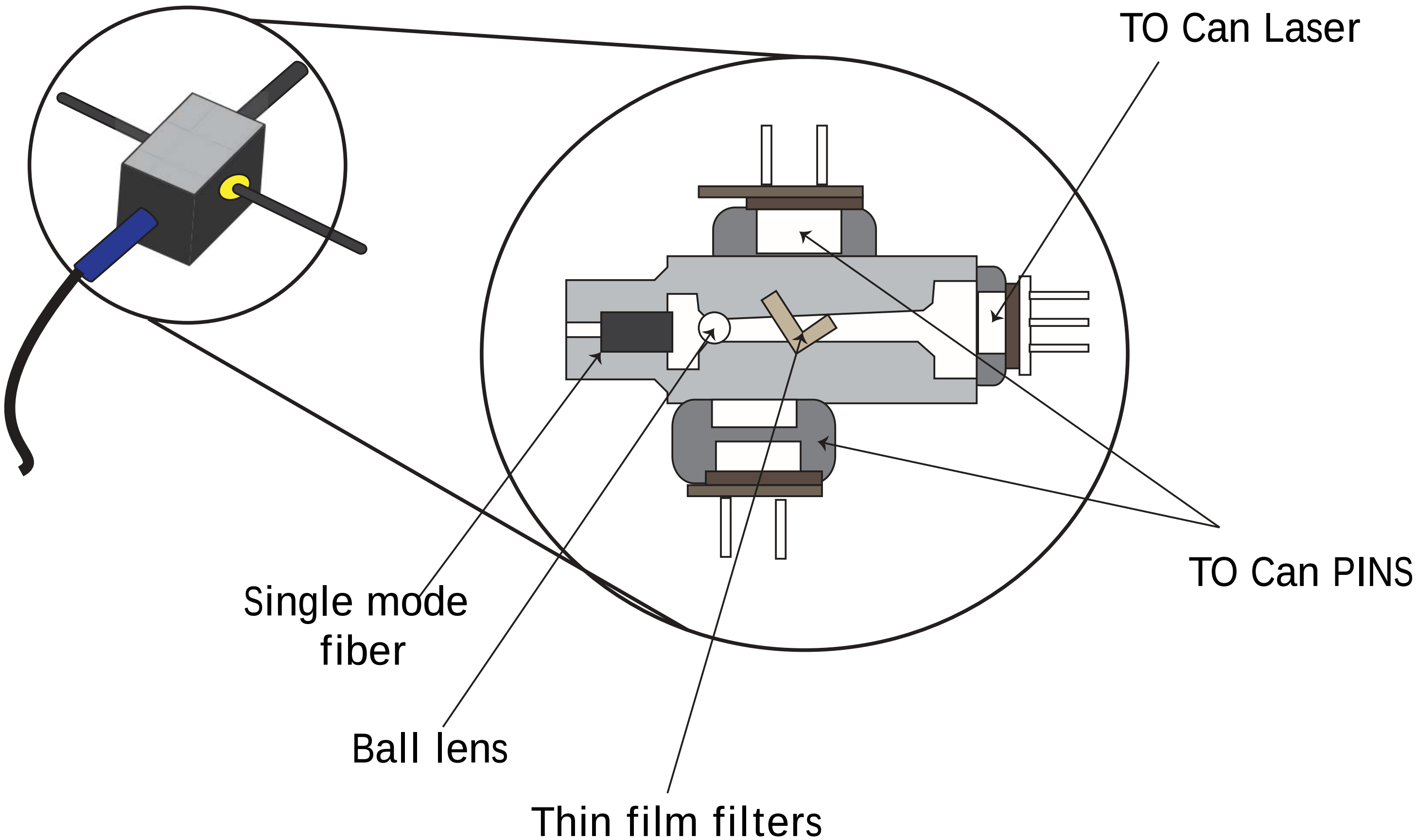
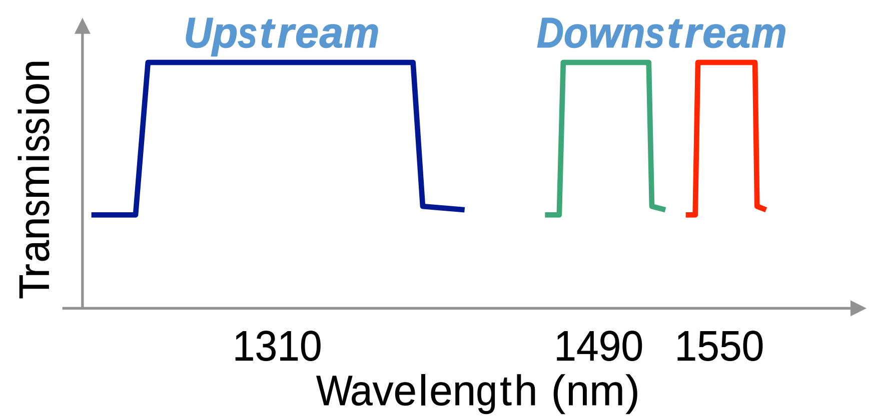
Optical Assembly

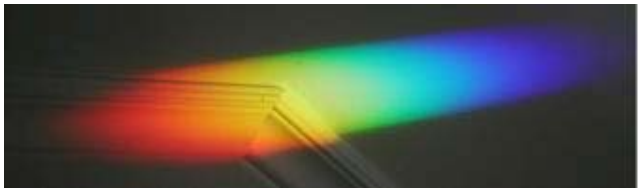


- **Channels downstream to each home**
  - λ = 1490 and λ = 1550 nm
- **Channel upstream from each home**
  - λ = 1310 nm



Separating Wavelengths

Dispersion



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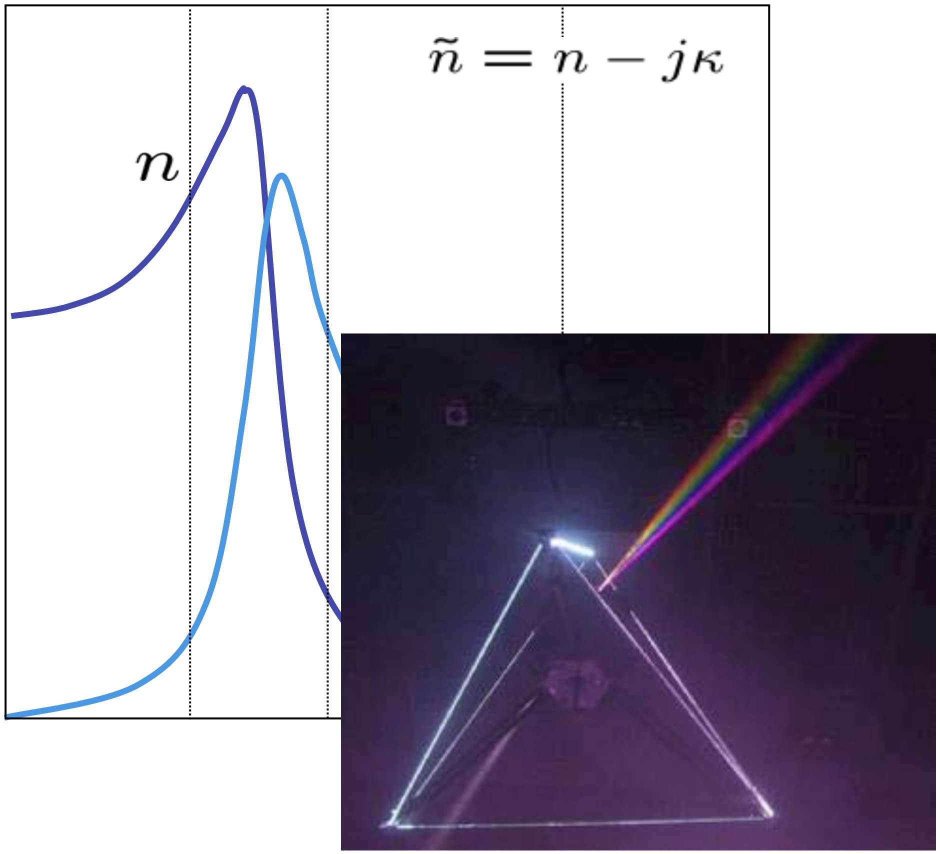

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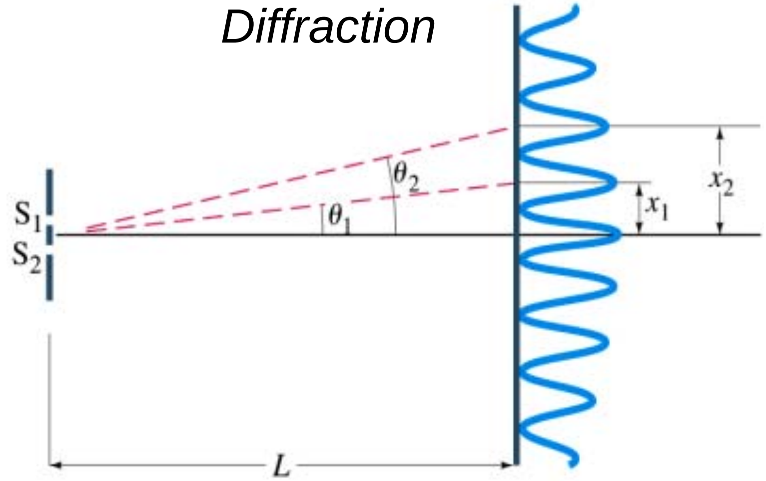
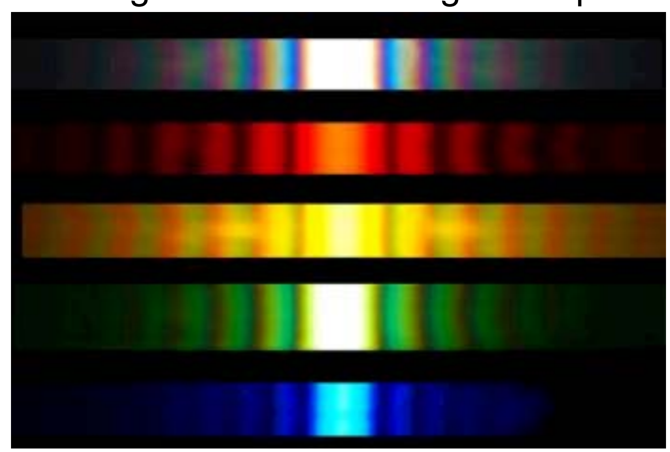
Diffraction



Sunlight diffracted through a 20 μm slit



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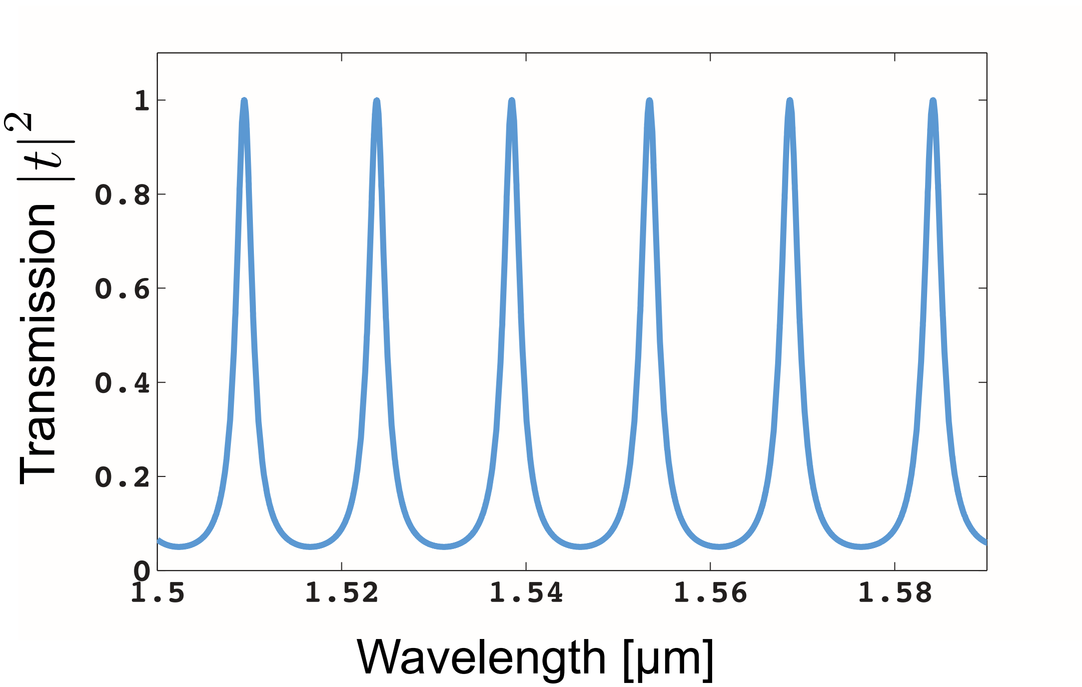
Fabry-Perot Resonance

$$t = \frac{t_{12}t_{21}e^{-jkL}}{1 - r_{12}r_{21}e^{-2jk}}$$

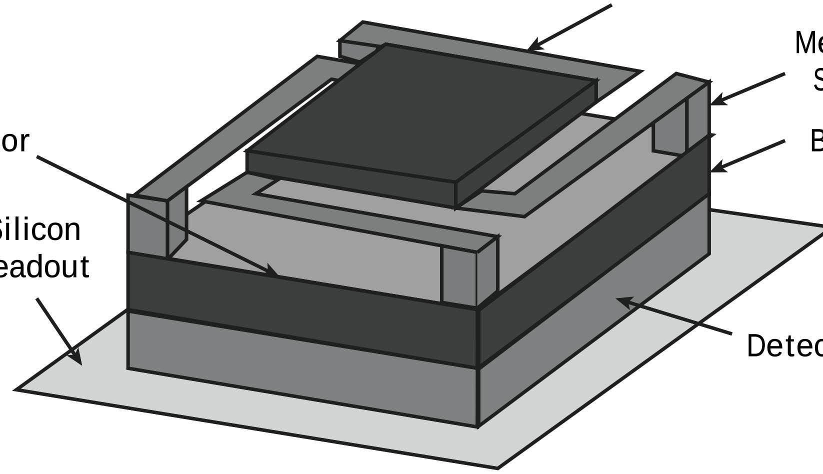


Fabry-Perot Resonance: $\max\{e^{-2jk_2L}\} = 1$ maximum transmission

$\min\{e^{-2jk_2L}\} = -1$ minimum transmission

General concept of a
MEMS Fabry-Perot filter
formed on a detector

(by applying voltage between the top and bottom mirror the distance L between the mirrors can be adjusted)



General concept of a
Mach-Zehnder Modulator

(phase shifters change the phase of the light beam in one of the waveguide arms with respect to the other beam, so that they can constructively or destructively interfere)