Problem Set #5

Issued: November 8, 2006
Due: November 20, 2006

Problem 1

Consider a 1-mile-by-2-mile homogeneous service region served by two mobile patrolling servers as shown in Figure 1. Here are the assumptions of the model:

1. Customer locations are uniformly independently located over the entire rectangular service region.
2. Over time, customers arrive as a homogeneous Poisson process at aggregate rate λ=2 customers per hour.
3. While not busy serving customers, each server patrols its sector (sector 1 or 2, respectively). Under this circumstance, the server's location at a random time is uniformly distributed over its sector. Each sector is a one-mile-by-one-mile square.
4. Travel distance is right-angle ("Manhattan metric"), with speed equal to 1000 mi./hr.
5. The on-scene time to serve a customer is a random variable having a negative exponential probability density function with mean equal to 30 minutes. Upon completion of service of a customer at the scene, the server resumes random patrol of his/her sector.
6. This is the dispatch strategy: For a customer from sector i (i=1,2), dispatch server i if available. Else dispatch the other server if available; Else the customer is lost forever.

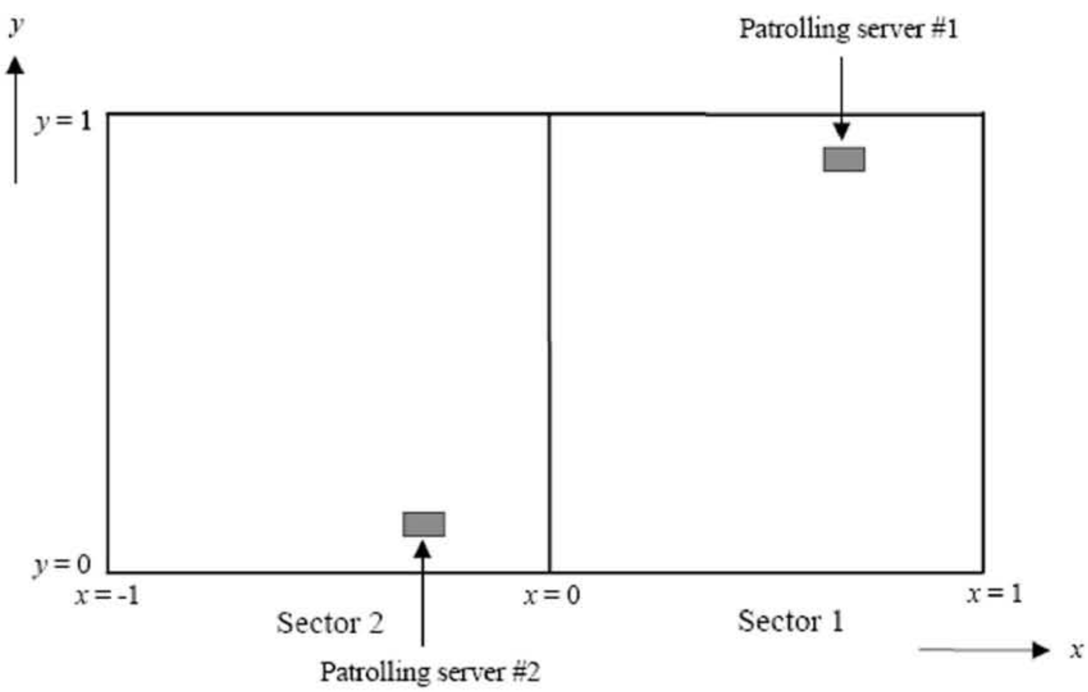

Figure 1

(a) Is it true that the workload (fraction of time busy) of each server is equal to 1/2? If true, briefly explain why. If false, derive the correct figure.
(b) Determine the fraction of dispatches that take server 1 to sector 2.
(c) Determine the mean travel time to a random served customer.

Now consider the situation as shown in Figure 2. Assumptions 1 through 5 above remain correct. However, Assumption 6 is altered as follows:

> *Case in which both servers are available:* For a customer from a part of sector i not in Buffer Zone i, dispatch server i. For a customer in Buffer Zone i, dispatch the other server only if that other server is within its own buffer zone and server i is not within its buffer zone; else dispatch server i to that customer.
> *Case in which only one server is available:* Dispatch that server.
> *Else the customer is lost forever.*

(d) Under this new dispatch policy, determine the fraction of dispatch assignments that send server 1 to sector 2.
(e) Without doing the detailed calculations, describe briefly how you would compute the mean travel time. How would the magnitude of the numerical answer compare to that of part (c)?
(f) Suppose under the simpler dispatch policy #1 above, we find that the workload of Sector #1 is twice the workload of Sector 2, while λ remains the same at $\lambda = 2$. Without doing the calculations, briefly explain how you would find an optimal boundary line separating Sectors 1 and 2, where 'optimal' means minimizing mean travel time. Would it be to the left of right of $x = 0$? Why?

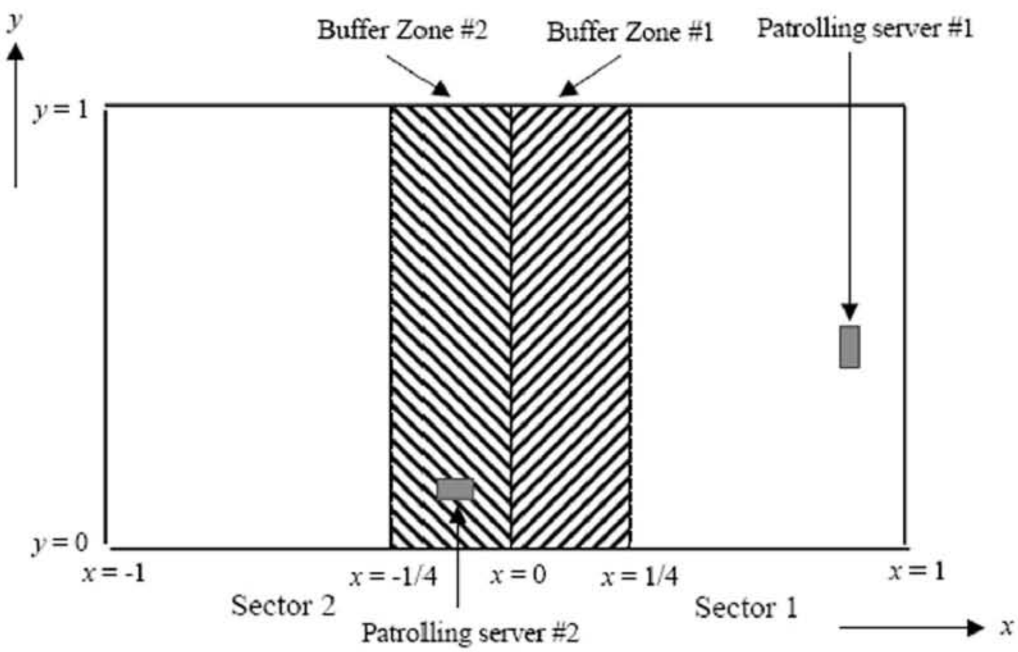

Figure 2

Problem 2

Consider a square one kilometer on a side, as shown in Figure 1. Geometrically, this is the same square that appeared in Problem 1 of Quiz #1. That is, emergency incidents can only occur on the perimeter of the square and travel can occur only along the perimeter of the square. There is no travel within the square. There are no emergency incidents within the square. U-turns are allowed and travel always occurs along the shortest path.

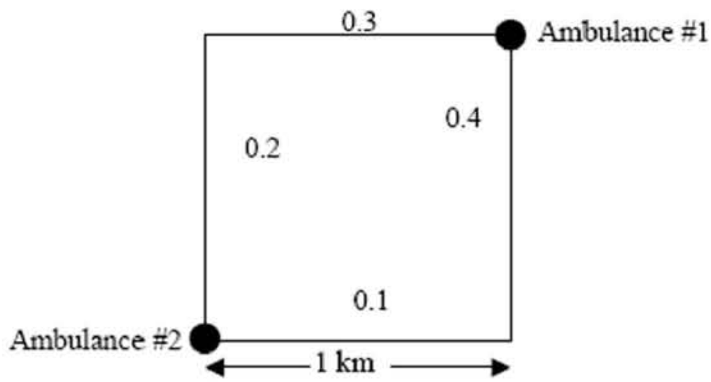

Figure 1

The square is served by two ambulances, ambulance #1 garaged in the northeast corner of the square and ambulance #2 garaged in the southwest corner of the square, as shown in Figure 1. Ambulances always return to their home garage locations after answering emergencies. So, an ambulance will never be dispatched directly from one call to another without returning to the home garage location.

Emergency incidents are ***not*** uniformly distributed over the square. The number adjacent to each link of the square is the probability that a random emergency incident will be generated on that link. Once the link of the incident is known, the conditional pdf of its location on the link is uniform over the link.

We model this system as an $N = 2$ server hypercube queueing model with μ^{-1} = mean service time per incident = 1 hour, λ = Poisson arrival rate of emergency incidents from entire square = 1/hour, and response travel speed = 100 km/hour. The usual assumptions related to negative exponential service times apply, as does the assumption that on scene time dominates the very small travel time component of the service time. Assume that the dispatcher dispatches the closest available ambulance (i.e. of the available ambulances, the one whose home garage location is closest to the emergency). Emergency incidents that occur while both ambulances are simultaneously busy are lost.

(a) (10 points) Given that an emergency incident occurs while ambulance #1 is busy and ambulance # 2 is available, find and plot the conditional pdf of the travel distance for ambulance #2 to travel to the scene of the emergency incident.
(b) (12 points) Find the workload (fraction of time busy) of each of the two ambulances.

(c) (13 points) If the dispatcher moves to an optimal dispatch strategy, i.e., one that minimizes mean travel time to a random incident, determine the optimal boundaries for response areas 1 and 2.

Problem 3

Problem 5.3 from the textbook.

Problem 4

Problem 5.6 from the textbook.

Problem 5

Consider a service facility at which Type 1 and Type 2 customers arrive in a Poisson manner at the rate of $\lambda_1 = 30$ per hour and $\lambda_2 = 24$ per hour, respectively. Service at the facility is FIFO and service times are constant and last exactly 1 minute for either type of customer. The cost of waiting in the queue per minute is \$2 and \$3 per Type 1 and Type 2 customers, respectively. Assume steady state conditions throughout the problem. Numerical answers are expected in all parts.

(a) What is the total expected cost of waiting time at this facility per hour?
(b) Find the internal cost and the external cost associated with a marginal Type 1 customer and repeat for the internal and external cost associated with a Type 2 customer. How do the external costs compare and why?

Assume now and for parts (c) – (f) of this problem that service to Type 1 customers lasts exactly 0.5 minute and to Type 2 customers exactly 1.625 minutes. The cost of waiting per minute is still \$2 and \$3 per Type 1 and Type 2 customers, respectively, and service is still FIFO.

(c) Repeat part (a). A numerical answer is expected once again.
(d) Repeat part (b) and comment on differences you see with the results of part (b).

Suppose now that you were asked to assign non-preemptive priorities to the two types of customers in a way that will minimize the total expected cost of waiting at the facility.

(e) Which type of customer should be assigned higher priority and why?
(f) Under the priorities assigned in part (e), what is the total expected cost of waiting at this facility per hour? How does this cost compare with the cost you found in part (c)?

Suppose now that the service times for Type 1 customers are negative exponential with an expected value of 0.5 minute and for Type 2 customers negative exponential with an expected value of 1.625 minutes. As for parts (e) and (f), the cost of waiting per minute is still \$2 and \$3 per Type 1 and Type 2 customers, respectively, and you are asked to assign non-preemptive priorities to the two types of customers in a way that will minimize the total expected cost of waiting at the facility.

(g) Which type of customer should be assigned higher priority and why?
(h) Under the priorities assigned in part (g), what is the total expected cost of waiting at this facility per hour? How does this cost compare with the cost you found in part (c) and part (f)?