1.033/1.57 Recitation: Stress & Strength

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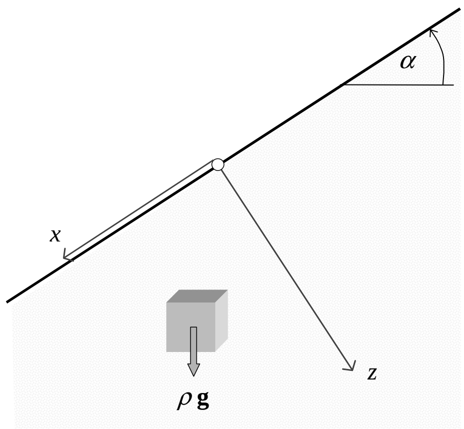
Why Sandcastles Fall? We want to study the stress fields in a dry and a humid sandpile, idealized as an inclined semi-infinite half-space oriented at an angle α to the horizontal (see figure below). We choose an $x - z$ coordinate system, in which z gives the distance from the surface of the pile ($z > 0$ down) and x gives the distance parallel to the surface (infinite extension in the $y-$ direction). The sandpile is subjected to its deadweight (volume mass density ρ, and $\mathbf{g}$ the earth acceleration vector), and static evolutions are assumed.



Problem Set: Mohr-Coulomb's problem — idealized problem of a sandpile.

1. **Dry Sandpile – The Mohr-Coulomb result**: We restrict ourselves to solutions which are functions of z alone, i.e.,

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}(z)$$

Furthermore, the sand behavior is assumed isotropic.

   (a) Determine precisely the conditions which stress field $\boldsymbol{\sigma}$ needs to satisfy in order to be statically admissible. Determine the non-zero stress components of $\boldsymbol{\sigma}$, and give a precise of the stress components of which the value is not given by static equilibrium (S.A.-stress conditions).

   (b) For a given distance $z > 0$ from the surface, represent the previously determined stress state in the Mohr Stress plane. In this plane, indicate the angle α.

   (c) We want to provide the critical angle $\alpha \leq \max \alpha$, by considering that the material in the sandpile obeys to the (dry sand) Mohr-Coulomb criterion:

$$|\tau| + \sigma \tan \varphi \leq 0$$

   where τ is the tangential stress across some plane interior to the sandpile, σ is the normal stress across the same plane, and $\tan \varphi$ is the internal friction angle. Show the criterion in the Mohr space, and determine the critical value of α at which the material reaches the Mohr-Coulomb criterion.

2. **Humid Sandpile**: Consider now a sandpile in which a normal adhesive stress s_A is exerted across every plane, in addition to whatever other stresses may exist due to body forces. This adhesive stress introduces a normal force between pairs of contiguous particles which allows the sandpile to support a finite shear stress (i.e. τ), even in the limit of zero applied compressive stresses (i.e. $\sigma = 0$). The maximum shear stress, in this case, is $\max |\tau| = s_A \tan \varphi$.

   (a) Propose a modified Mohr-Coulomb criterion, which for $s_A = 0$ gives the dry sand Mohr-Coulomb criterion.

   (b) In comparison with the dry sand criterion, how does the Mohr plane representation change in the case of a humid sandpile. Determine the critical angle at which the material reaches the humid sand failure criterion. In comparison with the dry sandpile, does $\max \alpha$ increase or decrease? Conclude by suggesting how sandcastles fall.