14.381 Practice Final

Fall 2005

Question 1. Relaxing assumptions on WLLN (Theorem 5.5.2).

a) Show that $p\lim \bar{X}_n = \mu$ for the same assumptions on X_i as in Theorem 5.5.2 except that now we replace the independence assumption with the assumption that the X_i are uncorrelated.

b) Further relax the assumptions so that rather than all of the X_i having the same variance, assume that all of the variances are bounded by $C < \infty$

Question 2. Consider the density $f_y(y) = \frac{2y}{\theta^2}$ where $0 < y < \theta.$, $\theta > 0$.For parts (b) onwards assume we observe a random sample of observations $Y_1, ...Y_n$ from this distribution.

(a) show that this is indeed a density.

(b) compute the method of moments estimator for θ based on the first moment only.

(c) compute the variance of this estimator.

(d) compute the Fisher information for estimating θ in the sample.

(e) contrast your results in (c) and (d) and comment.

Question 3. Suppose we have a random sample from the Poisson (λ) distribution. The probability density function for any X_i is

$$f_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

for $0 \leq \lambda < \infty$ and $x = 1, 2, 3, ...$ Both the mean and variance are equal to λ.

(a) Consider testing between $H_0 : \lambda = \lambda_0$ and $H_a : \lambda = \lambda_1 > \lambda_0$. Show that this test rejects for $\sum X_i$ large.

(b) Compute the sampling distribution for the test statistic (required for computation of the critical value). (note: you do not have to compute the critical value explicitly).

(c) Does a UMP test of the hypothesis $H_0 : \lambda = \lambda_0$ and $H_a : \lambda > \lambda_0$ exist? Explain.

Question 4. Consider a random variable X and the transformation $Y = a + bX$. What is the correlation between these two random variables?