

Crew Scheduling

Outline

- **Crew Scheduling**
- **Work Rules and Policies**
- **Manual Scheduling Process**
- **Model Formulation**
- **Automated Scheduling Experience**

Crew Scheduling Problem

Input

- A set of vehicle blocks each starting with a pull-out and ending with a pull-in at the depot
- Crew work rule constraints and pay provisions

Objective:

- Define crew duties (aka runs, days, or shifts) covering all vehicle block time so as to:
 - minimize crew costs

Crew Scheduling Problem

Constraints:

- **Work rules: hard constraints**
- **Policies: preferences or soft constraints**
- **Crews available: in short run the # of crews available are known**

Variations:

- **different crew types: full-time, part-time**
- **mix restrictions: constraints on max # of part-timers**

Typical Crew Scheduling Approach

Three-stage sequential approach:

1. Cutting long vehicle blocks into pieces of work
2. Combining pieces to form runs
3. Selection of minimum cost set of runs

Manual process includes only steps 1 and 2;
optimization process also involves step 3

Typical Crew Scheduling Approach

Cutting Blocks:

- each block consists of a sequence of vehicle revenue trips and non-revenue activities
- blocks can be cut only at relief points where one crew can replace another.
- relief points are typically at terminals which are accessible
- avoid cuts within peak period
- resulting pieces typically:
 - have minimum and maximum lengths
 - should be combinable to form legal runs

Vehicle Block Partitions

Definition: a partition of a block is the selection of a set of cuts each representing a relief

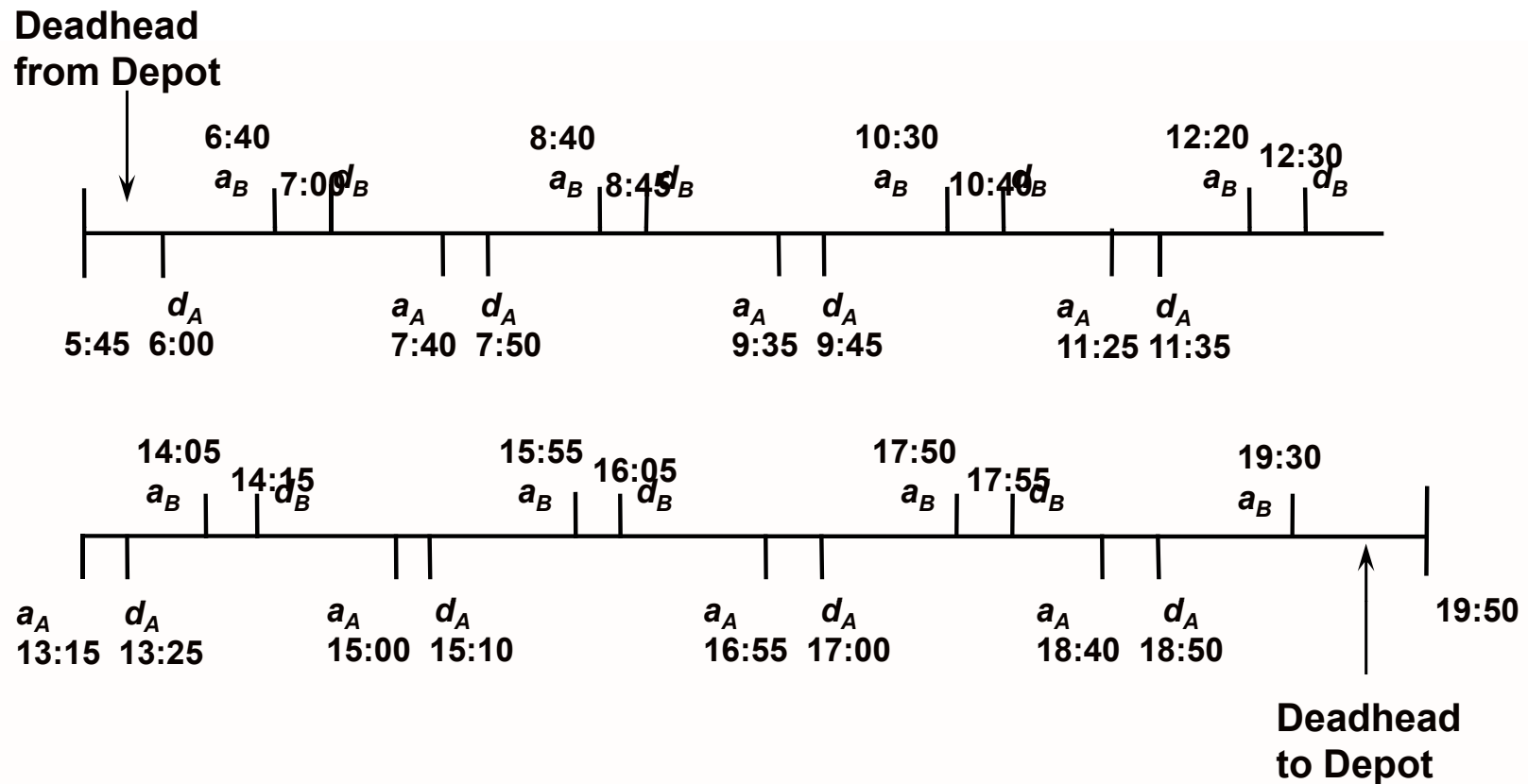
Key problems:

- very hard to evaluate a partition before forming runs
- many partitions are possible for any vehicle block

Possible Approaches:

- generate only one partition for each vehicle block
- generate multiple partitions for each vehicle block
- generate all possible partitions for each vehicle block

A Vehicle Block on Route AB



d_i = departure time from terminal i
 a_i = arrival time at terminal i

Combining Pieces of Work to Form Runs

- Large number of feasible runs by combining pieces of work
- Work rules are complex and constraining:
 - maximum work hours: e.g. 8 hrs 15 min
 - minimum paid hours - guarantee time: e.g. 8 hrs
 - overtime constraints and pay premiums: e.g. 50% pay premium
 - spread constraints and pay premiums: time between first report and last release for duty, e.g.,



has a spread of 12 hours

Combining Pieces of Work to Form Runs

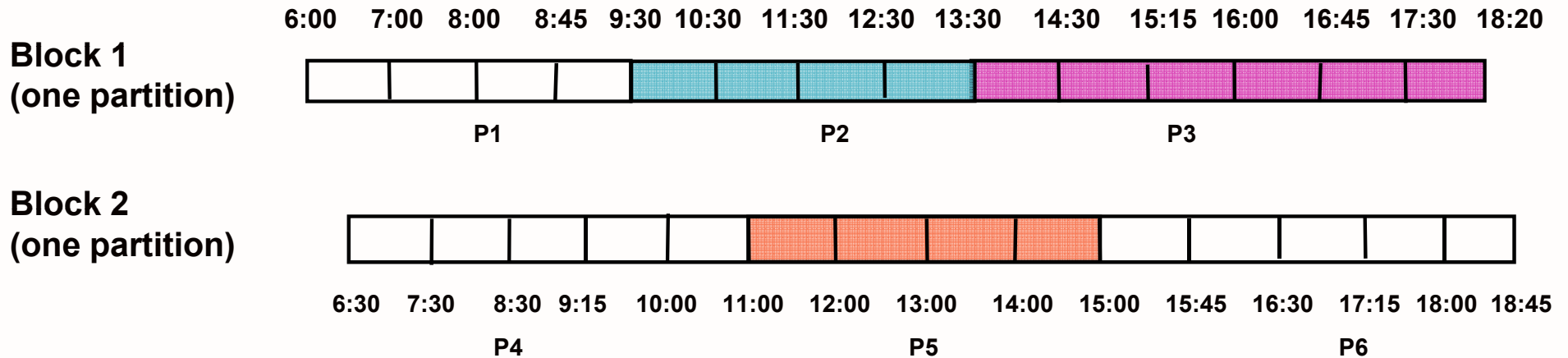
- swing pay premiums associated with runs with pieces which start and end at different locations, e.g.,



- different types of duties
 - split: a two-piece run
 - straight: a continuous run
 - trippers: a short run, usually worked on overtime

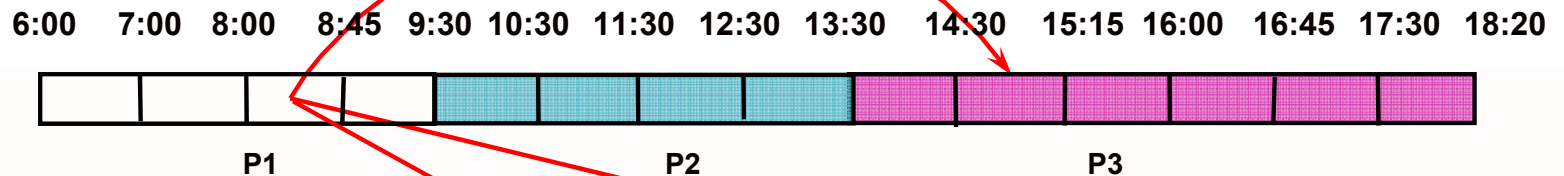
Approach: generate and cost out each feasible run

Combining Pieces of Work to Form Runs

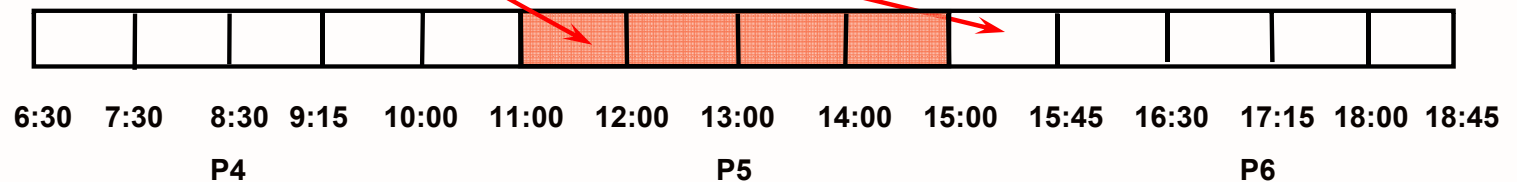


Combining Pieces of Work to Form Runs

Block 1
(one partition)



Block 2
(one partition)

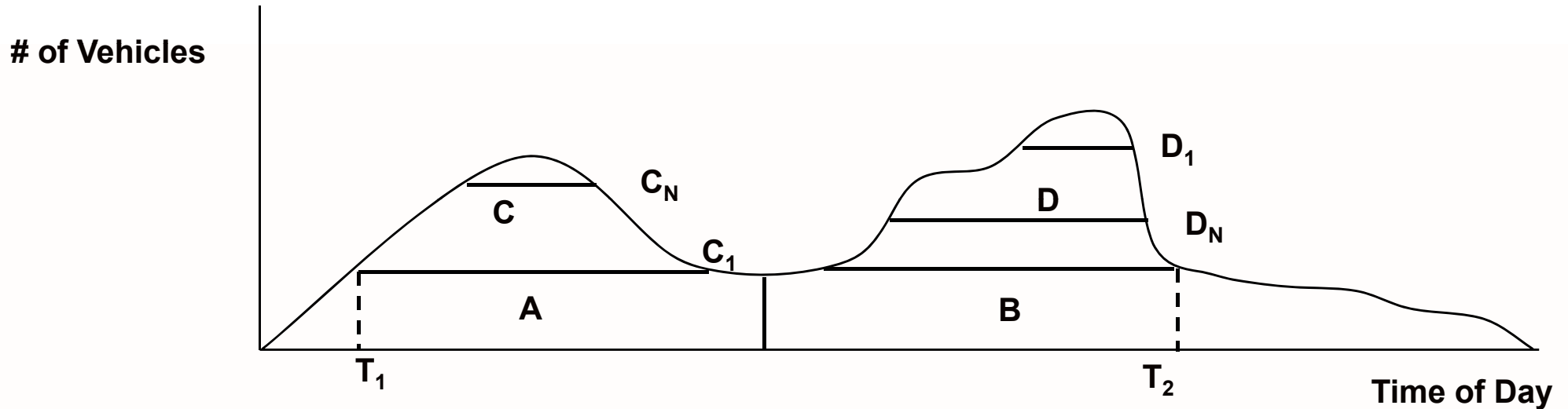


Possible Runs from defined pieces P1-P6:

Run #	1 st piece	2 nd piece	Spread Time	Work Time	Cost
1	P1	P2	7:30	7:30	C1
2	P1	P3	12:20	8:20	C2
3	P1	P5	9:00	7:30	C3
4	P1	P6	12:45	7:15	C4
5	P2	P3	8:50	8:50	C5
6	P2	P6	9:15	7:45	C6
7	P4	P3	11:50	9:20*	C7
8	P4	P5	8:30	8:30	C8
9	P4	P6	12:15	8:15	C9
10	P5	P6	7:45	7:45	C10

* illegal run:
max work time violation

Crew Scheduling: Manual Techniques



T_1 is earliest AM pullout which can still serve PM peak

T_2 is latest PM pullback which can still serve AM peak

A are AM straights (or short split runs)

B are PM straights (or short split runs)

C and D are long split runs

Typical Sequence

1. Based on total vehicle hours estimate total operators required
2. Determine # operators required in AM and PM peaks
3. Determine B based on: # of pull-ins after time T_2 .
4. Determine # split runs: (# of PM Peak Vehicles - B)
5. Determine A based on: # of AM Peak Vehicles - split runs
6. Combine earliest pullouts in C with earliest pull-ins in D to produce minimum spread split runs C_1D_1 . Iterate until all split runs are matched C_ND_N .

Examples

Time Period	# Vehicles	Period Length	# Vehicle Hours
AM Peak	8	3	24
			AM runs = 4
Base	4	6	24
			Split runs = 4
PM Peak	8	3	24
			PM runs = 4
Evening	4	6	24
			96, or 12 FTOs

Time Period	# Vehicles	Period Length	# Vehicle Hours
AM Peak	8	3	24
			AM runs = 3
Base	6	6	36
			Split runs = 5
PM Peak	8	3	24
			PM runs = 3
Evening	3	6	18
			102, or 13 FTOs

Selection of Minimum Cost Set of Runs

- Usually built around mathematical programming formulation

Problem Statement:

Given a set of m trips and a set of n feasible driver runs, find a sub-set of the n runs which cover all trips at minimum cost

Mathematical Model for Crew Scheduling Problem

A. Basic Model: Set Partitioning Problem

Notation:

P = set of trips to be covered

R = set of feasible runs

c_j = cost of run j

δ_i^j = binary parameter, if 1 means that trip i is included in run j , 0 o.w.

x_j = binary decision variable, if 1 means run j is selected, 0 o.w.

$$\begin{aligned} \text{Min} \quad & \sum_{j \in R} c_j x_j \\ \text{Subject to:} \quad & \sum_{j \in R} x_j \delta_i^j = 1 \quad \forall i \in P \\ & x_j \in \{0, 1\}, \quad \forall j \in R \end{aligned}$$

Mathematical Model for Crew Scheduling Problem

Problem size:

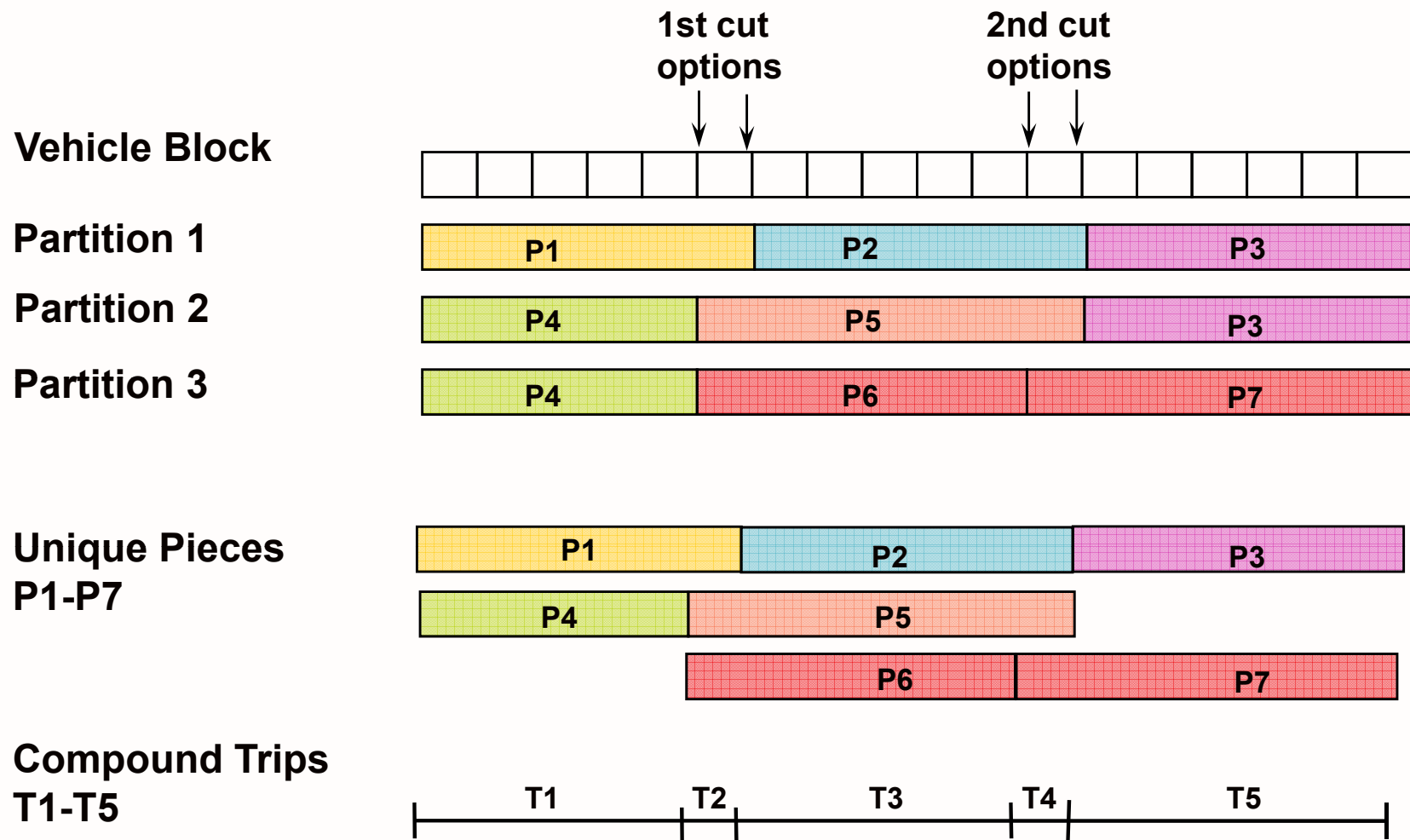
R decision variables (likely to be in millions)

P constraints (likely to be in thousands)

Problem size reduction strategy:

- replace individual trips with compound trips consisting of a sequence of vehicle trips which will always be served by a single crew.

Partitions of Vehicle Block, Pieces of Work and Compound Trip



May reduce the # of constraints but by less than one order of magnitude

Variations of Set Partitioning Problem

- 1. Set R consists of all feasible runs given all feasible partitions for all vehicle blocks**
 - size of model, specifically # of columns, explodes with problem size
 - only possible for small problems
- 2. Set R consists of a subset of all feasible runs**
 - not guaranteed to find an optimal solution
 - effectiveness will depend on quantity and quality of runs included
- 3. Column generation based on starting with a subset of runs and generating additional runs which will improve the solution as part of the model solution process.**

Model with Side Constraints

Often the number (or mix) of crew types is constrained in various ways which can be formulated as side constraints

Example: Suppose total tripper hours are constrained to be less than 25% of timetable time.

Let: WT = total timetable time
 R^T = set of tripper runs
 t_j = work time for tripper run j

Then the additional constraint is:
$$\sum_{j \in R^T} t_j x_j \leq 0.25 WT$$

Experience with Automated Crew Scheduling Systems

- **Virtually universally used in medium and large operators world-wide**
- **Two most widely used commercial packages are HASTUS (by GIRO Inc in Montreal) and Trapeze (by Trapeze Software Inc in Toronto/Phoenix), each with over 200 customers world-wide**
- **Typical cost ranges from \$100K to \$2 M for the software**
- **Key benefits of automated scheduling are:**
 - **scheduling process time reductions**
 - **improved accuracy**
 - **modest improvements in efficiency (typically 0-3%)**
 - **provides a key database for many other IT applications**

Experience with Automated Crew Scheduling Systems

- **Evolution of software has been from “black box” optimization/heuristics to highly interactive and graphical tools**
- **Current systems allow much greater ability to “shape” the solution to the needs of specific agencies**
- **One implication however is a profusion of these “soft” parameters which means greater complexity and it is very hard to get full value out of systems.**

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