

**7.014**  
**Lecture 29 & 30:**  
**Population Growth**  
**Lecture Slides**  
***April 25 & 27, 2005***

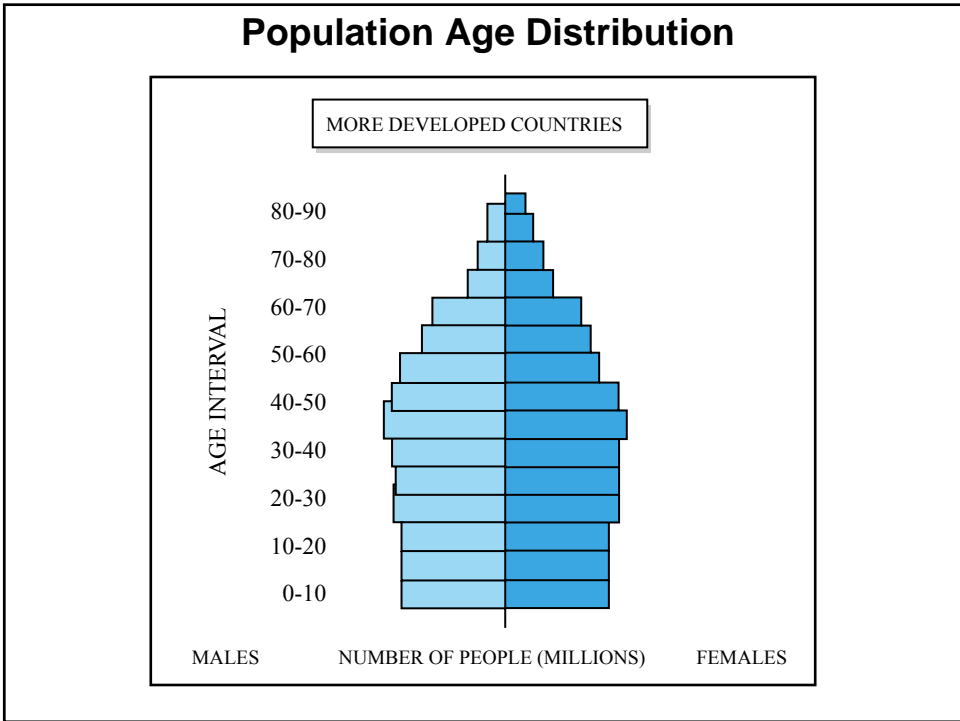


Figure by MIT OCW.

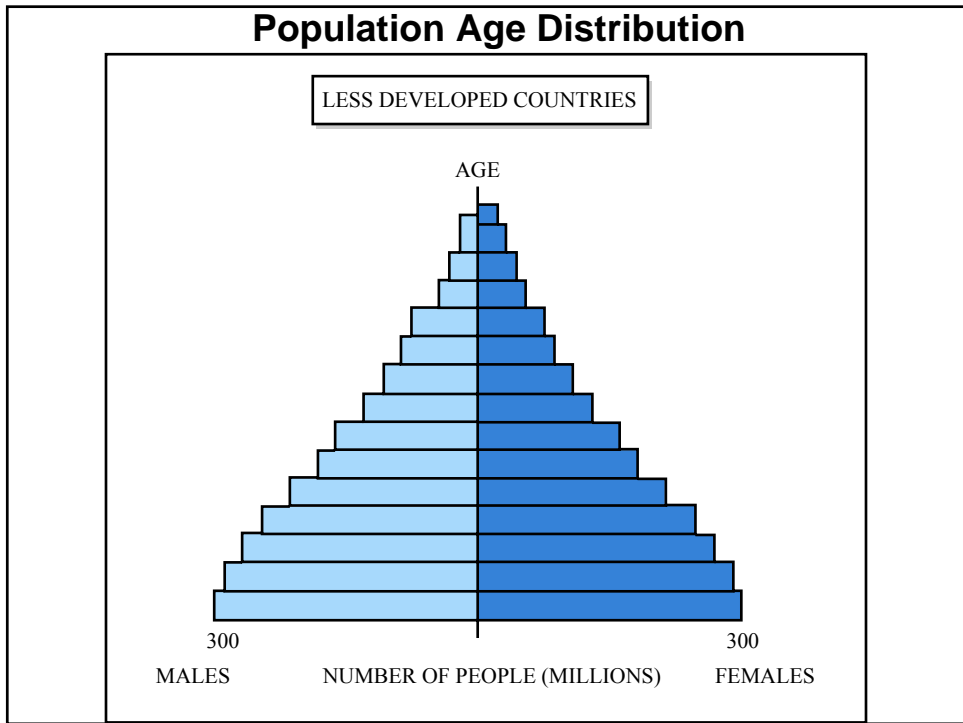
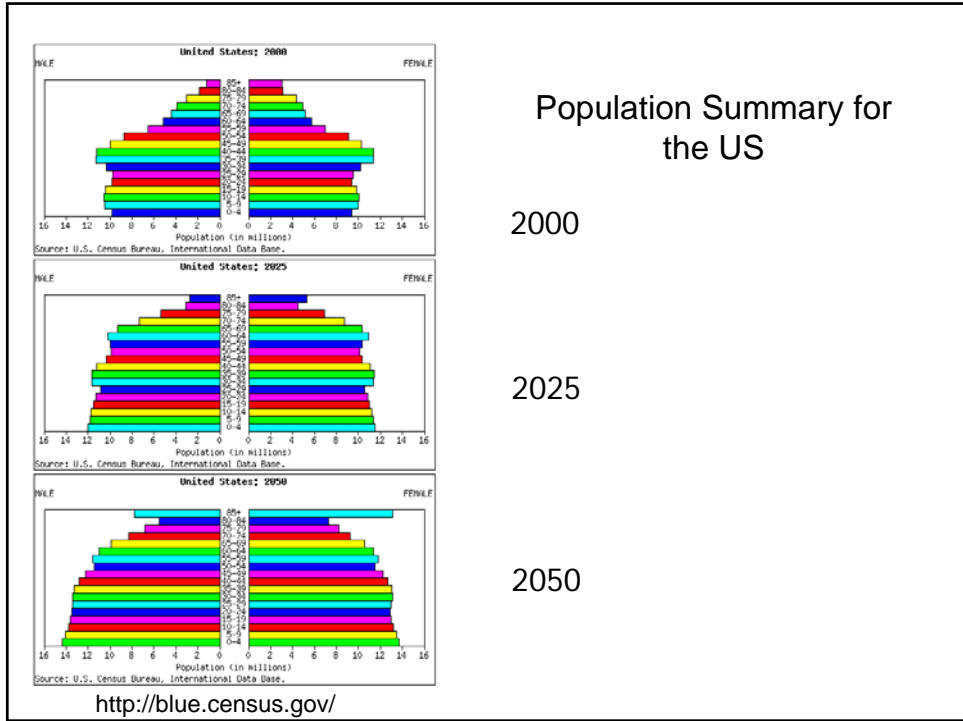
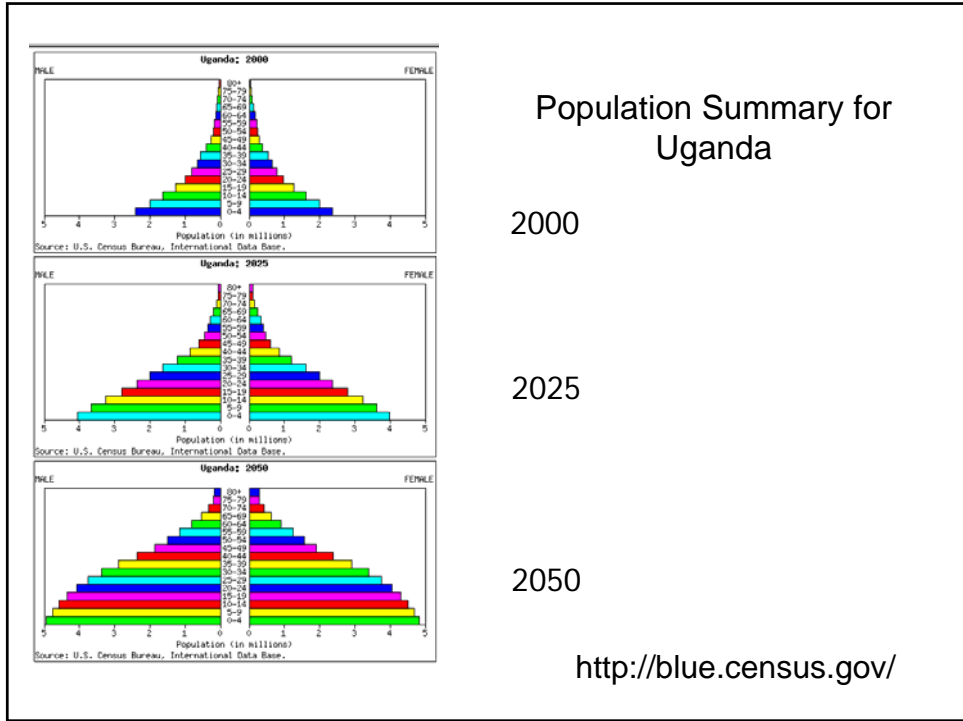


Figure by MIT OCW.



### Life Tables

$x$  = age or interval (defined)

$N_0$  = number of individuals in original cohort (defined)

$d_x$  = number of original cohort *dying* during interval

$N_x$  = number of individuals *surviving* to age  $x$  (measured)

$l_x$  = proportion of individuals surviving to age  $x$

$$l_x = N_x / N_0$$

$m_x$  = per capita births during age interval  $x$  to  $x+1$  (measured)

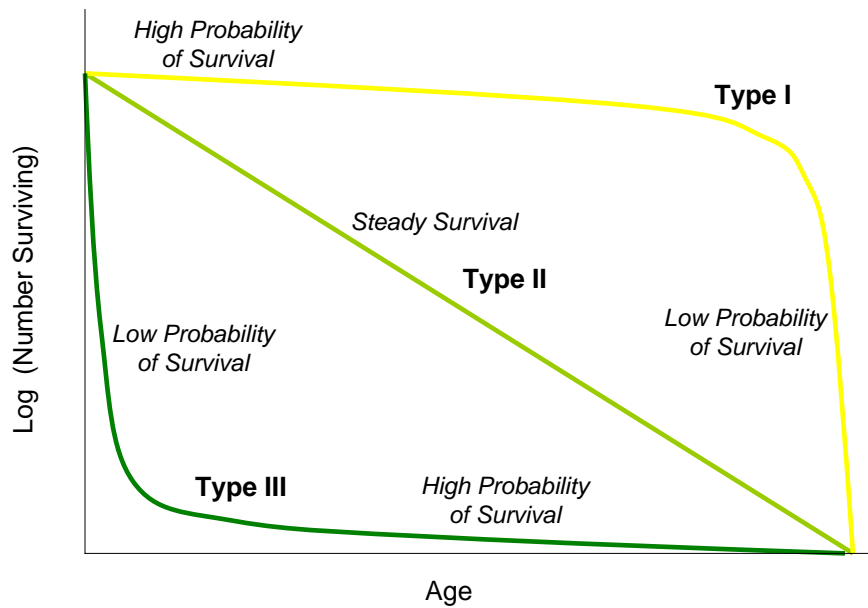
“age specific fecundity” = female offspring produced per female

### A COHORT LIFE TABLE (for Unicorns)

$N_0 = 100$

Age	Number Surviving	Proportion Surviving
<b>X</b>	<b>N<sub>x</sub></b>	<b>I<sub>x</sub></b>
0	100	1.0
1	50	.5
2	40	.4
3	30	.3
4	0	0

### SURVIVORSHIP CURVES



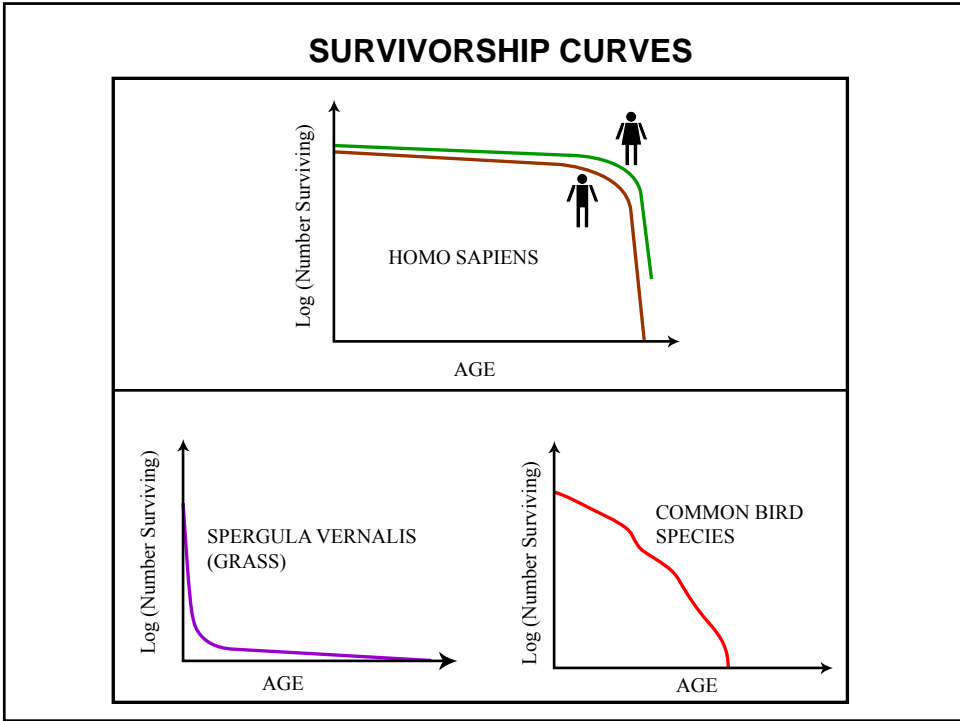


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### A COHORT LIFE TABLE (for Unicorns)

$N_0 = 100$

Age	Number Surviving	Proportion Surviving	Average Offspring per female of age x
$x$	$N_x$	$l_x$	$m_x$
0	100	1.0	0
1	50	.5	4
2	40	.4	2.5
3	30	.3	0
4	0	0	0

### A COHORT LIFE TABLE (for Unicorns)

$N_0 = 100$

Age	Number Surviving	Proportion Surviving	Average Offspring per female of age x	Realized Fecundity Values
<b>X</b>	<b>N<sub>x</sub></b>	<b>l<sub>x</sub></b>	<b>m<sub>x</sub></b>	<b>l<sub>x</sub>m<sub>x</sub></b>
0	100	1.0	0	0
1	50	.5	4	2
2	40	.4	2.5	1
3	30	.3	0	0
4	0	0	0	0

$$R_0 = \sum l_x m_x = 3$$

$R_0 =$  Net Replacement

### A STATIC LIFE TABLE (for Unicorns)

Sample of 100 unicorns  $N_0 = 100$

Age Interval	Number Surviving at Beginning of X	Number Dying	Proportion Surviving by Age X
X	N <sub>x</sub>	d <sub>x</sub>	l <sub>x</sub>
0	100	50	1.0
1	50	10	.5
2	40	10	.4
3	30	30	.3
4	0	0	0

### A COHORT LIFE TABLE (for Unicorns)

$N_0 = 100$

Age	Number Surviving	Proportion Surviving	Average Offspring per female of age x	Realized Fecundity Values	
X	$N_x$	$l_x$	$m_x$	$l_x m_x$	$l_x m_x X$
0	100	1.0	0	0	0
1	50	.5	4	2	2
2	40	.4	2.5	1	2
3	30	.3	0	0	0
4	0	0	0	0	0

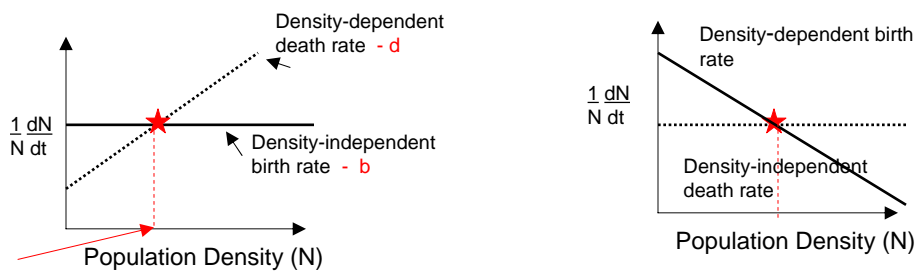
$$R_0 = \text{Net Replacement} = \sum l_x m_x = 3$$

$$G \sim (\sum l_x m_x X) / (\sum l_x m_x) = (\sum l_x m_x X) / R_0 = 4/3 \text{ years}$$

**Intrinsic Rate of Increase**

$$r \approx (\ln R_0) / G \approx (\ln 3) / 1.33 \approx 0.82 \text{ yr}^{-1}$$

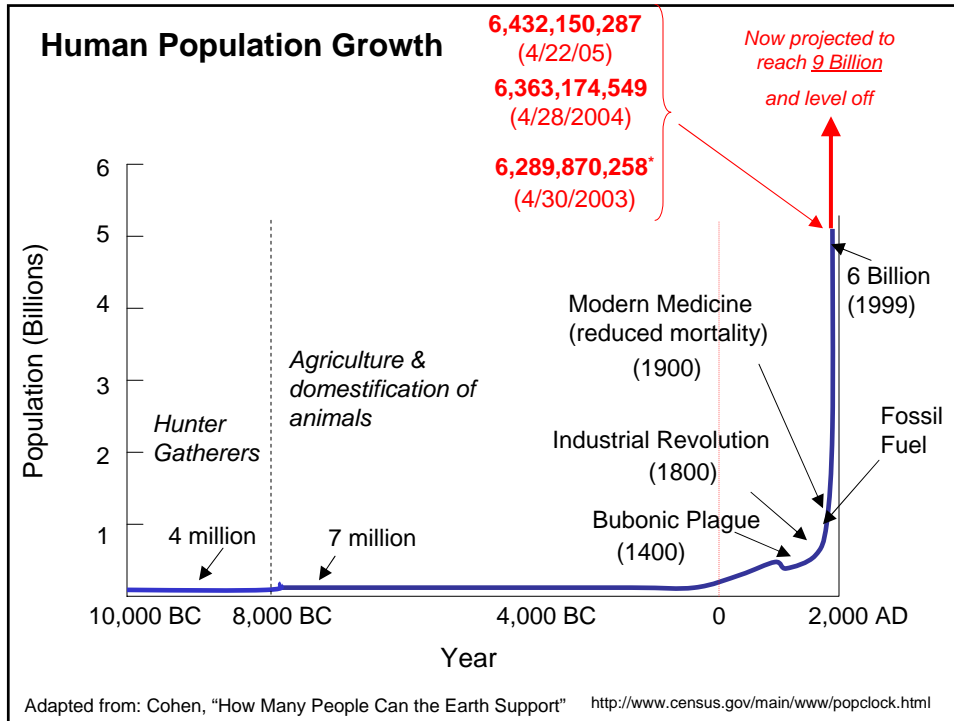
### Density-Dependant Factors Regulate Population Size



Equilibrium Density

$$r = b - d$$

as  $N \uparrow$   $r \downarrow$   
stabilizing



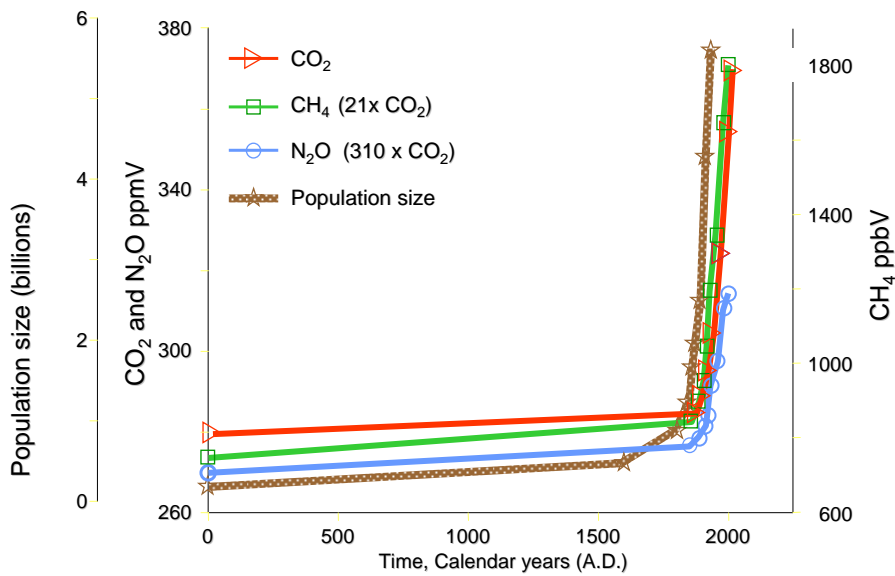
### Four "Evolution" in Human Population Growth

Evolution Driver	Midpoint	Population (billions)	Doubling Time (years) before	Doubling Time (years) after
Local Agriculture	8000 B.C.	0.005	40,000 - 300,000	1,400 - 3,000
Global Agriculture	1750 A.D.	0.75	750 - 1,800	100 - 130
Public Health	1950	2.5	87	36
Fertility Control	1970	3.7	34 (peak)	>40 (since 1990)

Adapted from: Cohen, "How Many People Can the Earth Support"



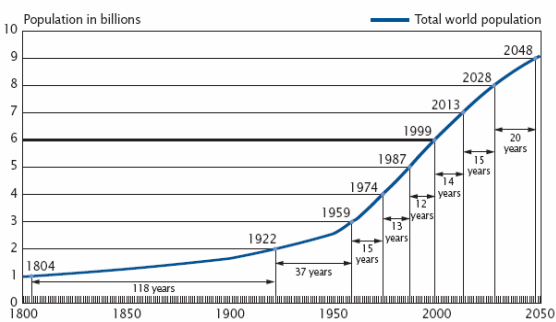
## and recall from lecture 20.....



Falkowski and Tchernov 2004

<http://www.census.gov/ipc/prod/wp02/wp02-1.pdf>

Figure 1.  
Annual Additions and the Annual Growth Rate of Global Population  
The growth of global population has peaked.



Source: United Nations, *World Population Prospects: The 1994 Revision*; U.S. Census Bureau, International Programs Center, International Data Base and unpublished tables.

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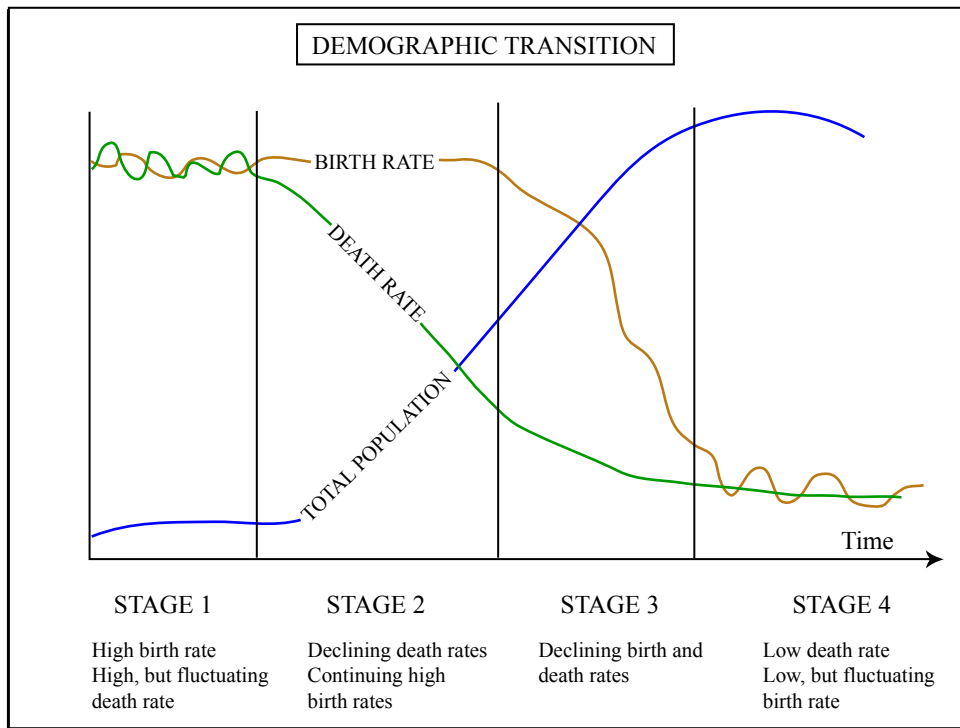


Figure by MIT OCW.