14.01 Principles of Microeconomics, Fall 2007
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Lecture 9

Preference Toward Risk, Risk Premium, Indifference Curves, and Reducing Risk

Outline

1. Chap 5: *Preference Toward Risk*

2. Chap 5: *Risk Premium*

3. Chap 5: *Indifference Curve*

4. Chap 5: *Reducing Risk: Diversification*

1 Preference Toward Risk - Risk Averse / Neutral / Seeking (Loving)

Three different kinds of behaviors:

Risk Averse (Figure 1)

- Facing two payoffs with the same expected value, prefer the less risky one.

- Diminishing marginal utility of income.

- Relation between the utility of expected value and expected utility

$$u(E(x)) > E(u(x)).$$

Example.
$$u(x) = \ln x.$$

Risk Neutral (Figure 2)

- Facing two payoffs with the same expected value, feel indifferent.

- Linear marginal utility of income.

- Relation between the utility of expected value and expected utility

$$u(E(x)) = E(u(x)).$$
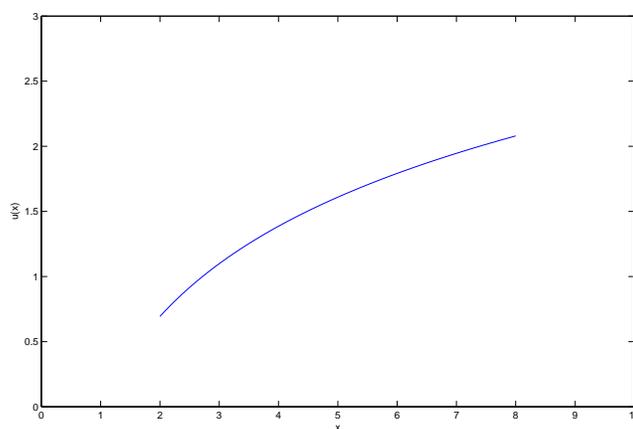
Figure 1: The Utility Function of Risk Averse.

Example.
$$u(x) = x.$$

Risk Seeking (Figure 3)

- Facing two payoffs with the same expected value, prefer the riskier one.

- Increasing marginal utility of income.

- Relation between the utility of expected value and expected utility

$$u(E(x)) < E(u(x)).$$

Example.
$$u(x) = x^2.$$

2   Risk Premium

Risk premium. The maximum amount of money that a risk-averse person
would pay to avoid taking a risk.

Example (Job Choice). Assume that a risk-averse person whose utility function
corresponds with the curve in Figure 4 has two possible incomes.
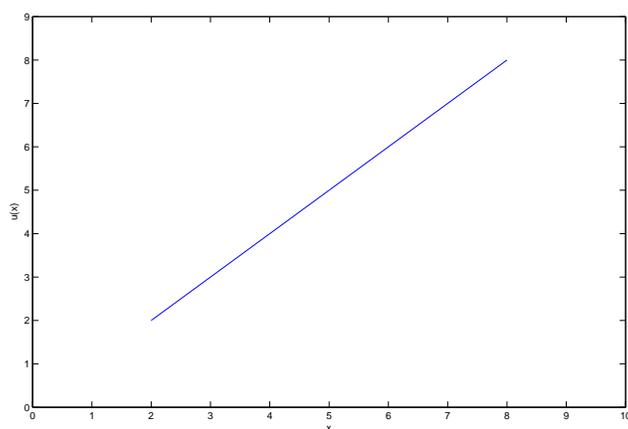
Figure 2: The Utility Function of Risk Neutral.
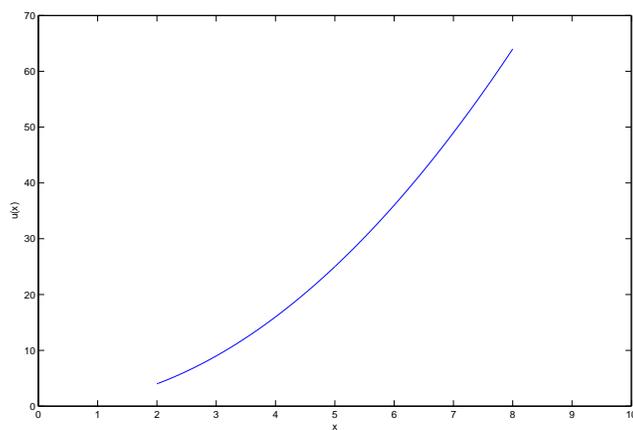
Figure 3: The Utility Function of Risk Seeking.
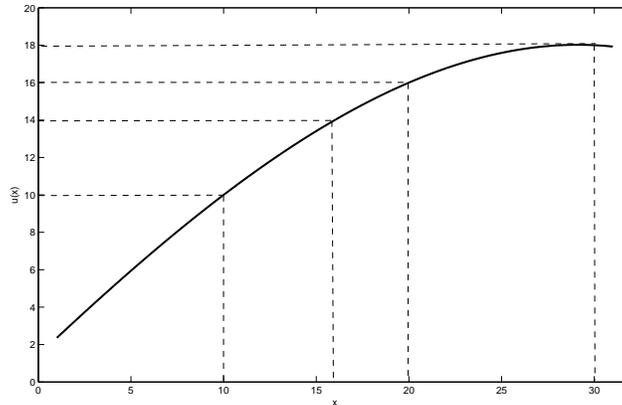
Figure 4: Risk Premium: A Utility Function.

- His income I might be 10 with probability 0.5 and 30 with probability 0.5. Then the expected value of income I is:

$$E(1) = 10 \times 0.5 + 30 \times 0.5 = 20,$$

with an expected utility:

$$E(u(I)) = u(10) \times 0.5 + u(30) \times 0.5 = 10 \times 0.5 + 18 \times 0.5 = 14.$$

- If we offer him a fixed income I', $I' = 16$, then his expected utility is:

$$E(u(I')) = u(16) \times 1 = 14 \times 1 = 14.$$

One can see that

$$E(u(I)) = E(u(I')).$$

However, $E(I) - E(I') = 4$. This means the person is willing to give up a value of 4 in exchange for a riskless income. Thus, the risk premium is

$$Risk\ Premium = E(I) - E(I') = 20 - 16 = 4.$$

3 Indifference Curve between Expected Value and Standard Deviation

The indifference curve we discussed before is about the quantities of two different goods, now we consider the indifference curve about expected value and standard deviation (Figure 5).
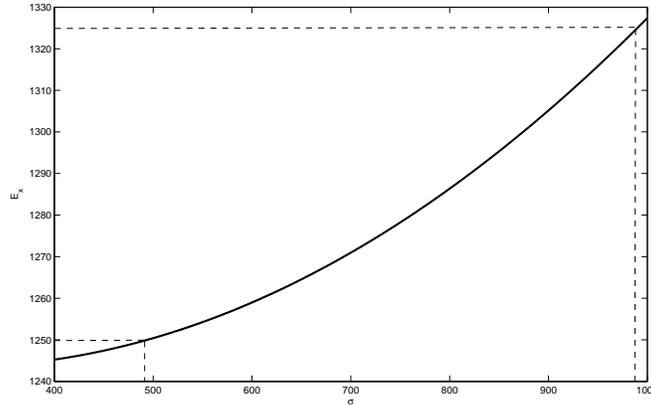
Figure 5: Indifference Curve between Expected Value and Standard Deviation.

	Probability 0.5	Probability 0.5
Job 1	900	1600
Job 2	625	2025

Table 1: The Income and Probability of Two Jobs.

Example (Job choice). Suppose one has the following utility function

$$u(x) = \sqrt{x}$$

and two job choices (see Table 1). Calculate expected utilities:

$$E(u(x_1)) = 0.5 \times \sqrt{900} + 0.5 \times \sqrt{1600} = 35,$$

$$E(u(x_2)) = 0.5 \times \sqrt{625} + 0.5 \times \sqrt{2025} = 35.$$

Thus, these two jobs give the person the same utility level, i.e. they are on a same indifference curve.

In order to plot the indifference curve, we should calculate their expected values and standard deviations.

$$E(x_1) = 1250$$

$$\sigma(x_1) = 494$$

$$E(x_2) = 1325$$

$$\sigma(x_2) = 990$$

Job 2 has higher expected value of income but it is riskier. (Figure 5)

Compare Figure 6 and Figure 7. The former is more risk averse since one must compensate more for more risk.
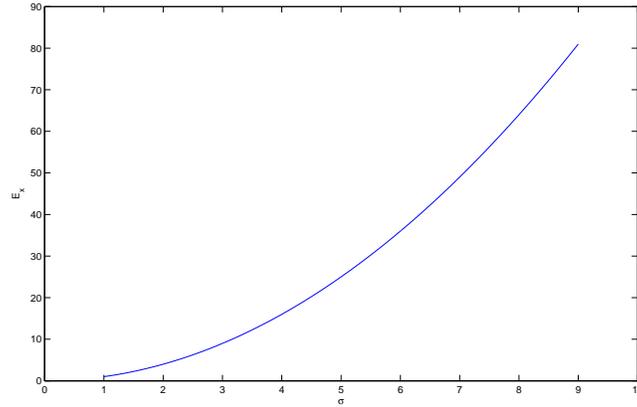
Figure 6: Indifference Curve between Expected Value and Standard Deviation, Larger Slope.
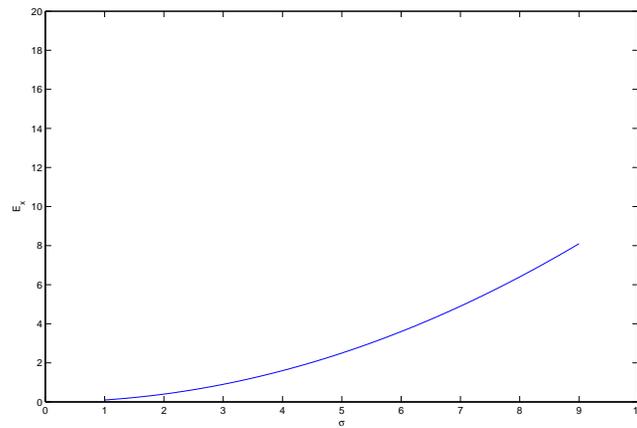
Figure 7: Indifference Curve between Expected Value and Standard Deviation, Smaller Slope.

4 Reducing Risk: Diversification

Diversification. Reducing risk by allocating resources to different activities whose outcomes are not closely related.

Example (Selling air conditioner and heater). Suppose that the weather has a probability 0.5 to be hot and 0.5 to be cold. Table 2 shows the company's profit if all its efforts in selling air conditioners (heaters) and the weather turns out to be hot (cold).

Weather	Hot	Cold
Air Conditioner	30,000	12,000
Heater	12,000	30,000

Table 2: Diversification: Selling Air Conditioners and Heaters.

- If one only sells air conditioners or heaters,

$$E(profit) = 21,000,$$

$$\sigma(profit) = 9,000.$$

- If the company puts half of its efforts in selling air conditioners and half of its efforts in selling heaters, then the profit is always 21,000 no matter the weather is cold or hot.

$$E(profit) = 21,000,$$

$$\sigma(profit) = 0.$$

Thus we should choose to sell both to reduce risk.

Example (Example: Stock versus mutual fund). Mutual fund may have the same return as stock but much less risk.

Example. "Don't put all your eggs in one basket."