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12.510 Introduction to Seismology  
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## Seismic tomography

We will study travel time tomography for body waves.

### Recall Geometric Ray Theory

*Plane wave superposition:*

$$\phi(x, y, z, t) = \phi(x, t) \approx \int_{-\pi}^{\pi} \int_{-\infty}^{+\infty} \phi(k_x, k_z, \omega) e^{i(kx - \omega t)} dk_x dk_z d\omega$$

The integration over frequencies is important, but for now we will consider the *high frequency approximation*, which leads to **ray theory**.

We can gain insight into the behavior of the seismic waves by considering the ray paths associated with them. This approach, studying wave propagation using ray path, is called **Geometric Ray Theory (GRT)**. Although it does not fully describe important aspects of wave propagation, it is widely used because it often greatly simplifies the analysis and gives a good approximation.

### *Eikonal Equation*

For the general case:

$$|\nabla T|^2 = \frac{1}{c^2} \Rightarrow |\nabla T(\underline{x})| = \frac{1}{c(\underline{x})}$$

$$\nabla T(x) = \frac{1}{c(x)} \underline{k} = \underline{s} = (p, \eta) = \text{slowness vector}$$

This equation implicates that

- ✧ Rays are perpendicular to wavefronts.
- ✧ The slowness gives the gradient of the travel time, and the gradient of the travel time specifies the direction of the ray. Each time  $c(x)$  changes, the gradient of  $T$  has to change, and the direction of propagation changes at the same time.
- ✧ If one knows  $c(x)$ , there is a way to reconstruct the direction of the ray: **Eikonal Ray Tracing**.

### Velocity Distribution and Travel Time Curves

Different distributions of velocity with depth produce characteristic travel time curves. The following figure shows the usual situation when velocity increases slowly with depth. Given two rays, the one with a smaller angle of incidence at the source has a smaller  $p$ , thus bottoms deeper at a point with smaller  $r_p$ , and eventually emerges further from the source. Thus the ray parameter decreases, and travel time increases, monotonically with distance.

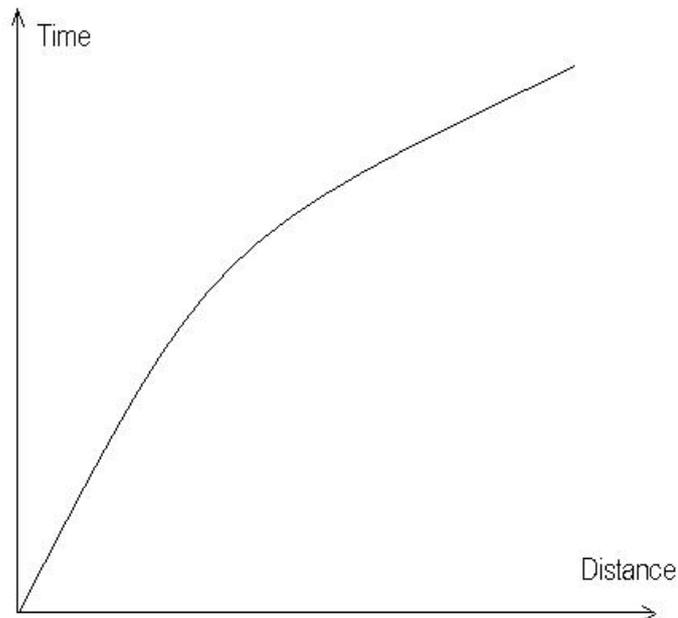


Figure 1 Travel time curve for velocity increasing slowing with depth

A more complicated situation occurs when velocity increases rapidly with depth. Rays that bottom either above or below the region of high velocity gradient behave as in the following figure, so the corresponding portions of the travel time and ray parameter curve show travel time increasing with distance. By contrast, rays that bottom in the region of high velocity gradient are bent upward more and emerge at smaller values of distance that would otherwise be the case. As a result, three rays with different ray parameters emerge at the same distance. As a result, three rays with different ray parameters emerge at the same distance, and the curve has three distinct branches, giving a characteristic **triplication**.

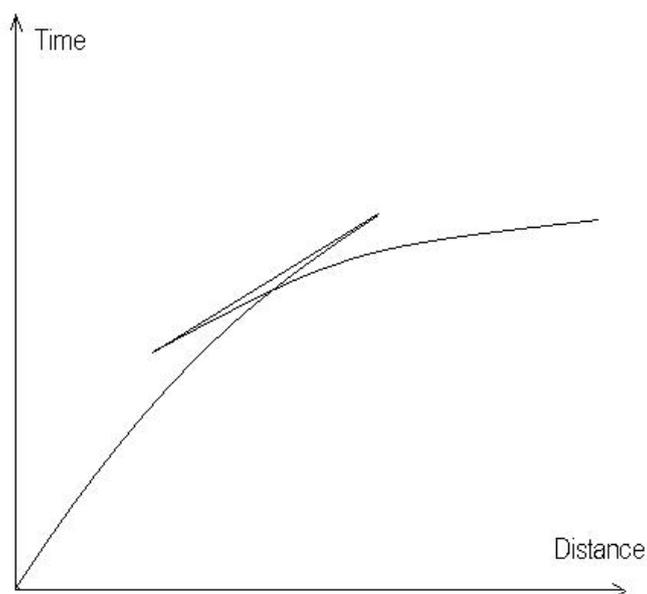


Figure 2 Travel time curve for velocity increasing rapidly

A third important case is a low velocity zone, where velocity decreases with depth and then increases. Rays entering the low-velocity zone bend down, rather than up, so no rays bottom there. This situation causes a **shadow zone**, a region of the earth's surface where no rays arrive. Just below the low-velocity zone, rays reach a given distance by two paths, giving two values of travel time for a given distance.

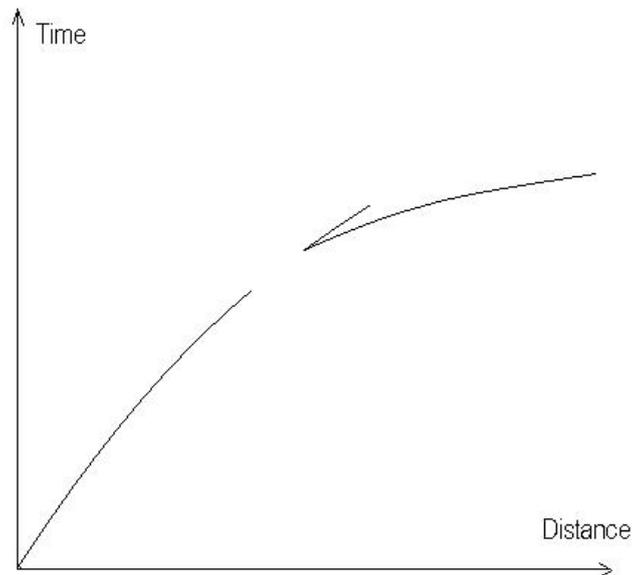


Figure 3 Travel time curve for a low-velocity zone

### **Nomenclatures of Body Waves**

At this stage, it is useful to introduce the jargon used to describe the different types of body wave propagation in Earth's interior. There are a few simple basic rules, but there are also some inconsistencies. Capital letters are used to denote body wave propagation (transmission) through a medium. On the other hand, lower case letters are used to indicate either reflections or upward propagation of body waves before they are reflected at Earth's surface. Note that this is always used in combination of a transmitted wave. The followings are summary of nomenclature of body wave.

- P a P wave in the mantle
- S an S wave in the mantle
- K a P wave through the outer core
- I a P wave through the inner core
- J an S wave through the inner core
- c a reflection from the mantle – outer core boundary
- i a reflection from the outer core – inner core boundary
- p a upgoing P wave from the earthquake focus to the surface of the earth

s a upgoing S wave from the earthquake focus to the surface of the earth

LR a Rayleigh wave

LQ a Love wave

The following figures describe the ray paths in terms of various phases. Note that the phases are in combination of a wave described above.

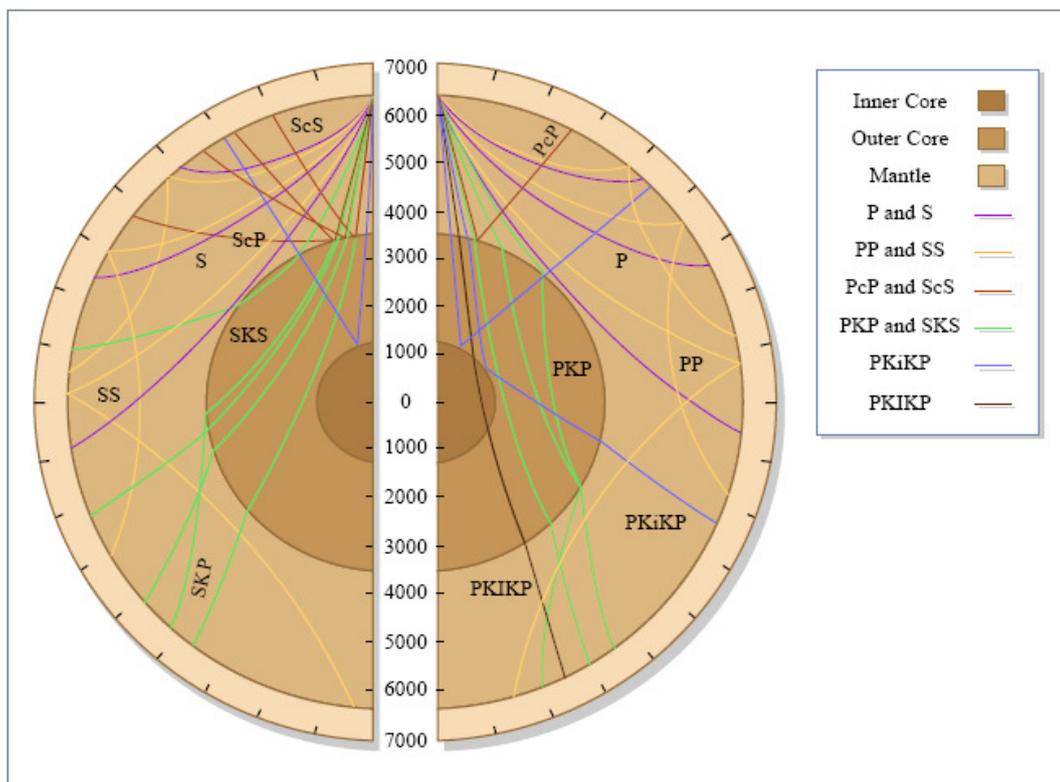


Figure by MIT OCW.

Figure 4 The main seismic phases in the earth

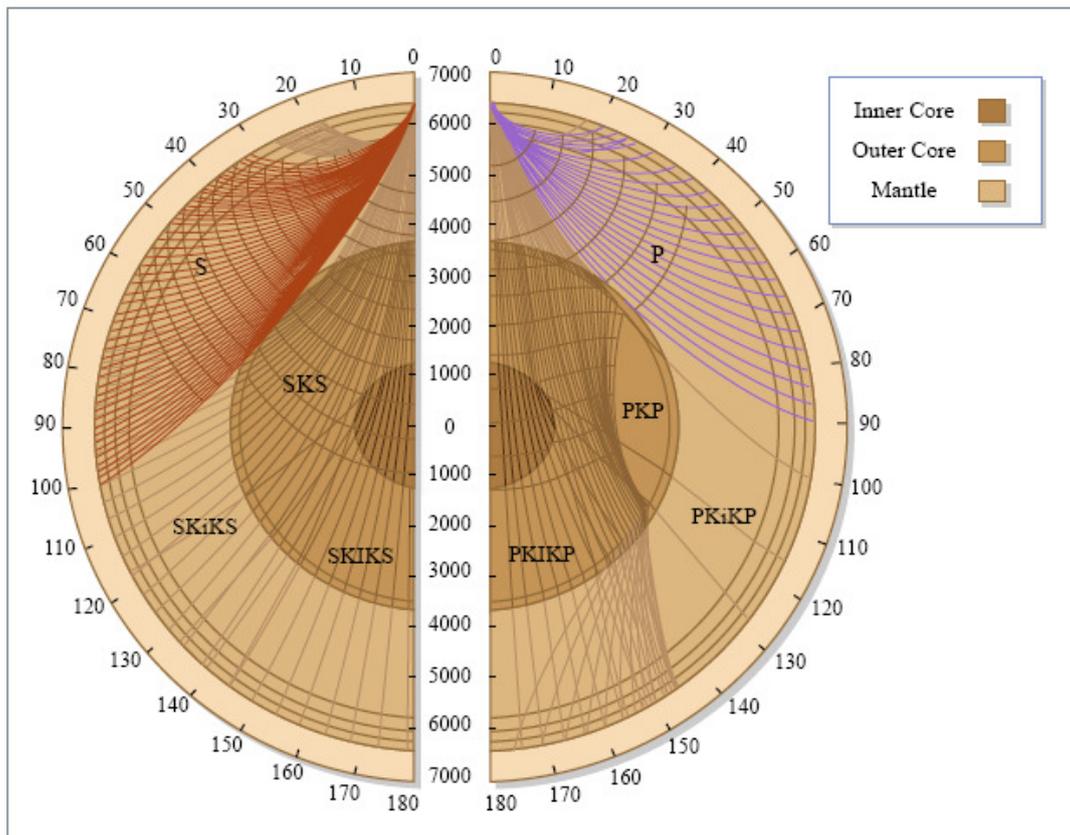


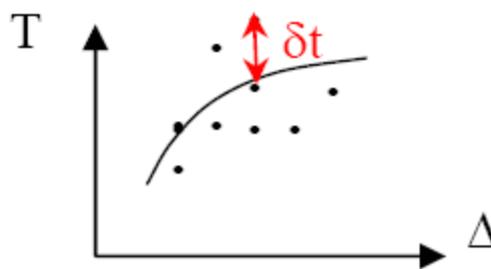
Figure by MIT OCW.

Figure 5 Ray paths for major P and S phases for the AKI35 model of seismic wave speeds

**Imaging the earth with seismic data**

**Steps:**

- ✧ Obtain observations, represented by dots on the figure below (picking of the different waves: P, S, PP...).



- ✧ Generate linear regression to give the best fit of these observations.
- ✧ Set up models (e.g.: IASP91, PREM...)
- ✧ Use an inversion technique to get the P & S wave velocity as a function of the depth.
- ✧ From these models, we can produce travel time curves again.