

12.540 Principles of the Global
Positioning System
Lecture 14

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<http://geoweb.mit.edu/~tah/12.540>

Propagation Medium

- Propagation:
 - Signal propagation from satellite to receiver
 - Light-time iteration
 - Basic atmospheric and ionospheric delays
 - Propagation near receiving antenna

Propagation

- Basics:
 - Signal, tagged with time from satellite clock, transmitted.
 - About 60 msec (20,000 km) later the signal arrives at GPS receiver. Satellite has moved about 66 m during the time it takes signal to propagate to receiver.
 - Time the signal is received is given by clock in receiver. Difference between transmit time and receive time is pseudorange.
 - During the propagation, signal passes through the ionosphere (10-100 m of delay, phase advance), and neutral atmosphere (2.3-30 m depending on elevation angle).

Propagation

- To determine an accurate position from range data, we need to account for all these propagation effects and time offsets.
- In later lectures, examine ionospheric and atmospheric delays, and effects near antenna.
- Basic clock treatment in GPS
 - True time of reception of signal needed
 - True time of transmission needed (af_0 , af_1 from broadcast ephemeris initially OK)
 - Position of satellite when signal transmitted

Times

- RINEX data files, tag measurements by reception time as given by the receiver clock. The error in the receiver time must be determined iteratively
- For linearized least squares or Kalman filter need to establish non-linear model and then estimator determines adjustments to parameters of model (e.g. receiver site coordinates) and initial clock error estimates that “best” match the data.

Non-linear model

- Basics of non-linear model:
 - Rinex data file time tags give approximate time measurement was made.
 - Using this time initially, position of satellite can be computed
 - Range computed from receiver and satellite position
 - Difference between observed pseudorange and computed ranges, gives effects of satellite and receiver clock errors. In point positioning, satellite clock error is assumed known and when removed from difference, error in receiver clock determined.
 - With new estimate of receiver clock, process can be iterated.
 - If receiver position poorly known, then whole system can be iterated with updated receiver coordinates.

Sensitivities

- Satellites move at about 1 km/sec, therefore an error of 1 msec in time results in 1 m satellite position (and therefore in range estimate and receiver position).
- For pseudo-range positioning, 1 msec errors OK. For phase positioning (1 mm), times needed to 1 μ sec.
- (1 μ sec is about 300 m of range. Pseudorange accuracy of a few meters in fine).

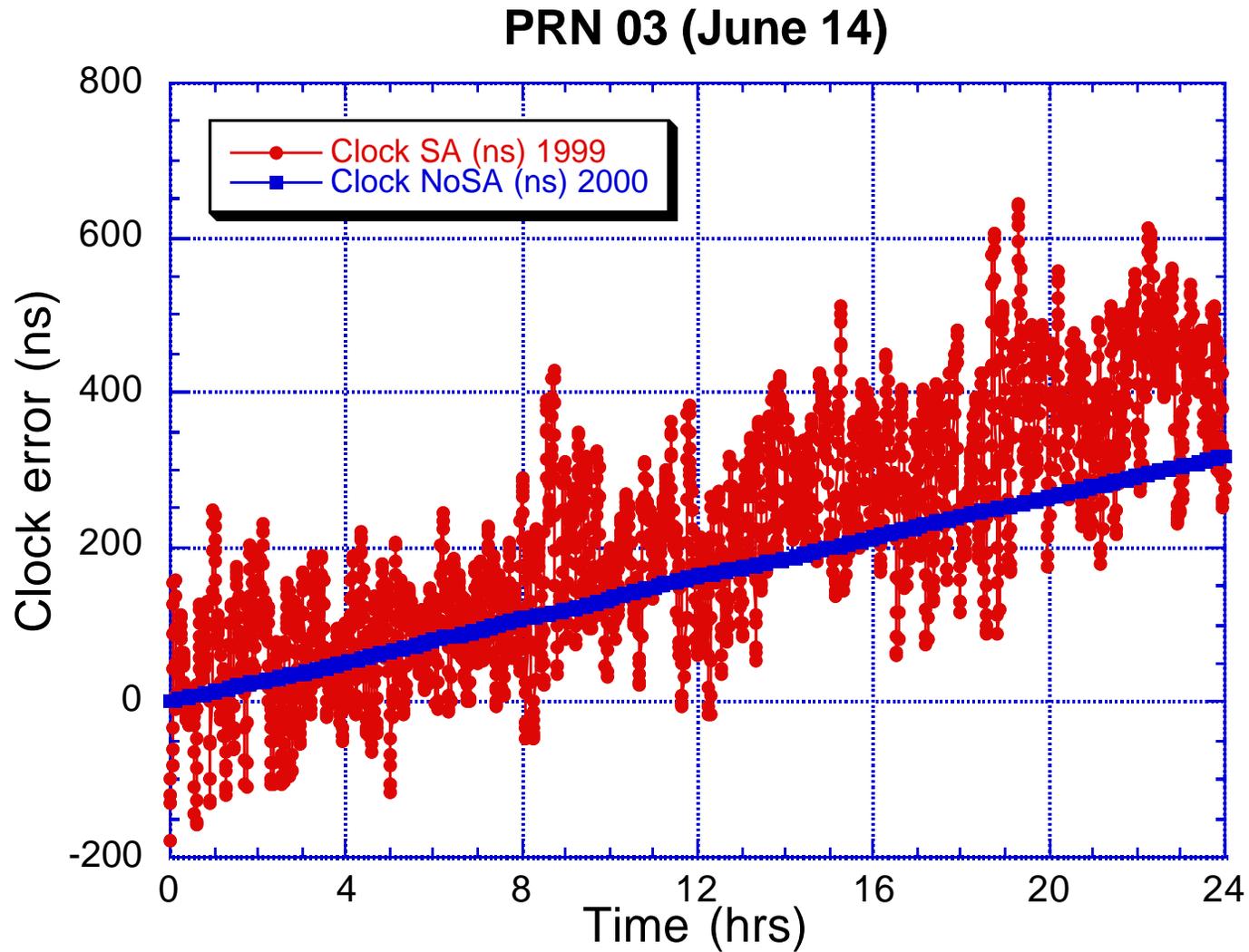
“Light-time-iteration”

- To compute theoretical range; two basic methods used
 - (a) “Doppler shift corrections” ie. Account for rate of change of range during propagation time
 - (b) “Light-time-iteration” Method most commonly used.
- Light time iteration: Basic process is to compute range using simple Cartesian geometry but with position of receiver at receive time and position of transmitter at transmit time.

Light-time-iteration

- Light time iteration must be computed in a non-rotating frame
- Reason: Consider earth-fixed frame: one would simply compute Earth fixed coordinates at earlier time. In non-rotating frame, rotation to inertial coordinates would be done at two different time (receiver when signal received; transmitter when signal transmitted).
- Difference is rotation of Earth on ~ 60 msec. Rotation rate ~ 460 m/sec; therefore difference is about 30 meters.

Clock errors



Relativistic effects

- General relativity affects GPS in three ways
 - Equations of motions of satellite
 - Rates at which clock run
 - Signal propagation
- In our GPS analysis we account for the second two items
- Orbits only integrated for 1-3 days and equation of motion term is considered small

Clock effects

- GPS is controlled by 10.23 MHz oscillators
- On the Earth's surface these oscillators are set to $10.23 \times (1 - 4.4647 \times 10^{-10})$ MHz (39,000 ns/day rate difference)
- This offset accounts for the change in potential and average velocity once the satellite is launched.
- The first GPS satellites had a switch to turn this effect on. They were launched with “Newtonian” clocks

Propagation and clock effects

- Our theoretical delay calculations are made in an Earth centered, non-rotating frame using a “light-time” iteration i.e., the satellite position at transmit time is differenced from ground station position at receive time.
- Two corrections are then applied to this calculation

Corrections terms

- Propagation path curvature due to Earth's potential (a few centimeters)

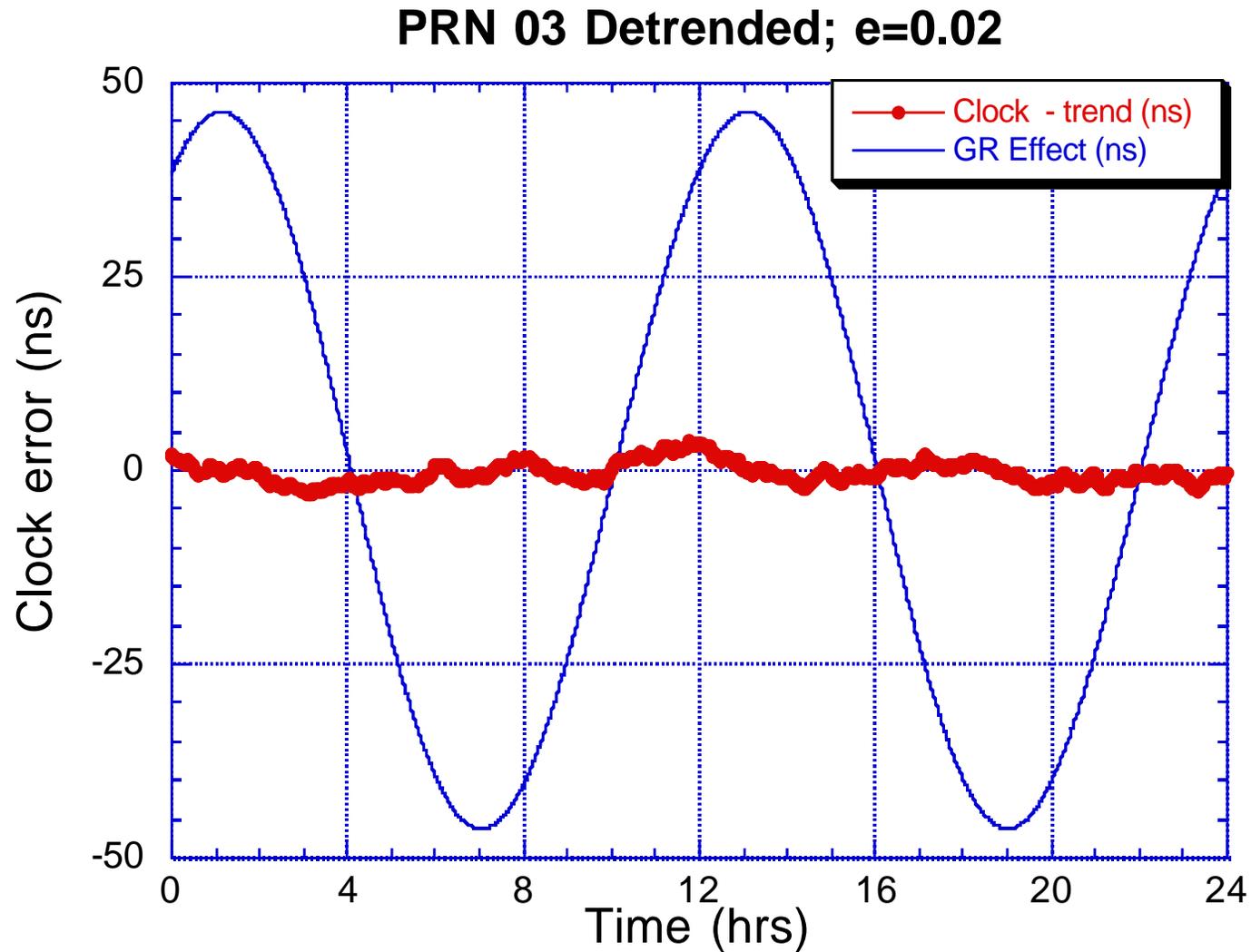
$$\Delta\tau = \frac{2GM}{c^3} \ln\left(\frac{R_r + R_s + \rho}{R_r + R_s - \rho}\right)$$

- Clock effects due to changing potential

$$\Delta\tau = \frac{-\sqrt{GM}}{c^2} e\sqrt{a} \sin E$$

- For $e=0.02$ effect is 47 ns (14 m)

Effects of General Relativity



Tests of General Relativity

- After some back of the envelope derivations, in the parameterized post-Newtonian formulation, the time delay expression* becomes:

$$\Delta\tau = \frac{-2\sqrt{GM}}{c^2} \frac{(2 + \gamma)}{3} e\sqrt{a} \sin E$$

- In PPN, γ is the gravitational curvature term. In general relativity $\gamma = 1$. It usually appears as $(1+\gamma)/2$ but this has been modified as explained next.
- The clock estimates from each GPS satellite allow daily estimates of γ

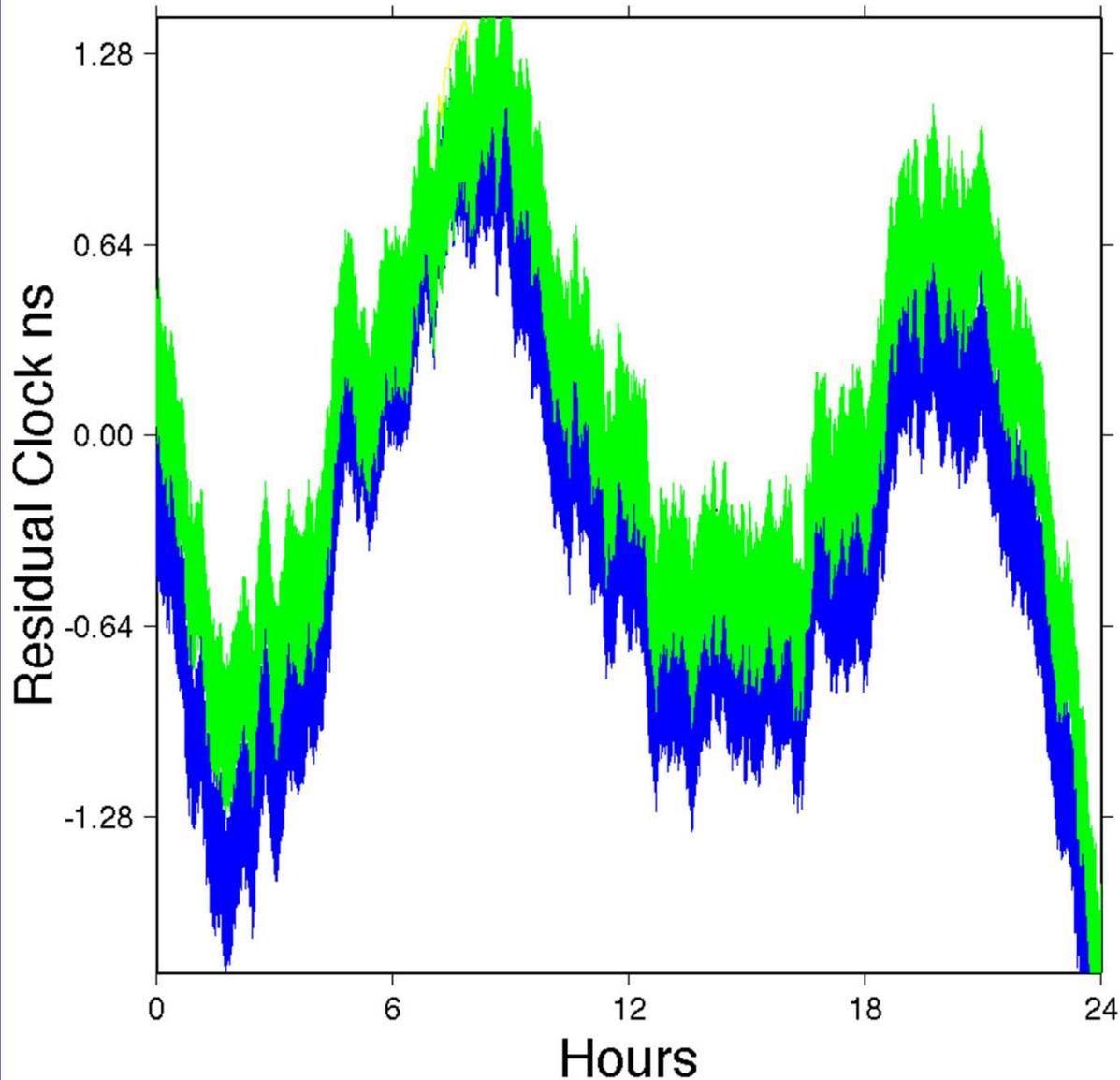
PPN formulation

- The clock rate correction is made of two parts:
 - Special relativity term that depends on $v^2/2c^2$ where v is the satellite velocity and c is the speed of light.
 - General relativity contribution that depends on $2U/c^2$ where U is potential
- The periodic variations arise from the eccentricity of the orbit: At perigee
 - v is fastest which slows clock
 - U is largest which also slows clock (decreasing potential increases the clock rate).
- The contribution to $\Delta\tau$ from the special relativity term is half that of the general relativity term and thus the γ formulation.

Using GPS to determine γ

- Each day we can fit a linear trend and once-per-revolution sin and cos terms to the clock estimates of each of the 28-32 GPS satellites.
- Comparison between the amplitude and phase (relative to $\sin(E)$) allows and estimate of gamma to be obtained
- Quadrature estimates allows error bound to be assessed ($\cos(E)$ term)
- **Problems:**
 - Once-per-orbit perturbations are common. However should not be proportional to eccentricity.
 - Also possible thermal effects on the clocks in the satellites. (This is probably largest effect as we will see).
 - General quality of clocks in the satellites.
- There are multiple groups that process global GPS data using different processing software and analysis methods (double differencing versus one-way clock estimation).

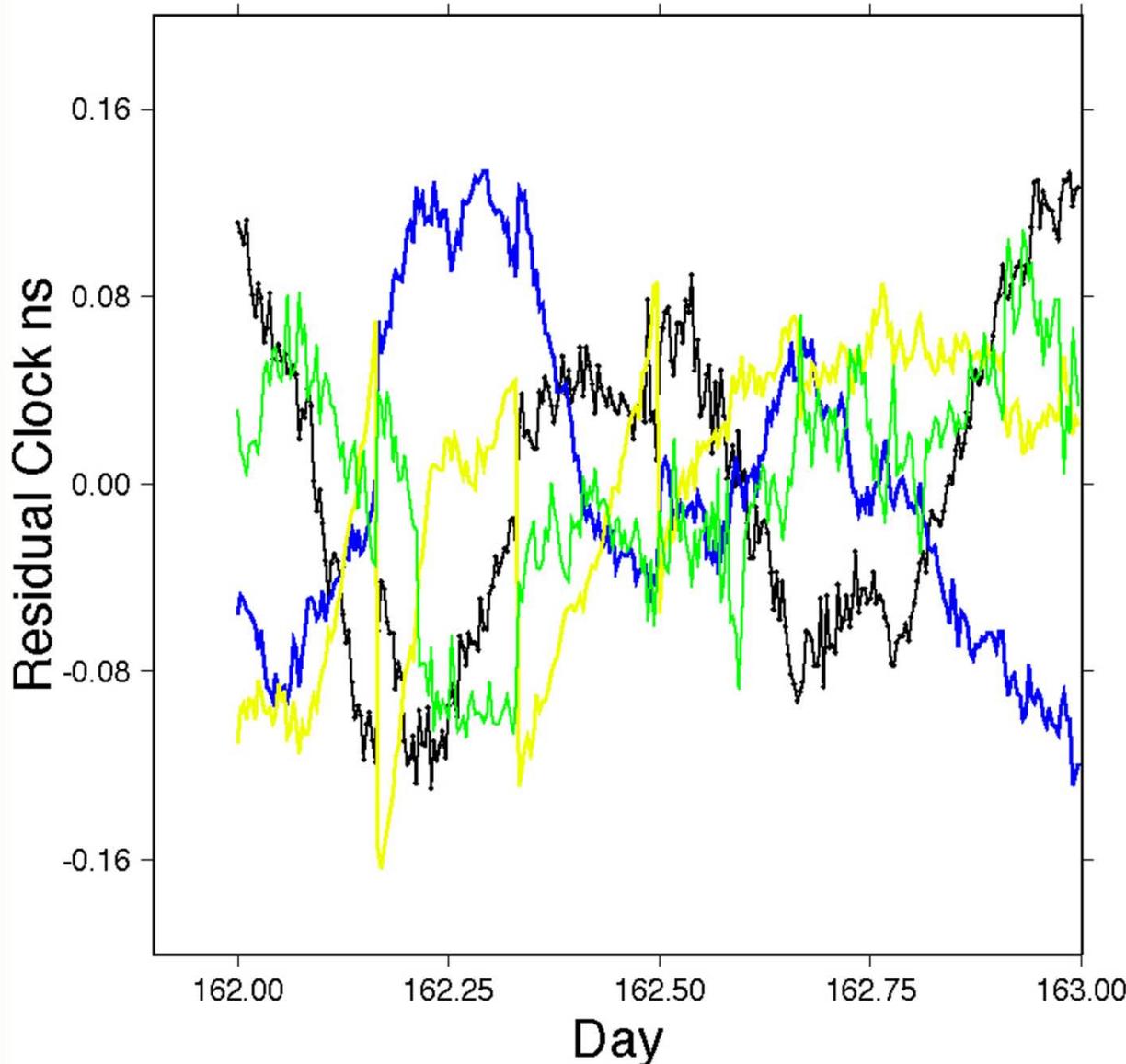
MIT: Wmean 509059.939 ns, Rate -55578.020 +- 8.505 ps/hr, RMS 634.0 ps # 288
IGS: Wmean 509058.756 ns, Rate -55533.300 +- 8.505 ps/hr, RMS 645.0 ps # 288
COD: Wmean 509025.632 ns, Rate -55548.159 +- 20.648 ps/hr, RMS 705.0 ps # 288
GFZ: Wmean 509060.400 ns, Rate -55563.757 +- 8.505 ps/hr, RMS 649.0 ps # 288
jpl: Wmean 509061.284 ns, Rate -55528.817 +- 2.690 ps/hr, RMS 661.0 ps # 2880
CLOCK COMPARISON GPS WEEK 1274 DAY 4 SITE G07



Example
satellite clock
(PRN 07)

Results from
different analysis
groups
Eccentricity is
 0.011 ; $\tau_g \sim 25$ ns.

MIT: Wmean -0.550 ns, Rate 0.000 +- 0.000 ps/hr, RMS 67.0 ps # 288
COD: Wmean 0.051 ns, Rate 0.000 +- 0.000 ps/hr, RMS 66.0 ps # 288
GFZ: Wmean 0.122 ns, Rate 0.000 +- 0.000 ps/hr, RMS 61.0 ps # 288
JPL: Wmean 0.101 ns, Rate 0.000 +- 0.000 ps/hr, RMS 48.0 ps # 288
CLOCK DIFFERENCE GPS WEEK 1274 DAY 4 SITE G07

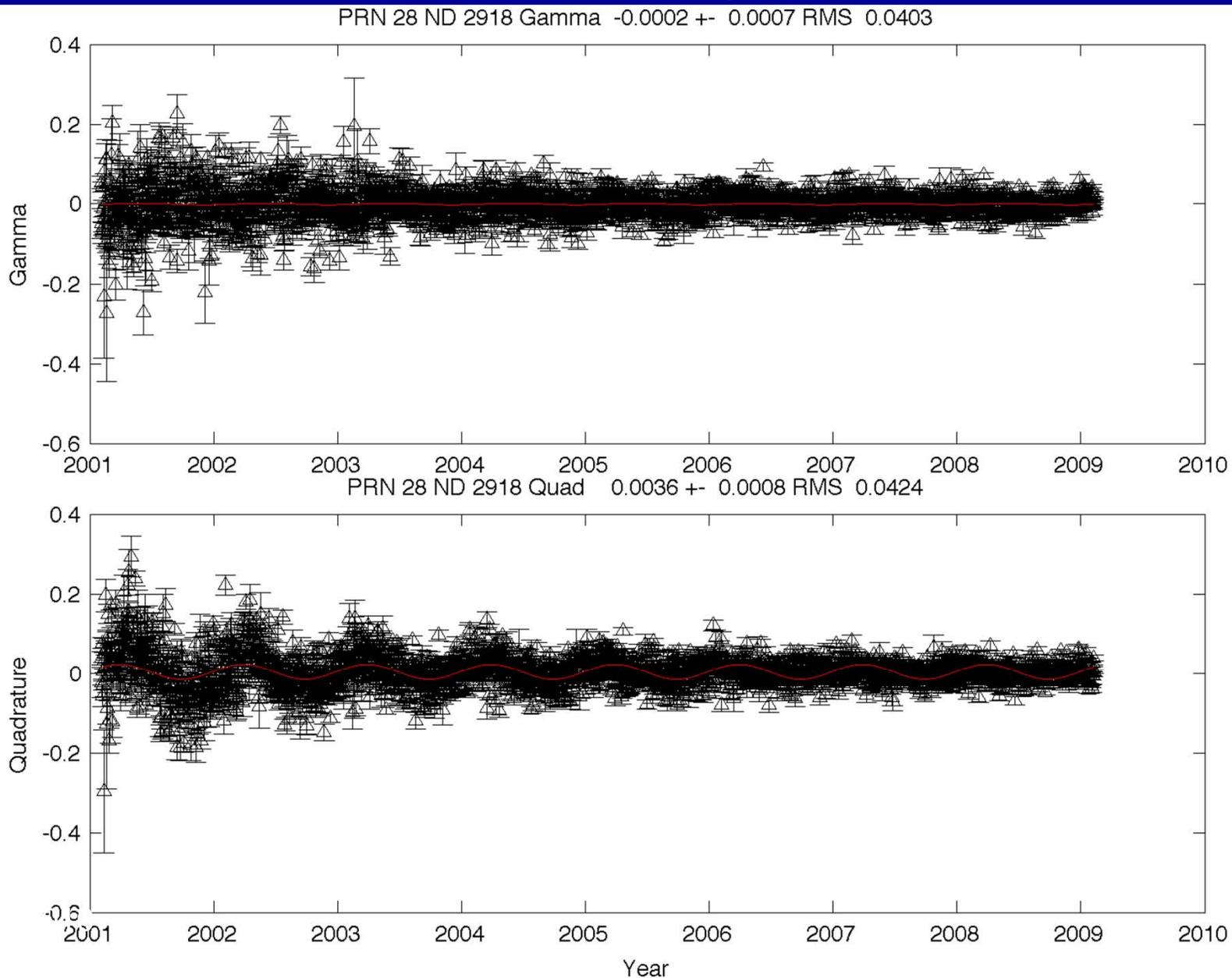


Differences between groups

- RMS differences are about 0.2% of relativistic effect (measure of “processing” noise).
- Clock variation is ~4%.
- Eccentricity ~0.011: $\Delta\tau$ 25 ns

PRN 28 MIT Analysis

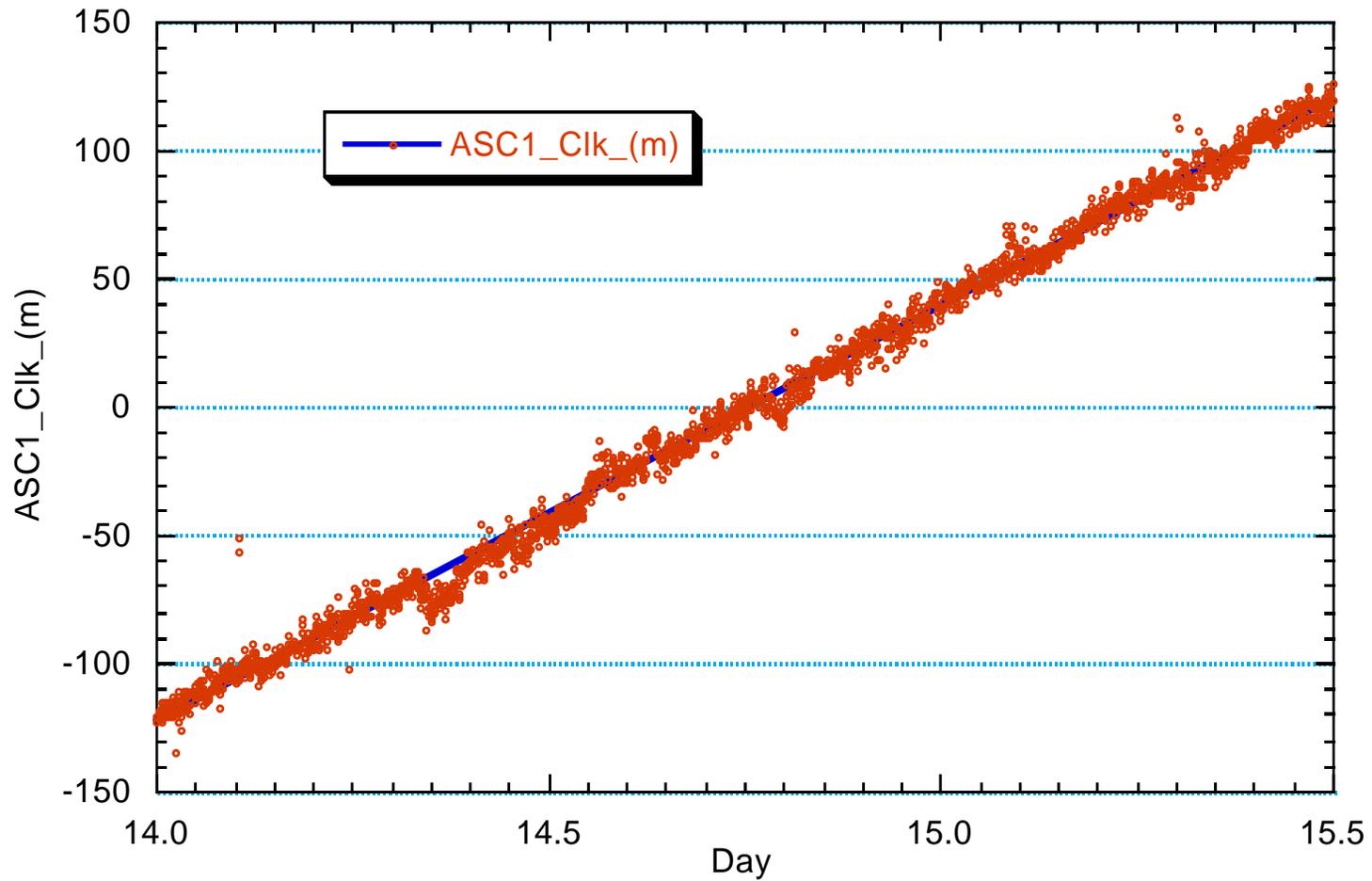
Error bars from RMS fit to clock estimates (linear, sinE and cosE terms only). Red line is fit to annual signal.



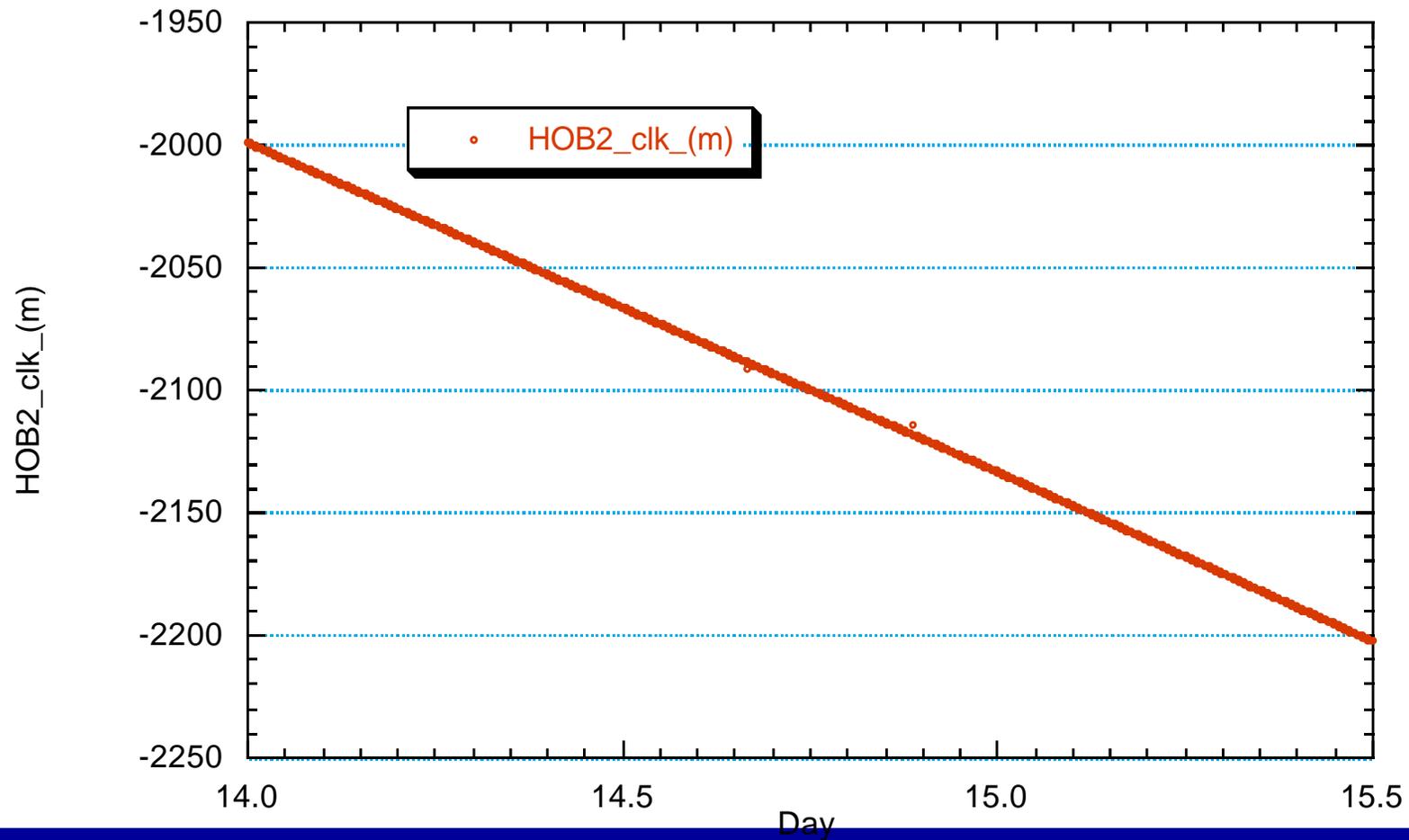
Examples of receiver clock behavior

- Examples of satellite and station clock behaviors can be found at:
- <http://geoweb.mit.edu/~tah/MITCk>
- IGS Time Standards are given at:
<https://goby.nrl.navy.mil/IGStime/index.php>
- Directories are by GPS week number and directories ending in W are total clock estimates; folders ending in D are differences between IGS analysis centers
- Now examine some examples

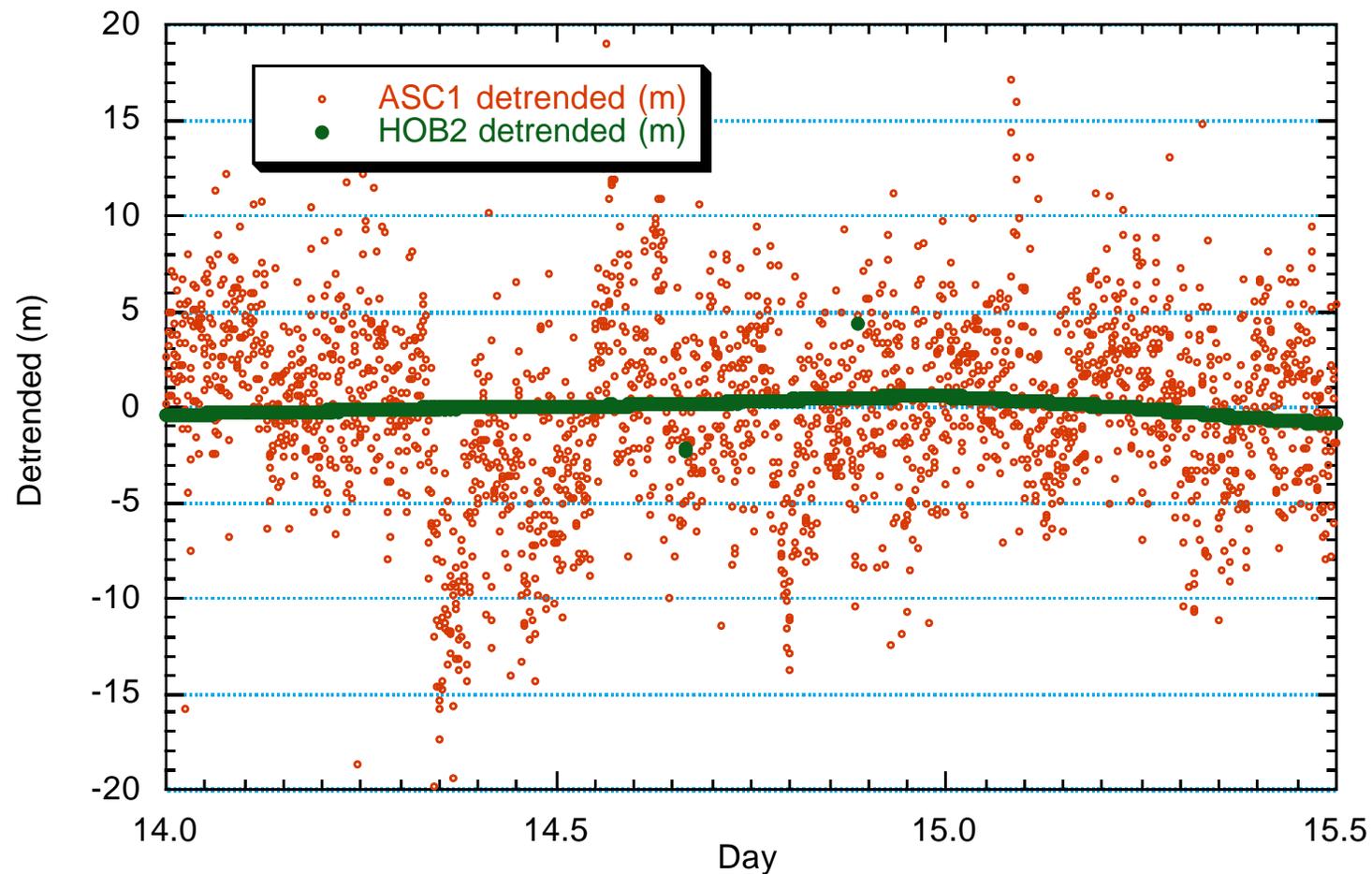
Receiver clocks: ASC1



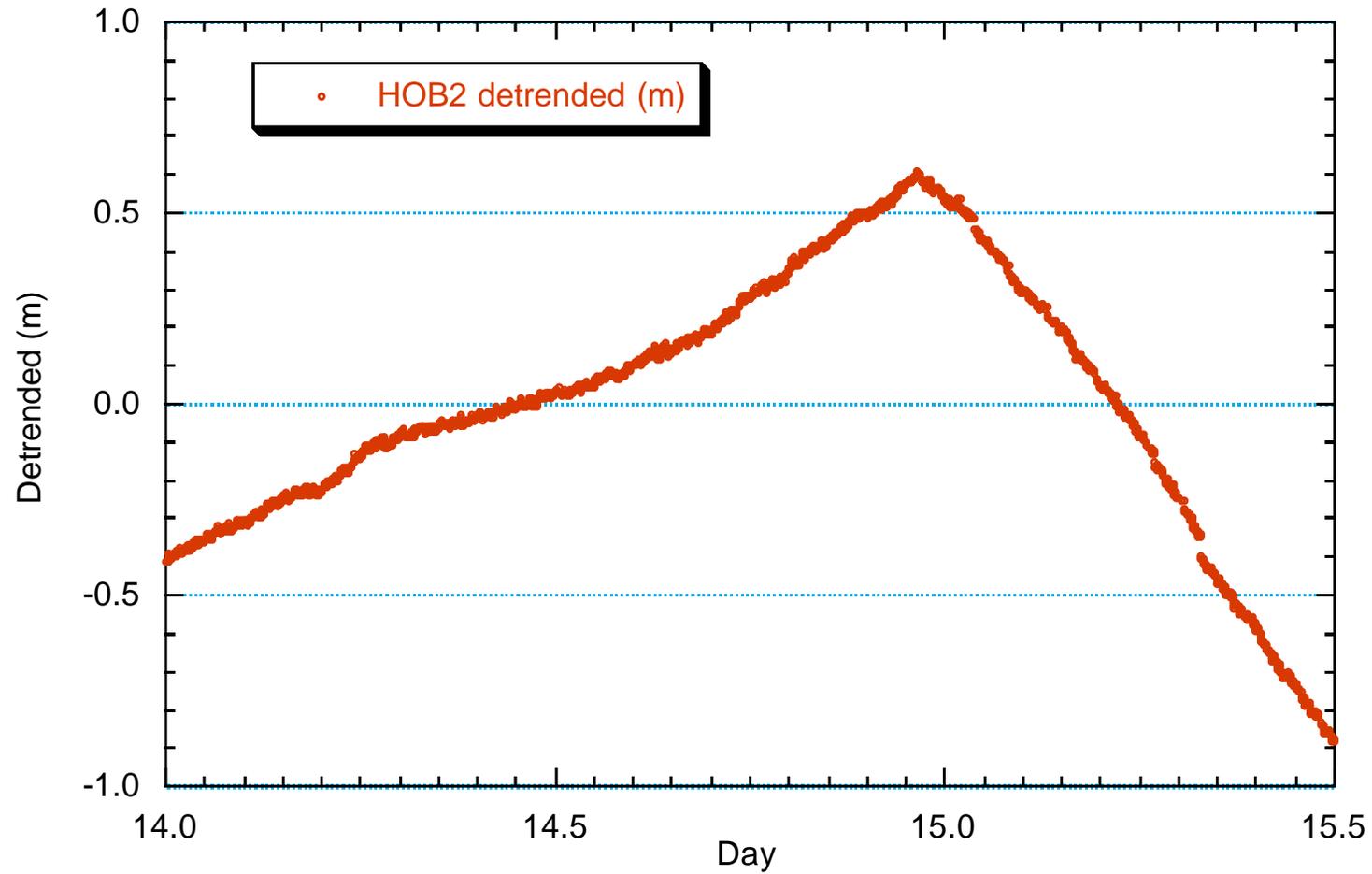
Receiver Clock: HOB2 Hydrogen Maser



ASC/HOB2 Linear trends removed



HOB2 only



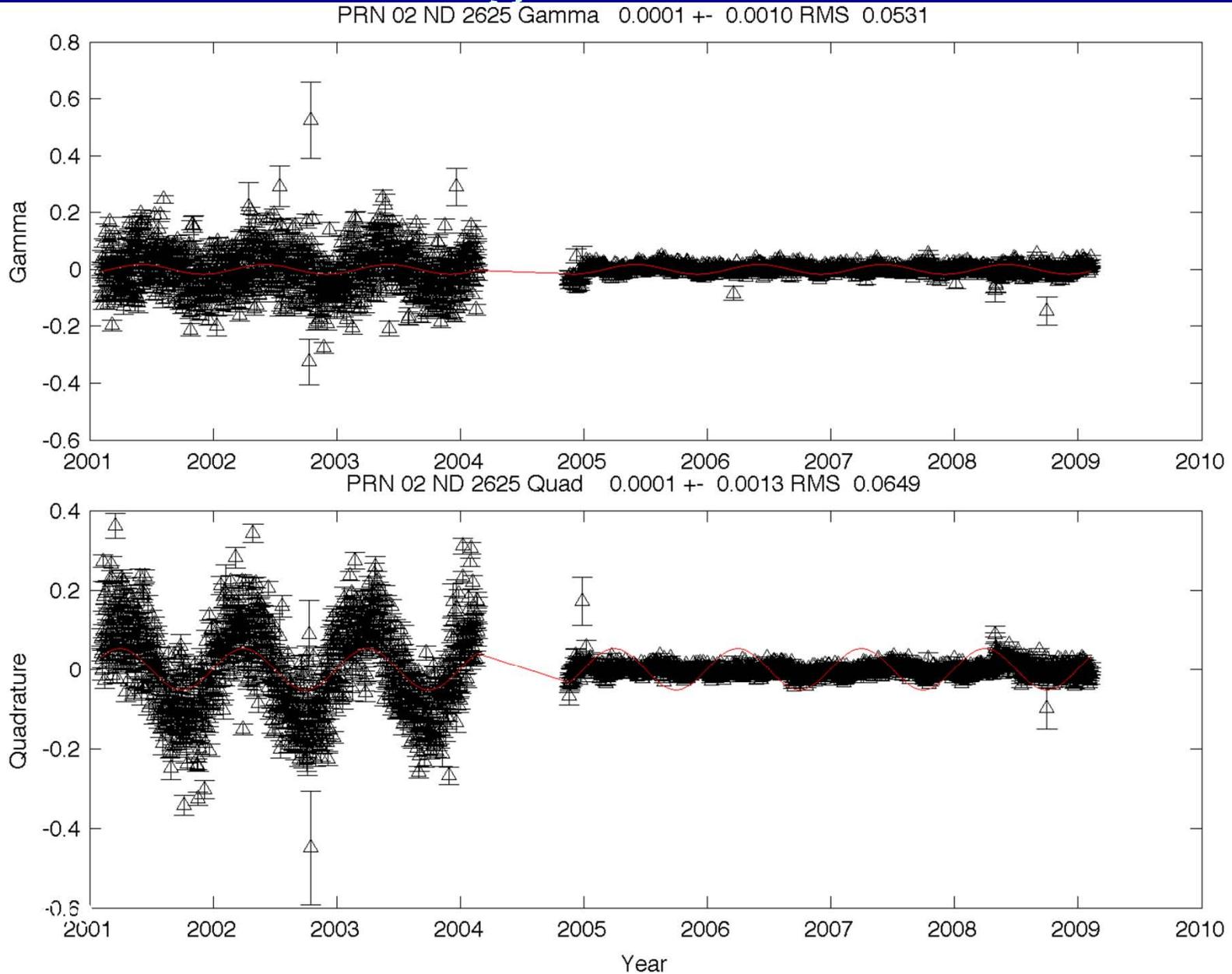
Summary of clocks

- In some cases; clock are well enough behaved that linear polynomials can be used.
- Most commonly: receiver clocks are estimated at every measurement epoch (white noise clocks) or GPS data is differenced to remove clock (as in question 2 of HW 2).
- Errors in receiver clocks are often thousands of km of equivalent time.
- More detailed analysis of gamma estimates in following slides.

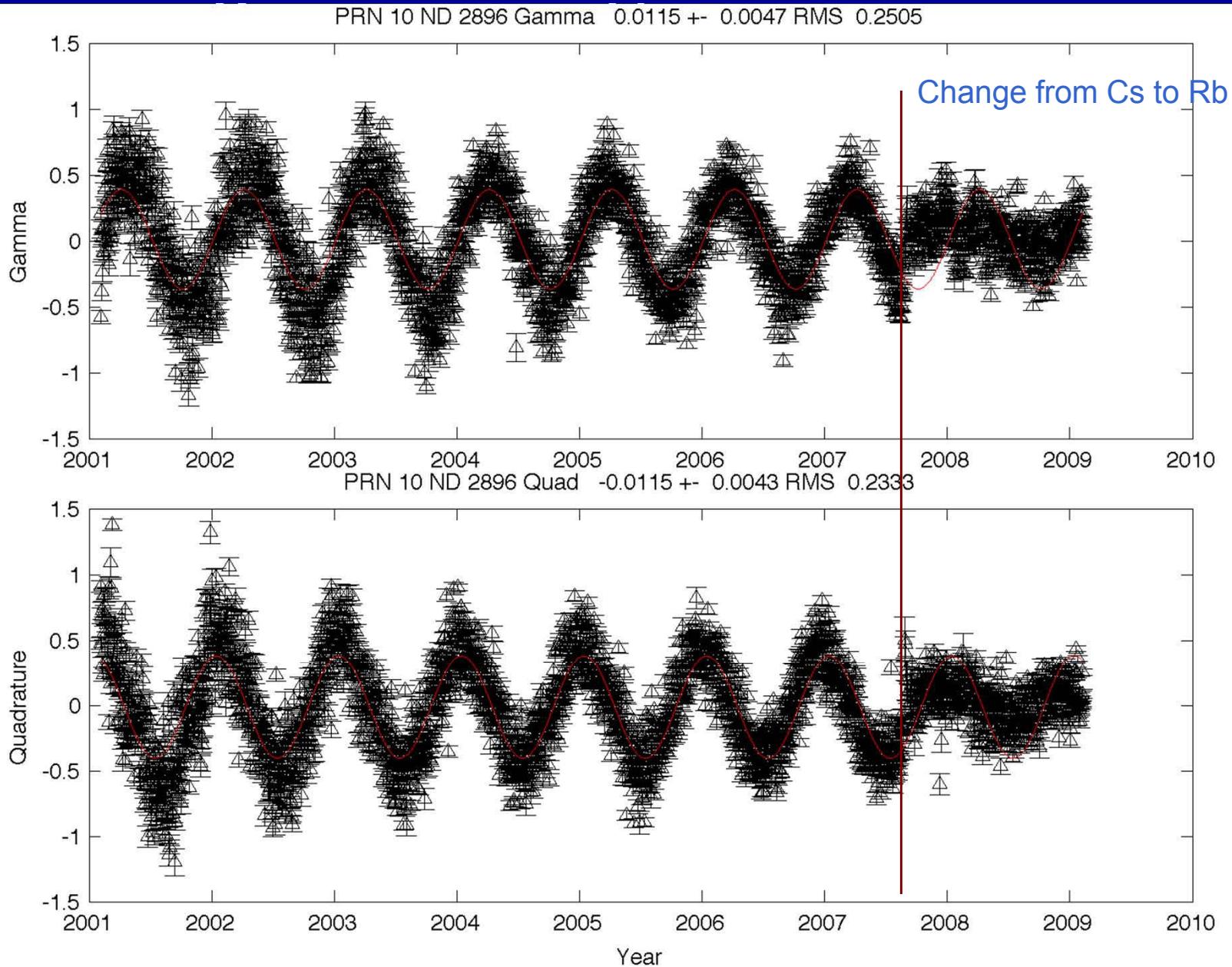
Analysis of result

- Not all satellites look like PRN 28. Only a few satellites are so well behaved.
- The following figures show some examples that show:
 - Change in character when satellite changed (PRN02)
 - Effects of change from Cs to Rb clocks (PRN10)
 - Effects of changing eccentricity of orbit (PRN18) - change from 0.001 to 0.010
 - Period of oscillation: Draconic period: Period of orbital plane relative to sun (precession of node makes this different from 365.25 days). Since all satellites in ~55 deg inclination and precess at the same rate, period is 351.4 days.

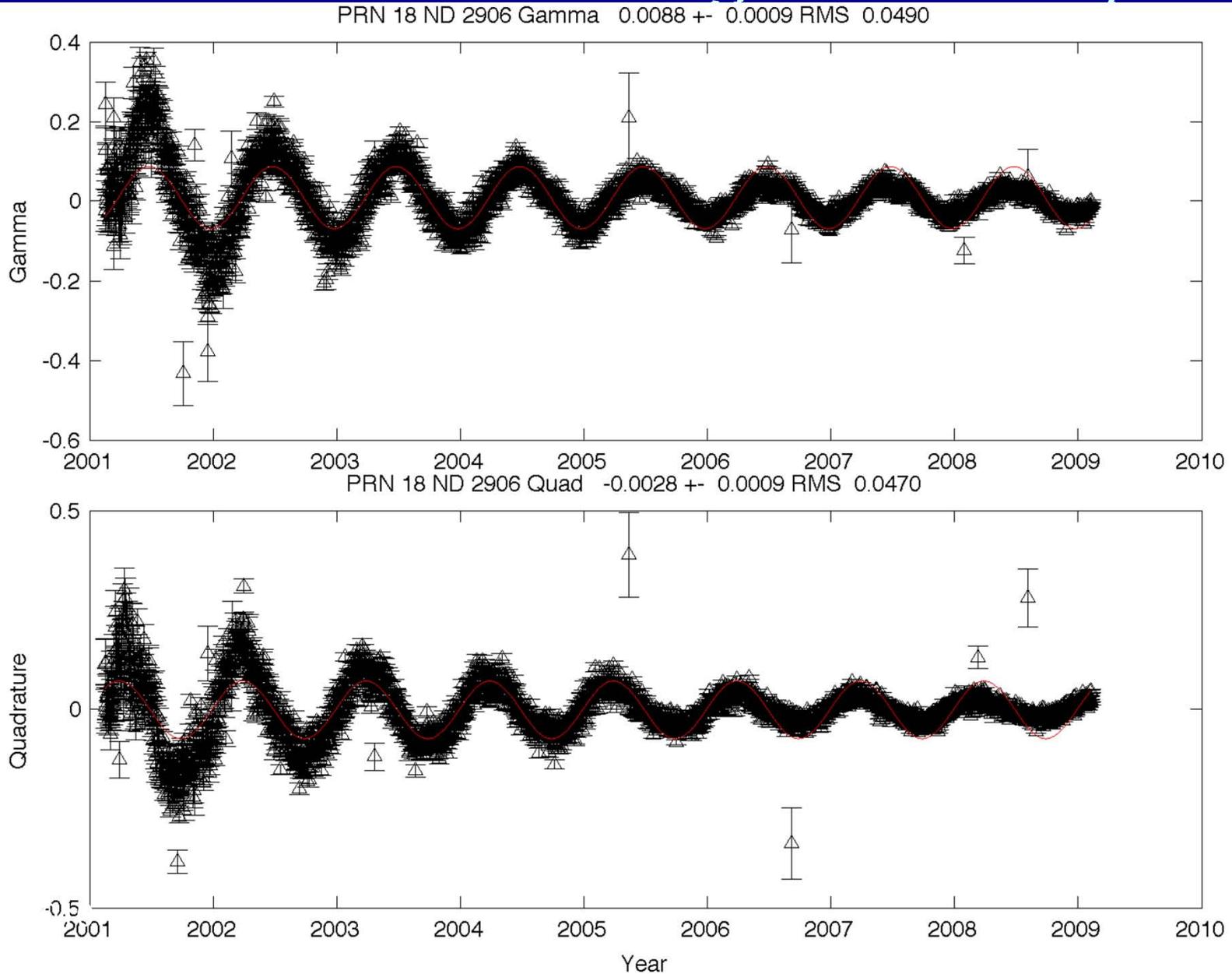
Change of satellite



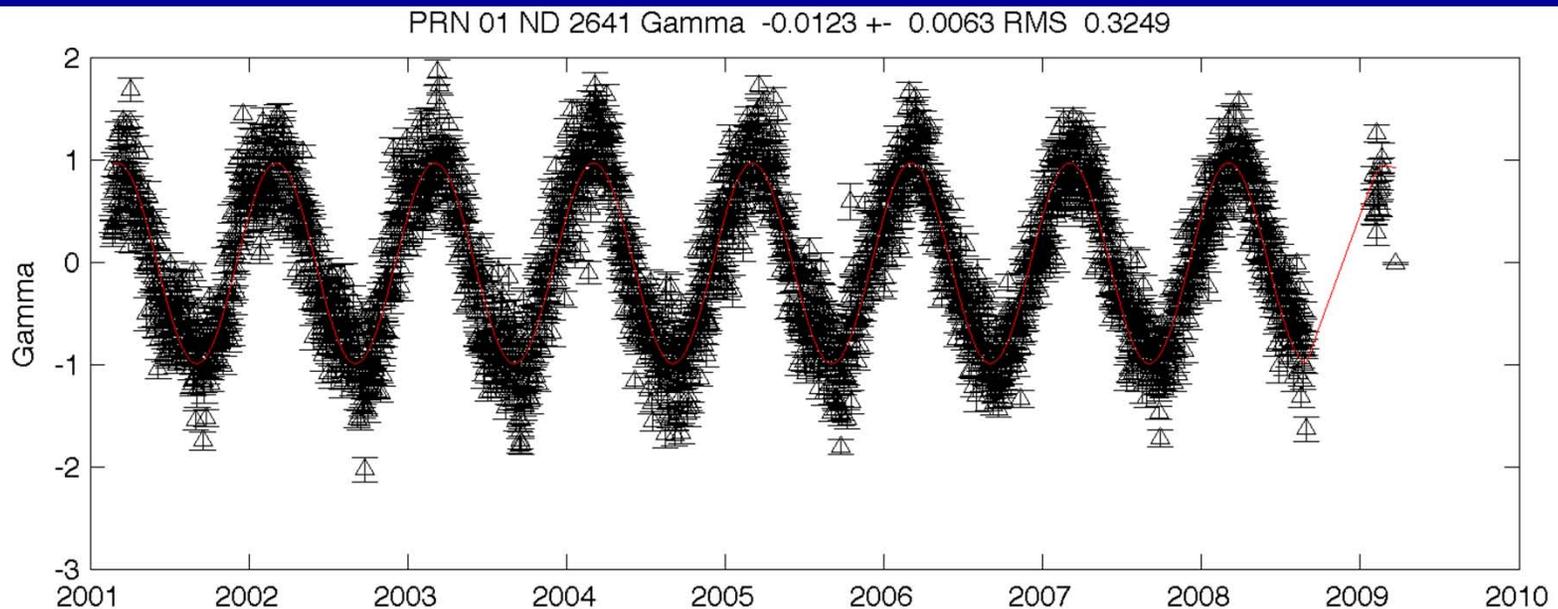
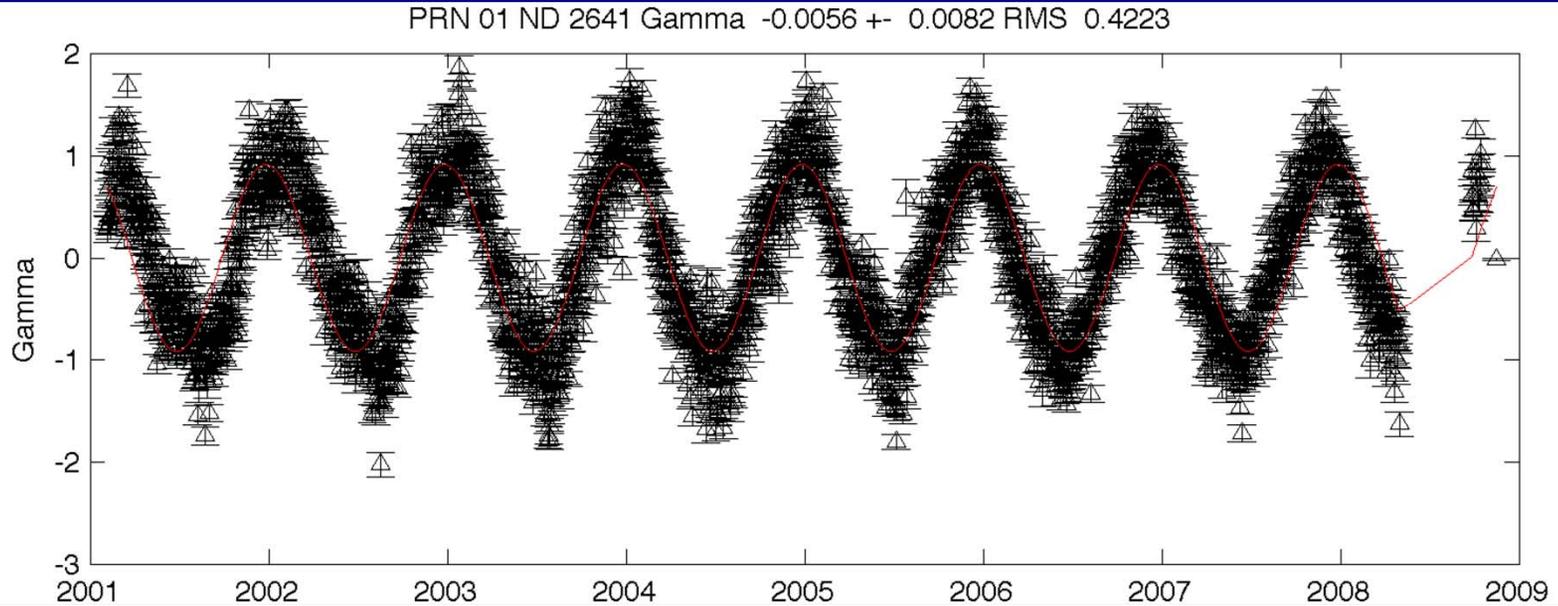
Change of clock type from Cs to Rb



Effects of increasing eccentricity



Annual versus Draconic Period



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