## 14.01 Fall 2010
## Problem Set 5 Solutions

1. (24 points) You manage a factory that produces cans of peanut butter. The current market price is $10/can, and you know the following about your costs:

$$MC(5) = 10, \qquad\qquad ATC(5) = 6$$
$$MC(4) = 4, \qquad\qquad ATC(4) = 4$$

(a) (8 points) A case of food poisoning breaks out due to your peanut butter, and you lose a lawsuit against your company. As punishment, Judge Judy decides to take away all of your profits, and considers the following two options to be equivalent:

   i. Pay a lump sum in the amount of your profits.
   ii. Impose a tax of $[P - ATC(q^*)]$ per can since that is your current profit per can, where $q^*$ is the profit maximizing output before the lawsuit.

Explain briefly why she is right or wrong.

You maximize profits where $P = MC$, and since $P = 10 = MC(5)$ you would set $q^* = 5$.

$$\pi/q = (P - ATC) = (10 - 6) = 4$$

The tax would be $4/can.

(b) (8 points) Judge Judy gives you the option of choosing either plan. Which plan would you choose? Provide intuition. Hint: a clear diagram may be helpful.

The lump sum transfer will result in $\pi = 0$.

With the tax, $MC_t(4) = 8$ and since $P = 10 > 8$, this implies that the optimal $q_t$ is such that $4 < q_t < 5$. We also know that $8 < ATC_t(q_t) < 10$, since average total cost also shifts up by 4. This implies $\pi_t = q_t(P - ATC_t) > 0$.

Intuition: Since we know that $MC(4) = ATC(4)$, the average total cost is minimized at $q = 4$, both with and without the tax. By imposing a $4/can tax, the firm reduces its quantity from 5 to $q_t$, which decreases average total cost. $(P - ATC)$ or the profit/can increases, so that the firm is still making positive profits despite the $4/can tax.
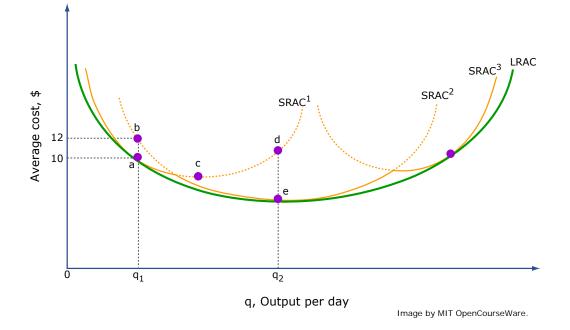
Conclusion: You would prefer to have the tax plan.

(c) (8 points) In the following year, you finally pay off the court ordered amount and everything returns to normal. However, an employee tells you that at current levels of production, actual costs are double what you initially thought. After carefully looking at the cost information, you decide to continue producing the exact same amount. Could you still be maximizing profits?

Yes, the increase in costs could be completely due to fixed costs. This would not affect your behavior in any way in the short run. However, in the long run, since $\pi = q(P - ATC) = 5(10 - 12) = -10 < 0$, you would exit the market.

Problem 1 solution courtesy of Edward Cho. Used with permission.

2. (32 points) Some questions about short-term and long-term costs:

(a) (8 points) What is the relationship between the long-run average cost curve and the short-run average cost curve? Please show graphically.

Image by MIT OpenCourseWare.

(b) (8 points) What algebraic condition describes a firm that is at an output level that maximizes its profits, given its capital in the short-term?

The required condition is $MR = SRMC$, or marginal revenue is equal to short-run marginal cost.

(c) (8 points) What two algebraic conditions describe a firm that is at a capital level that minimizes its costs in the long-term?

The required conditions are $SRAC = LRAC$ and $SRMC = LRMC$, or both marginal cost and average cost are equal to their long-run levels.

(d) (8 points) If a firm is characterized by short-run marginal cost that is greater than long-run marginal cost and short-run average cost greater than long-run average cost, how should it change its capital level in the long-run to minimize costs?

It should increase its capital input in the long-run in order to move cost curves to the right.
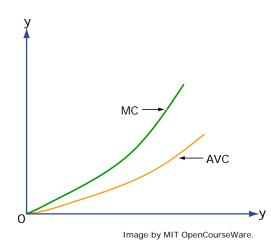
Problem 2 solution by MIT OpenCourseWare.

3. (6 points) Suppose, in the short run, the output of widgets is supplied by 100 identical competitive firms, each having a cost function:

$$c_s(y) = \frac{1}{3}y^3 + 2$$

The demand for widgets is given by:

$$y^d(p) = 6400/p^{\frac{1}{2}}$$

(a) (3 points) Obtain the short run industry supply function for widgets.

Since $P = MC = y^2$, the supply function of each firm is given by $y_i^s = p^{\frac{1}{2}}$. In the figure below, $MC = y^2$ and $AVC = \frac{1}{3}y^{\frac{1}{2}}$.



Image by MIT OpenCourseWare.

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The industry supply function is $y^s(p) = 100y_i^s(p) = 100p^{\frac{1}{2}}$.

(b) (3 points) Obtain the short run equilibrium price of widgets, and the output of widgets supplied by each firm.

$$y^s = y^d \longrightarrow 100p^{\frac{1}{2}} = \frac{6400}{p^{\frac{1}{2}}} \longrightarrow p = 64. \text{ Hence } y^* = 100 \cdot 8 = 800 \text{ and } y_i = 8. \text{ Finally, } p^* = 64.$$

4. (22 points) Sebastian owns a coffee factory in Argentina. His production function is:

$$F(K, L) = (K - 1)^{\frac{1}{4}} L^{\frac{1}{4}}$$

Consider the cost of capital to be $r$ and the wage to be $w$. Both inputs are variable, and Sebastian faces no fixed costs.

(a) (2 points) What is the $MRTS$ of labor for capital?

$$MRTS = \frac{MP_L}{MP_K} = \frac{(K - 1)}{L}$$

(b) (5 points) What are Sebastian's input demands, conditional on the quantity $(q)$ he wants to produce? [Hint: Treat $w$ and $r$ as parameters.]

Optimality Conditions:

$$\frac{K - 1}{L} = \frac{w}{r}$$

$$q = (K - 1)^{\frac{1}{4}} L^{\frac{1}{4}}$$

Conditional Demand Functions:

$$L^d = \left(\frac{r}{w}\right)^{\frac{1}{2}} q^2$$

$$K^d = \left(\frac{w}{r}\right)^{\frac{1}{2}} q^2 + 1$$

(c) (3 points) Show that Sebastian's long run cost function is $C(q) = r + 2(wr)^{\frac{1}{2}} q^2$.

$$C(q) = wL^d + rK^d = w\left(\frac{r}{w}\right)^{\frac{1}{2}} q^2 + r\left[\left(\frac{w}{r}\right)^{\frac{1}{2}} q^2 + 1\right] = r + 2(wr)^{\frac{1}{2}} q^2$$

(d) (5 points) What is the supply function of Sebastian's firm?

The inverse supply function for a firm is given by $P = MC(q)$, for any $q$ such that $P \geq \min AC(q)$. This is true when $MC = AC$.

$$4(wr)^{\frac{1}{2}} q = \frac{r}{q} + 2(wr)^{\frac{1}{2}} q$$

$$q = \frac{\left(\frac{r}{w}\right)^{\frac{1}{4}}}{2^{\frac{1}{2}}}$$

Therefore, the inverse supply function is $P = 4(wr)^{\frac{1}{2}} q$, for all $q \geq \left(\frac{r}{w}\right)^{\frac{1}{4}}/2^{\frac{1}{2}}$.

When $q = \left(\frac{r}{w}\right)^{\frac{1}{4}}/2^{\frac{1}{2}}$, $P = 2^{\frac{3}{2}}(wr^3)^{\frac{1}{4}}$, and therefore, Sebastian's supply function is:

$$\begin{cases} q = \frac{P}{4(wr)^{\frac{1}{2}}}, & \text{when } P \geq 2^{\frac{3}{2}}(wr^3)^{\frac{1}{4}} \\ q = 0, & \text{otherwise} \end{cases}$$

Consider now that $r = 4, w = 1$, and that the market demand for coffee is $Q^d = 20 - P$. There are 7 other companies operating in this market, all with cost structures identical to Sebastian's company.

(e) (6 points) What is the aggregate supply in this market?

As shown in (c), $C(q) = r + 2(wr)^{\frac{1}{2}} q^2$. Therefore, $MC = 4(wr)^{\frac{1}{2}} q = 8q$.

Inverse supply of one firm:

$P = MC = 8q$, for any $q$ such that $P \geq \min AC$. At $\min AC, MC = AC$. Therefore,

$$8q = \frac{4}{q} + 4q \longrightarrow q = 1$$

$$\min AC = 8$$

Supply of one firm:

$$\begin{cases} q = \frac{P}{8}, & \text{for } P \geq 8 \\ q = 0, & \text{otherwise} \end{cases}$$

Aggregate supply:

$$\begin{cases} Q^s = 8 \cdot \left(\frac{P}{8}\right) = P, & \text{for } P \geq 8 \\ Q^s = 0, & \text{otherwise} \end{cases}$$

(f) (6 points) Calculate the equilibrium price, aggregate quantity sold, quantity sold by each firm, and economic profit of each firm.

$$Q^s = Q^d$$
$$P = 20 - P \longrightarrow P = 10$$
$$Q = 10$$
$$q = \frac{10}{8} = \frac{5}{4}$$

Economic profit of one firm $= 10 \cdot \frac{5}{4} - 4 - 4 \cdot \left(\frac{5}{4}\right)^2 = 2.25$

(g) (5 points) Can this be a long run equilibrium? Why? How will the supply side of the market adjust in the long run?

No, because the economic profits of the firms in this market will attract the entry of other firms, until the economic profits are driven to 0.

(h) (6 points) What is going to be the price in the long run? How many firms will be present in this market in the long run? How much will each firm produce?

In the long run, we must have $P = \min AC = 8$. When $P = 8, Q^d = 20 - 8 = 12$. So we will need to have 12 firms in this market in the long run, each producing $q = 1$.

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