Lecture Packet #3: Hydraulic Head and Fluid Potential

What makes water flow?

Consider pressure
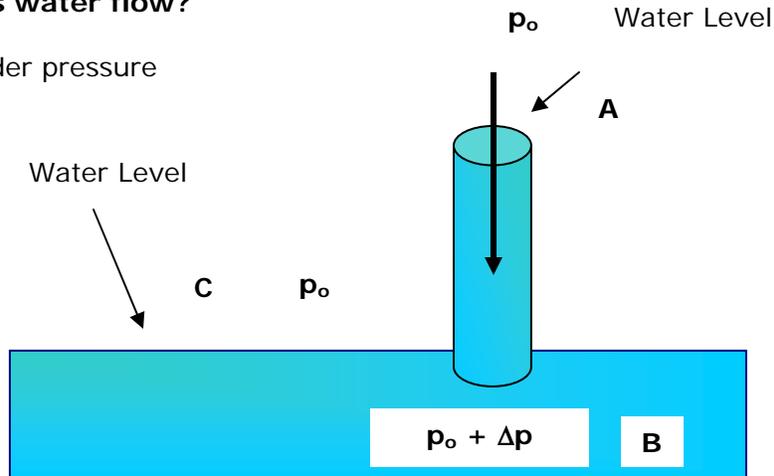
Pressure at A = atmospheric (p_o)
Pressure at B > atmospheric ($p_o + \Delta p$)
Pressure at C = atmospheric (p_o)

But flow is not from A to B to C.
Flow is not "down pressure gradient."

Hubbert (1940) – **Potential**

> A physical quantity capable of measurement at every point in a flow system, whose properties are such that flow always occurs from regions in which the quantity has less higher values to those in which it has lower values regardless of the direction in space.

Examples:
Heat conducts from high temperature to low temperature
- Temperature is a potential

Electricity flows from high voltage to low voltage
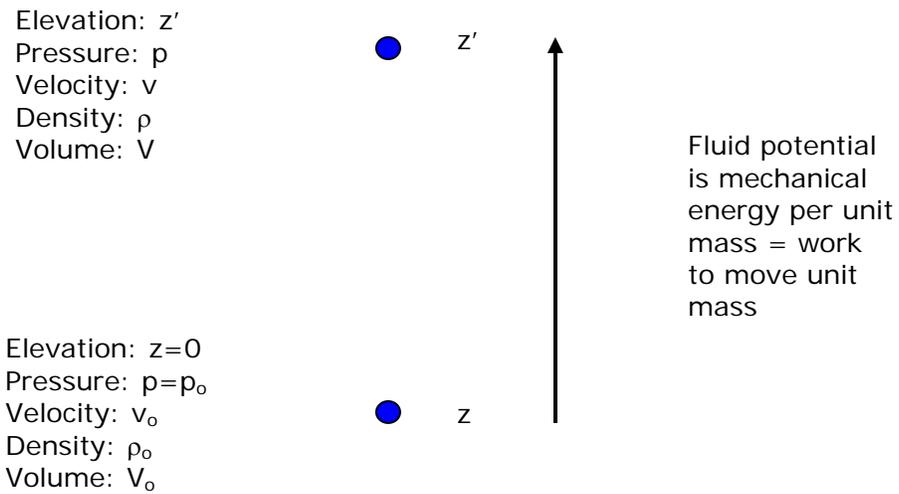- Voltage is a potential

Fluid potential and hydraulic head
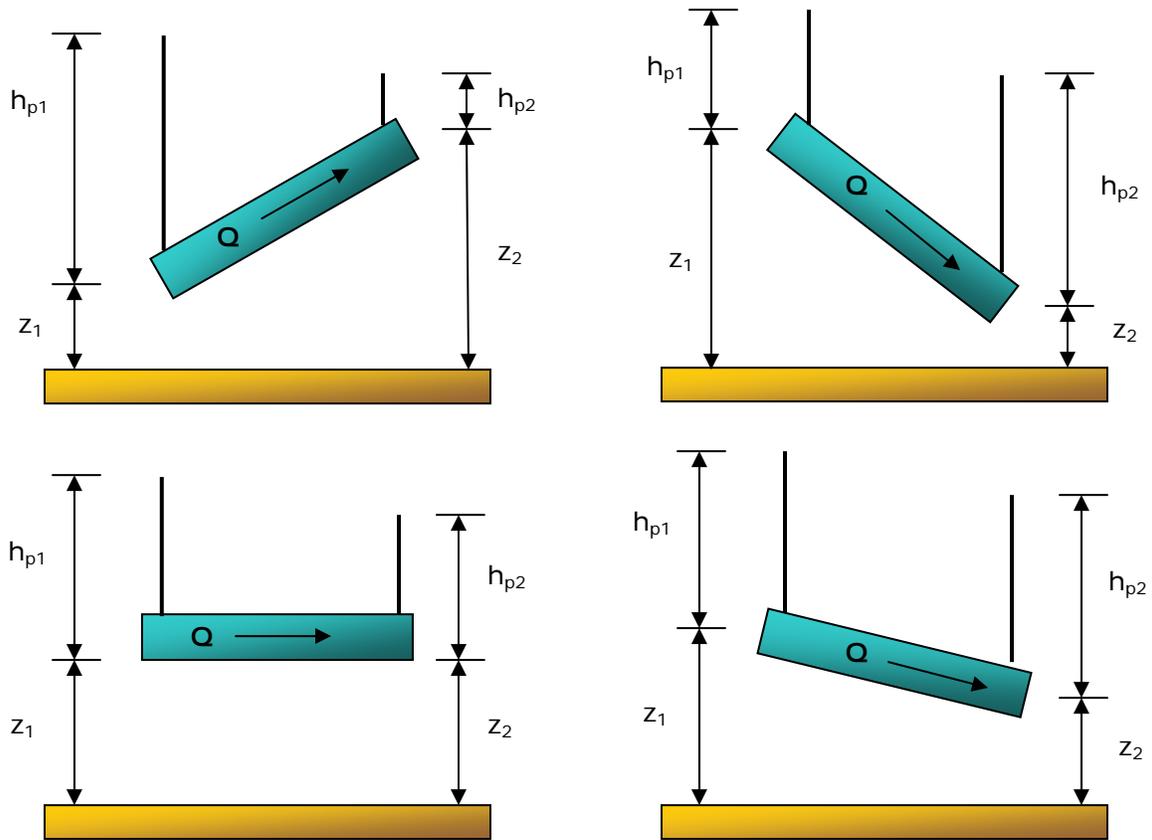
Fluids flow from high to low fluid potential
- Flow direction is away from location where mechanical energy per unit mass of fluid is high to where it is low.
- How does this relate to measurable quantity?

Groundwater flow is a **mechanical process** – forces driving fluid must overcome <u>frictional forces</u> between porous media and fluid. (generates thermal energy)

Work – mechanical energy per unit mass required to move a fluid from point z to point z'.

Elevation: z'
Pressure: p
Velocity: v
Density: ρ
Volume: V

z'

Fluid potential is mechanical energy per unit mass = work to move unit mass

Elevation: z=0
Pressure: $p=p_o$
Velocity: v_o
Density: ρ_o
Volume: V_o

z

Which way does water flow (function of Z and P) ?

Fluid potential is the mechanical energy per unit mass of fluid potential at z' = fluid potential at datum + work from z to z'.

The work to move a unit mass of water has three components:

1) Work to lift the mass (where z = 0)

$$w_2 = mgz'$$

2) Work to accelerate fluid from v=0 to v'

$$w_2 = \frac{mv^2}{2}$$

3) Work to raise fluid pressure from $p=p_o$ to p

$$w_3 = \int_{P_o}^{p} Vdp = m\int_{P_o}^{p} \frac{V}{m}dp = m\int_{P_o}^{p} \frac{dp}{p}$$

Note that a unit mass of fluid occupies a volume V = 1/ρ

The Fluid Potential (the mechanical energy per unit mass, m=1)

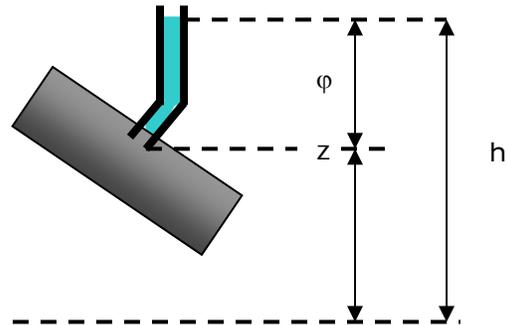$$\phi = gz + \frac{v^2}{2} + \int_{p_o}^{p} \frac{dp}{p} +$$

$$\phi = gz + \frac{v^2}{2} + \frac{p - p_0}{p} \qquad \text{for incompressible fluid (ρ is constant)}$$

This term is almost always unimportant in groundwater flow, with the possible exception of where the flow is very fast, and Darcy's Law begins to break down.

How does potential relate to the level in a pipe?

At a measurement point pressure is described by:

P = ρg(depth) + p$_o$
p = ρgφ + p$_o$
p = ρg(h-z) + p$_o$

Return to fluid potential equation

$$\phi = gz + \frac{v^2}{2} + \frac{p - p_0}{p}$$

Neglect velocity (kinetic) term, and substitute for p

$$\phi = gz + \frac{\rho g (h - z) + p_0 - p_0}{\rho}$$

So,

$$\phi = gh \qquad \text{or} \qquad h = \phi / g$$

Thus, head h is a fluid potential.

- Flow is always from high h to low h.
- H is energy per unit weight.
- H is directly measurable, the height of water above some point.

$$h = z + \varphi$$

Thermal Potential

$$\varphi_t = \text{Thermal Potential}$$

Temperature can be an important driving force for groundwater and soil moisture.
- Volcanic regions
- Deep groundwater
- Nuclear waste disposal

Can cause heat flow and also drive water.

Chemical Potential
Adsorption Potential

Total Potential is the sum of these, but for saturated conditions for our initial cases we will have:

$$\varphi = \varphi_g + \varphi_p$$

$$q^* = -L_1 \frac{\partial h}{\partial l} - L_2 \frac{\partial T}{\partial l} - L_3 \frac{\partial c}{\partial l}$$

Derivation of the Groundwater Flow Equation

Darcy's Law in 3D

Homogeneous vs. Heterogeneous
Isotropic vs. Anisotropic

Isotropy – Having the same value in all directions. K is a scalar.

K

$$q_x = -K \frac{\partial h}{\partial x} \qquad q_y = -K \frac{\partial h}{\partial y} \qquad q_z = -K \frac{\partial h}{\partial z}$$

Anisotropic – having directional properties. **K** is really a tensor in 3D

Gradient

K

Flow

The value that converts one vector to another vector is a **tensor**.

$$q_x = -K_{xx}\frac{\partial h}{\partial x} - K_{xy}\frac{\partial h}{\partial y} - K_{xz}\frac{\partial h}{\partial z}$$

$$q_y = -K_{yx}\frac{\partial h}{\partial x} - K_{yy}\frac{\partial h}{\partial y} - K_{yz}\frac{\partial h}{\partial z}$$

$$q_z = -K_{zx}\frac{\partial h}{\partial x} - K_{zy}\frac{\partial h}{\partial y} - K_{zz}\frac{\partial h}{\partial z}$$

- The first index is the direction of flow
- The second index is the gradient direction

Interpretation - K_{xx} is a coefficient along the x-direction that contributes a component of flux along the x-axis due to the coefficient along the z-direction that contributes a component of flux along the z-axis due to the component of the gradient in the y-direction.

The conductivity ellipse (anisotropic vs. isotropic)
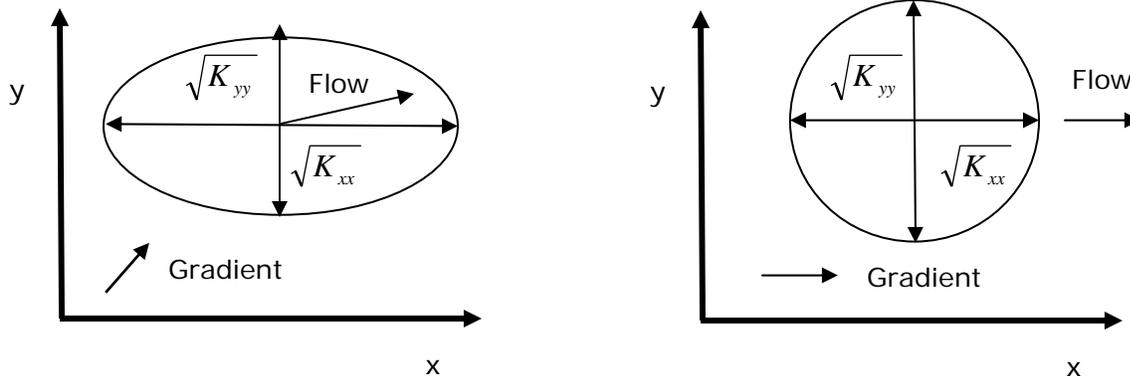
If $K_{yy} = K_{xx}$ then the media is isotropic and ellipse is a circle.

It is convenient to describe Darcy's law as:

$$\vec{q} = -\overline{\overline{K}}\nabla h$$

Where ∇ is called del and is a gradient operator, so ∇h is the gradient in all three directions (in 3D). $\overline{\overline{K}}$ is a matrix.

$$q_x = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ \\ K_{yx} & K_{yy} & K_{yz} \\ \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \begin{matrix} \dfrac{\partial h}{\partial x} \\ \\ \dfrac{\partial h}{\partial y} \\ \\ \dfrac{\partial h}{\partial z} \end{matrix}$$

$q_x =$ $q_y = -$ $q_z =$

The magnitudes of **K** in the principal directions are known as the principal conductivities.

If the coordinate axes are aligned with the principal directions of the conductivity tensor then the cross-terms drop out giving:
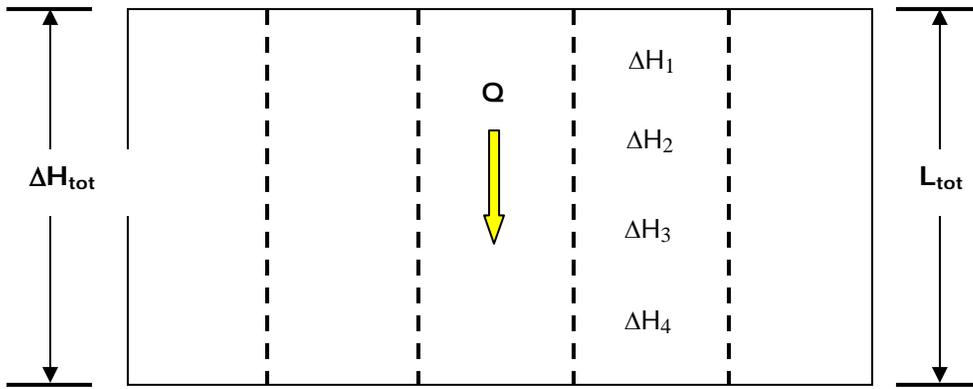
$$q_x = \begin{bmatrix} K_{xx} & & \\ \\ & K_{yy} & \\ \\ & & K_{zz} \end{bmatrix} \begin{matrix} \dfrac{\partial h}{\partial x} \\ \\ \dfrac{\partial h}{\partial y} \\ \\ \dfrac{\partial h}{\partial z} \end{matrix}$$

$q_x =$ $q_y = -$ $q_z =$

Effective Hydraulic Conductivity
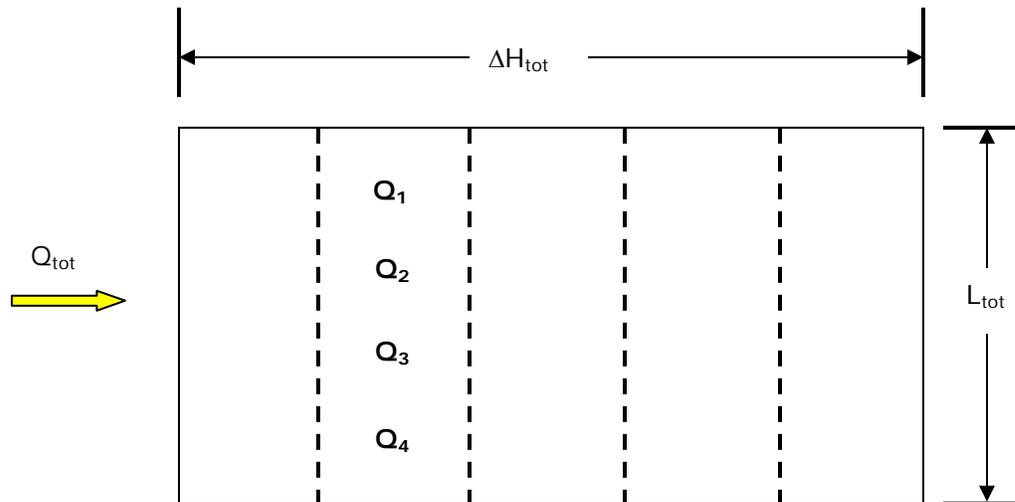
Q_{tot} $=$ Q_1 + Q_2 + Q_3 + Q_4

$=$ $\dfrac{\Delta H}{\Delta x}L_1 K_1 + \dfrac{\Delta H}{\Delta x}L_2 K_2 + \dfrac{\Delta H}{\Delta x}L_3 K_3 + \dfrac{\Delta H}{\Delta x}L_4 K_4$

$=$ $\dfrac{\Delta H}{\Delta x}\displaystyle\sum_{i}^{n} L_i K_i$

$=$ $\dfrac{\Delta H}{\Delta x}L_{tot} K_{eff}$

$$K_{eff} = \frac{\displaystyle\sum_{i}^{n} L_i K_i}{\displaystyle\sum_{i}^{n} L_i}$$

$\Delta H_{tot} = \Delta H_1 + \Delta H_2 + \Delta H_3 + \Delta H_4$

$$q = \frac{\Delta H_{tot}}{L_{tot}} K_{eff} = \frac{\Delta H_1}{L_1} K_1 = \frac{\Delta H_2}{L_2} K_2 = \frac{\Delta H_3}{L_3} K_3 = \frac{\Delta H_4}{L_4} K_4$$

$$\frac{qL_{tot}}{K_{eff}} = \sum_i^n \frac{qL_i}{K_i}$$

$$K_{eff} = \frac{\sum\limits_i^n L_i}{\sum\limits_i^n \dfrac{L_i}{K_i}}$$