1.060 Engineering Mechanics II	Spring 2006

Recitation 5 - Problems

March 16th and 17th

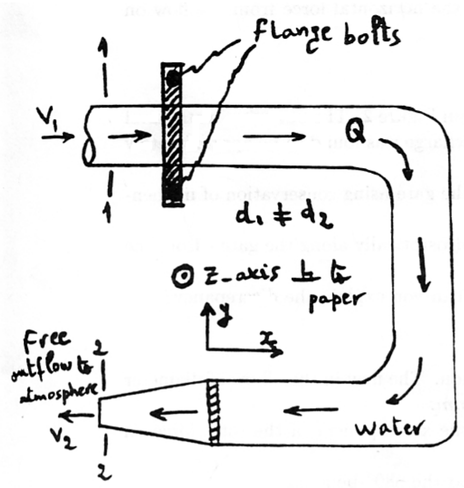


Figure 1: Horizontal elbow and nozzle in Problem 1.

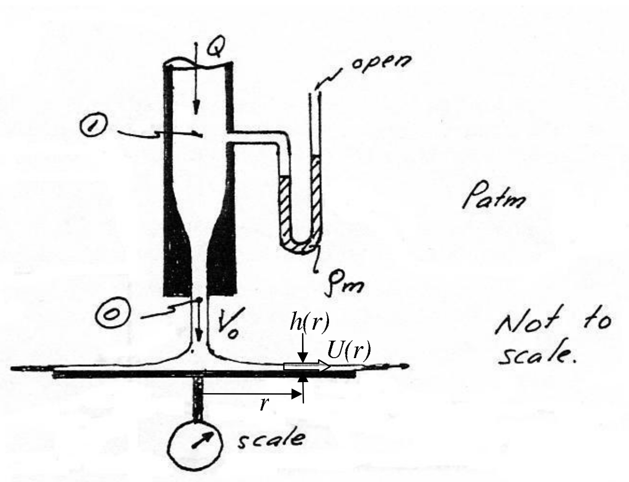


Figure 2: Diffuser in Problem 2.

Problem 1

Figure 1 shows a horizontal elbow and a nozzle combination. The flow in the elbow of diameter $d_1 = 300\ mm$ is $Q = 90\ l/s$. The nozzle has a diameter $d_2 = 100\ mm$ and discharges into the atmosphere.

a) Given that the pressure at section 1 is $p_1 = 70\ kPa$, find the x-component of the total force on the flange bolts (F_x).

b) Determine the head loss associated with the flow around the 180^o-bend.

(NOTE: $1\ l = 1\ liter = 1\ dm^3 = 0.001\ m^3$).

Problem 2

Figure 2 illustrates a classic fluid mechanics experiment. A flow of water, $\rho = 1000\ kg/m^3$, exits vertically from a diffuser –a smooth contraction from diameter $D_1 = 3\ cm$ to $D_0 = 1\ cm$– into the atmosphere a short distance, $5\ cm$, above a horizontal plate. The horizontal plate is sufficiently large to completely deflect the flow so that this leaves the plate with a purely horizontal velocity. The pressure immediately before the diffuser ($10\ cm$ above the exit) is measured by a mercury manometer ($\rho_m = 13.6\ \rho$).

a) How are the velocities V_1, before the diffuser, and V_0, at the diffuser exit, related?

b) Why is it reasonable to apply Bernoulli principle without headloss to relate conditions at the manometer pressure tap and the jet exit?

c) If the fluid velocities of interest are of the order of 5 m/s or greater, why would it be reasonable to neglect elevation differences of the order of 10 cm or smaller?

d) For a manometer reading of $\Delta z_m = 10\ cm$ estimate the pressure, p_1, at the entrance of the diffuser.

e) Use Bernoulli, neglecting elevation differences and headlosses, to estimate the jet velocity, V_0, at the exit from the diffuser.

f) Estimate the total vertical force exerted by the jet impacting on the horizontal plate.

g) If gravity (i.e., elevation head differences) and losses are neglected, obtain an expression for the velocity, U(r), and thickness, $h(r)$, of the fluid on the plate, as a function of the radial coordinate, r.

(NOTE: This is an old test problem).

Problem 3

The vertical velocity distribution in a wide rectangular duct of height H can be expressed as

$$u(z) = U + u'(z)$$

where $-H/2 \leq z \leq H/2$ is the vertical coordinate, U is the depth-averaged velocity, and $u'(z)$ is the velocity deviation with respect to the average. $|u'(z)/U|$ is much smaller than 1 for most of the depth, as represented in Figure 3.

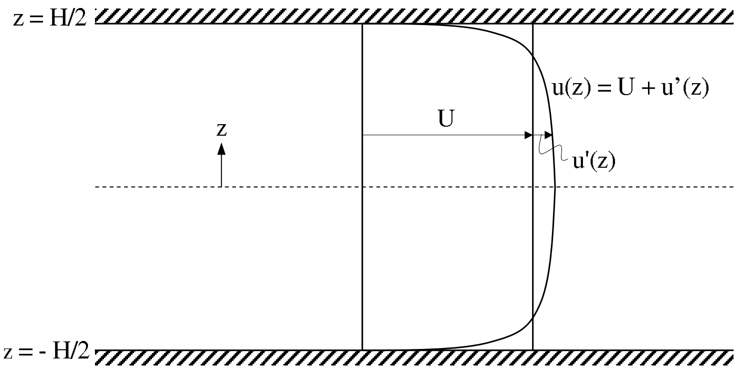


Figure 3: Vertical velocity distribution in a rectangular duct (Problem 3).

a) What is the discharge in the duct (per unit width into the paper)?

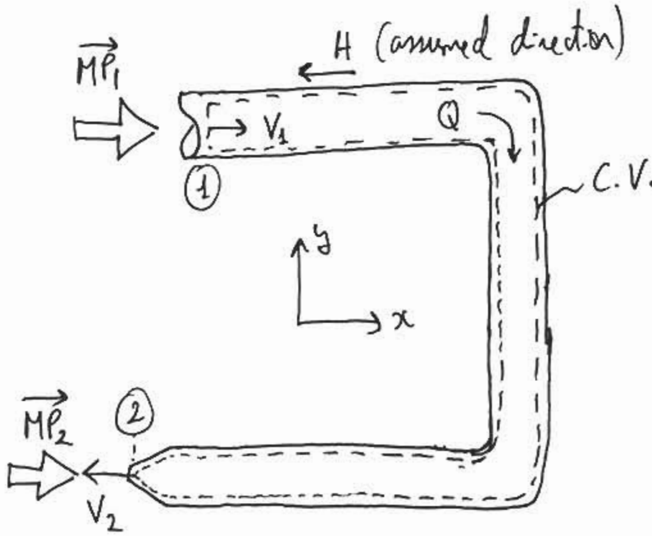
b) Show that the momentum coefficient is $K_m = 1 + \delta_m^2$, where

$$\delta_m^2 = \frac{1}{H} \int_{-H/2}^{H/2} \left(\frac{u'}{U}\right)^2 dz \ll 1.$$

c) Show that the energy coefficient is $K_e = 1 + \epsilon_e$, where $\epsilon_e \simeq 3\delta_m^2$.

RECITATION 5 – SOLUTIONS

– PROBLEM Nº 1:



H: Force exerted by the bolts on the control volume (C.V.)

Pipe is on the x-y plane (no gravity).

a) Continuity: $Q = V_1 A_1 = V_2 A_2 = 0'090 \ m^3/s$

$$V_1 = \frac{0'090}{\frac{\pi}{4} 0'3^2} = 1'273 \ m/s$$

$$V_2 = \frac{0'090}{\frac{\pi}{4} 0'1^2} = 11'46 \ m/s$$

From the problem statement: $p_1 = 70 \ kPa = 70000 \ Pa$, $p_2 = p_{atm} = 0$

Conservation of linear momentum for steady flow

$$\vec{0} = \sum \overrightarrow{MP} + \overrightarrow{gravity} + \overrightarrow{\sum \text{all other forces on C.V.}}$$

x-axis: $0 = MP_1 + MP_2 + 0 - H$

$$H = MP_1 + MP_2 = (\rho V_1^2 + p_1) A_1 + (\rho V_2^2 + p_2) A_2 = 6094 \ N \ \text{(to the left)}$$

By action and reaction principle, $\underline{\underline{F_x = 6094 \ N \text{ to the right}}}$

(F_x is the force exerted by the CV on the bolts).

b)

Energy equation from point 1 to point 2:

$$H_1 = H_2 + \sum \Delta H_{losses}$$

$$z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + \sum \Delta H_{losses}$$

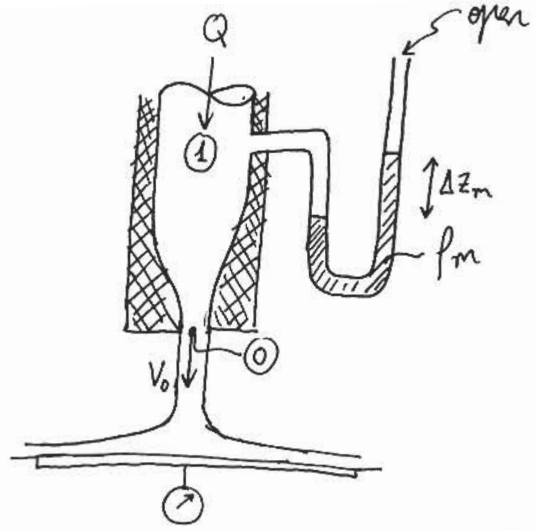
$$\underline{\underline{\sum \Delta H_{losses}}} = \left(0 + \frac{70000}{9800} + \frac{1'273^2}{2 \cdot 9'8}\right) - \left(0 + 0 + \frac{11'46^2}{2 \cdot 9'8}\right) =$$

$$= \underline{\underline{0'525 \ m}}$$

$\sum \Delta H_{losses}$ is the sum of all headlosses:

1) Headloss due to friction
2) Headloss due to curvature and separation in the 90° corners.
3) Headloss due to the nozzle.

- PROBLEM N°2:



a)

Continuity: $V_1 A_1 = V_0 A_0 \Rightarrow \underline{\underline{V_1}} = V_0 \dfrac{A_0}{A_1} = \underline{\underline{V_0 \left(\dfrac{D_0}{D_1}\right)^2 = 9 V_0}}$

b)

Flow is converging (velocity increases from ① to ⓪), so we can neglect localized "minor" losses, and path is relatively short (compared to D), so we can neglect friction losses.

c)

For Δz to be negligible : $\dfrac{V^2}{2g} \gg \Delta z$

$V \approx 5\ m/s \Rightarrow V^2/2g \simeq 5^2/2 \cdot 10 = 1'25\ m \gg \Delta z \simeq 10\ cm$, or neglecting $\Delta z = 10$ cm produces error in V of the order $\sqrt{2g}\left(\sqrt{1'25 \pm 0'1} - \sqrt{1'25}\right) = \pm\ 20\ cm \simeq \pm 4\%$. Not much.

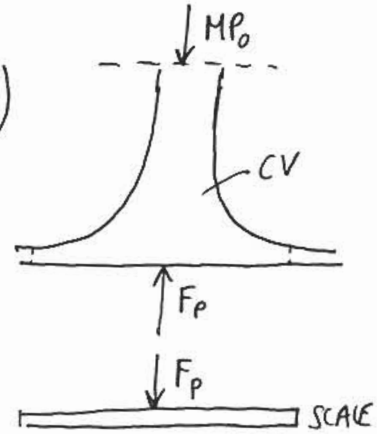
d) From manometer reading, neglecting elevation difference between pressure tap and mercury in right leg of manometer

$\underline{\underline{p_1}} = (\rho_m - \rho) g \Delta z_m = (13'6 - 1) \cdot 10^3 \cdot 9'8 \cdot 0'1 = \underline{\underline{12'35\ kPa}}$

e) $V_1^2/(2g) + p_1/(\rho g) + z_1 = V_0^2/(2g) + \overset{=0}{p_0}/\rho g + \overset{\simeq z_1}{z_0}$

$V_0^2\left(1 - \underbrace{\left(\dfrac{V_1}{V_0}\right)^2}_{=(D_0/D_1)^4}\right) = 2\ \dfrac{p_1}{\rho} \Rightarrow \underline{\underline{V_0}} = \left\{2 \cdot \dfrac{12'35\ kPa}{10^3}\ \dfrac{1}{1 - \left(\dfrac{0'01}{0'03}\right)^4}\right\}^{1/2} = \underline{\underline{5'0\ m/s}}$

f)

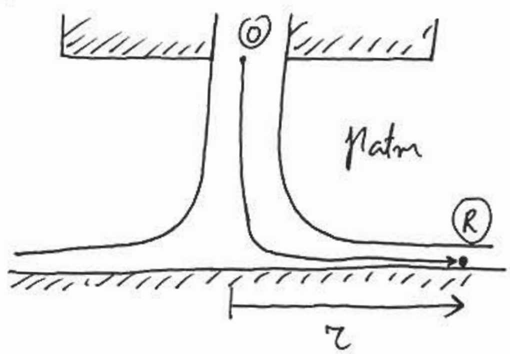


$$\underline{\underline{F_p}} = MP_o = (\rho V_o^2 + \underbrace{\cancel{p_o}}_{=0}) A_o = \rho V_o^2 \left(\frac{\pi}{4} D_o^2\right) =$$

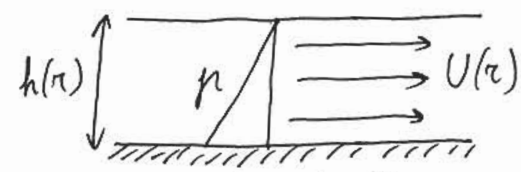
$$= 10^3 \cdot 5^2 \cdot \frac{\pi}{4} 0'01^2 = \underline{\underline{1'96 \text{ N}}}$$

Force on plate is <u>directed downwards</u>

g)



Detail of point R:



Well behaved flow:
Hydrostatic pressure

Neglecting losses, we apply Bernoulli between 0 and R:

$$p_0 + \rho g z_0 + \frac{V_0^2}{2g} = p_R + \rho g z_R + \frac{V_R^2}{2g}$$

$p_0 = 0$ (atmospheric pressure)

$$\left.\begin{array}{l} p_R \simeq \rho g h \\ \rho g z_0 \\ \rho g z_R \end{array}\right\} \text{Neglected (since we neglect gravity effects)}$$

Therefore, $V_R = \underline{\underline{U(r)}} = V_0 = \underline{\underline{5'0 \text{ m/s}}}$

Due to continuity,

$$Q = V_0 A_0 = V_R A_R = U(r)\, 2\pi r\, h(r) \quad \text{(radial symmetry)}$$

$$\underline{\underline{h(r)}} = \frac{V_0 A_0}{U(r)\, 2\pi r} = \frac{\pi/4\ 0'01^2}{2\pi r} = \underline{\underline{\frac{1'25 \cdot 10^{-5}}{r}}} \quad (S.I.)$$

- PROBLEM N°3 :

a)

$$\underline{\underline{Q}} = U \cdot A = U \cdot (H \cdot 1) = \underline{\underline{U \cdot H}}$$ (per unit width into the paper)

b)

$$\underline{\underline{K_m}} = \frac{\int_A q_\perp^2 \, dA}{U^2 A} = \frac{\int_{-H/2}^{H/2} (U+u')^2 dz}{U^2 H} =$$

$$= \frac{1}{U^2 H}\left[U^2 H + 2U \underbrace{\int_{-H/2}^{H/2} u' \, dz}_{\substack{=0 \text{ by} \\ \text{definition of } u'}} + \int_{-H/2}^{H/2} u'^2 dz \right] =$$

$$= 1 + \underbrace{\frac{1}{H}\int_{-H/2}^{H/2} \left(\frac{u'}{U}\right)^2 dz}_{\text{Call this } \delta^2} = 1 + \delta^2$$

$\uparrow$ small quantity

c)

$$\underline{\underline{K_e}} = \frac{\int_A q_\perp^3 \, dA}{U^3 A} = \frac{\int_{-H/2}^{H/2} (U+u')^3 dz}{U^3 H} =$$

$$= \frac{1}{U^3 H}\left[U^3 H + 3U^2 \underbrace{\int_{-H/2}^{H/2} u' \, dz}_{=0} + 3U \int_{-H/2}^{H/2} u'^2 dz + \int_{-H/2}^{H/2} u'^3 dz \right] =$$

$$= 1 + \underbrace{3\frac{1}{H}\int_{-H/2}^{H/2}\left(\frac{u'}{U}\right)^2 dz}_{=3\delta^2} + \underbrace{\frac{1}{H}\int_{-H/2}^{H/2}\left(\frac{u'}{U}\right)^3 dz}_{\simeq O(\delta^3) \ll 3\delta^2} \approx 1 + 3\delta^2$$