Incremental Path Planning

Continuous Planning and Dynamic A*

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16.412/6.834 Cognitive Robotics
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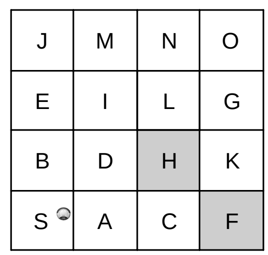
Outline

- Optimal Path Planning in Partially Known Environments.
- Continuous Optimal Path Planning
  - Dynamic A*
  - Incremental A* (LRTA*) [Appendix]

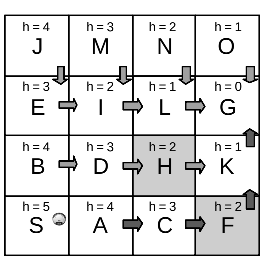
[Zellinsky, 92]

1. Generate global path plan from initial map.
2. Repeat until goal reached or failure:
   - Execute next step in current global path plan
   - Update map based on sensors.
   - If map changed generate new global path from map.

Compute Optimal Path

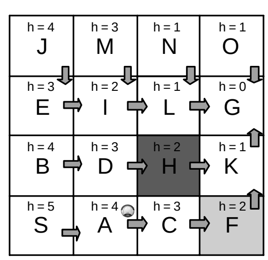


Begin Executing Optimal Path



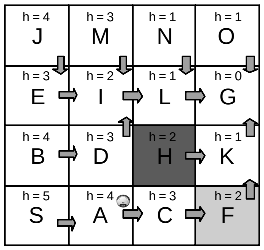
- Robot moves along backpointers towards goal.
- Uses sensors to detect discrepancies along way.

Obstacle Encountered!



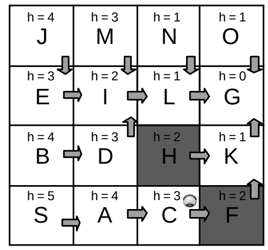
- At state A, robot discovers edge from D to H is blocked (cost 5,000 units).
- Update map and reinvoke planner.

Continue Path Execution



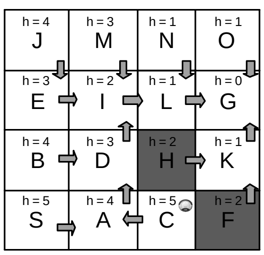
- A's previous path is still optimal.
- Continue moving robot along back pointers.

Second Obstacle, Replan!



- At C robot discovers blocked edges from C to F and H (cost 5,000 units).
- Update map and reinvoke planner.

Path Execution Achieves Goal



- Follow back pointers to goal.
- No further discrepancies detected; goal achieved!

Outline

- Optimal Path Planning in Partially Known Environments.
- Continuous Optimal Path Planning
  - Dynamic A*
  - Incremental A* (LRTA*) [Appendix]

What is Continuous Optimal Path Planning?

- Supports search as a repetitive online process.
- Exploits similarities between a series of searches to solve much faster than solving each search starting from scratch.
- Reuses the identical parts of the previous search tree, while updating differences.
- Solutions guaranteed to be optimal.
- On the first search, behaves like traditional algorithms.
  - D* behaves exactly like Dijkstra's.
  - Incremental A* A* behaves exactly like A*.

Dynamic A* (aka D*) [Stenz, 94]

1. Generate global path plan from initial map.
2. Repeat until Goal reached, or failure.
   - Execute next step of current global path plan.
   - Update map based on sensor information.
   - Incrementally update global path plan from map changes.

➔ 1 to 3 orders of magnitude speedup relative to a non-incremental path planner.

Map and Path Concepts

- *c(X,Y) :*
  Cost to move from Y to X.
  c(X,Y) is undefined if move disallowed.
- *Neighbors(X)* :
  Any Y such that c(X,Y) or c(Y,X) is defined.
- *o(G,X)* :
  True optimal path cost to Goal from X.
- *h(G,X)* :
  Estimate of optimal path cost to goal from X.
- *b(X) = Y : backpointer* from X to Y.
  Y is the first state on path from X to G.

D* Search Concepts

- *State tag t(X)* :
  - *NEW* : has no estimate h.
  - *OPEN* : estimate needs to be propagated.
  - *CLOSED* : estimate propagated.
- *OPEN list* :
  States with estimates to be propagated to other states.
  - States on list tagged OPEN
  - Sorted by key function k (defined below).

D* Fundamental Search Concepts

- k(G,X) : key function

  Minimum of
  - h(G,X) before modification, and
  - all values assumed by h(G,X) since X was placed on the OPEN list.
- *Lowered state* : k(G,X) = current h(G,X),
  - Propagate decrease to descendants and other nodes.
- *Raised state* : k(G,X) < current h(G,X),
  - Propagate increase to dscendants and other nodes.
  - Try to find alternate shorter paths.

Running D* First Time on Graph

Initially
- Mark G Open and Queue it
- Mark all other states New
- Run Process_States on queue until path found or empty.

When edge cost c(X,Y) changes
- If X is marked Closed, then
  - Update h(X)
  - Mark X open and queue with key h(X).

Use D* to Compute Initial Path

J NEW	M NEW	N NEW	O NEW
E NEW	I NEW	L NEW	G NEW
B NEW	D NEW	H NEW	K NEW
S NEW	A NEW	C NEW	F NEW

States initially tagged NEW (no cost determined yet).

Use D* to Compute Initial Path

J NEW	M NEW	N NEW	O NEW
E NEW	I NEW	L NEW	h = 0 G OPEN
B NEW	D NEW	H NEW	K NEW
S NEW	A NEW	C NEW	F NEW

	OPEN List
1	(0,G)

```
if kold = h(X) then
 for each neighbor Y of X:
  if t(Y) = NEW or
    (b(Y) = X and h(Y) ? h(X) + c(X,Y)) or
    (b(Y) ? X and h(Y) > h(X) + c(X,Y)) then
    b(Y) = X; Insert(Y,h(X) + c(X,Y))
```

- Add Goal node to the OPEN list.
- Process OPEN list until the robot's current state is CLOSED.

Process_State: New or Lowered State

- Remove from Open list , state X with lowest k
- If X is a new/lowered state, its path cost is optimal! Then propagate to each neighbor Y
  - If Y is New, give it an initial path cost and propagate.
  - If Y is a descendant of X, propagate any change.
  - Else, if X can lower Y's path cost, Then do so and propagate.

Use D* to Compute Initial Path

J NEW	M NEW	N NEW	O NEW
E NEW	I NEW	L NEW	h = 0 G OPEN
B NEW	D NEW	H NEW	K NEW
S NEW	A NEW	C NEW	F NEW

	OPEN List
1	(0,G)

```
if kold = h(X) then
  for each neighbor Y of X:
    if t(Y) = NEW or
      (b(Y) = X and h(Y) ? h(X) + c(X,Y)) or
      (b(Y) ? X and h(Y) > h(X) + c(X,Y)) then
      b(Y) = X; Insert(Y,h(X) + c(X,Y))
```

- Add new neighbors of G onto the OPEN list
- Create backpointers to G.

Use D* to Compute Initial Path

J NEW	M NEW	N NEW	h = 1 O OPEN
E NEW	I NEW	h = 1 L OPEN	h = 0 G CLOSED
B NEW	D NEW	H NEW	h = 1 K OPEN
S NEW	A NEW	C NEW	F NEW

	OPEN List
1	(0,G)
2	(1,K) (1,L) (1,O)

```
if kold = h(X) then
  for each neighbor Y of X:
    if t(Y) = NEW or
      (b(Y) = X and h(Y) ? h(X) + c(X,Y)) or
      (b(Y) ? X and h(Y) > h(X) + c(X,Y)) then
      b(Y) = X; Insert(Y,h(X) + c(X,Y))
```

- Add new neighbors of G onto the OPEN list
- Create backpointers to G.

Use D* to Compute Initial Path

J NEW	M NEW	N NEW	h = 1 O OPEN
E NEW	I NEW	h = 1 L OPEN	h = 0 G CLOSED
B NEW	D NEW	h = **2** H OPEN	h = 1 K CLOSED
S NEW	A NEW	C NEW	h = 2 F OPEN

	OPEN List
1	(0,G)
2	(1,K) (1,L) (1,O)
3	(1,L) (1,O) (2,F) (2,H)

```
if kold = h(X) then
  for each neighbor Y of X:
    if t(Y) = NEW or
      (b(Y) = X and h(Y) ? h(X) + c(X,Y)) or
      (b(Y) ? X and h(Y) > h(X) + c(X,Y)) then
      b(Y) = X; Insert(Y,h(X) + c(X,Y))
```

- Add new neighbors of K on to the OPEN list
- Create backpointers.

Use D* to Compute Initial Path

J NEW	M NEW	h = **2** N OPEN	h = 1 O CLOSED
E NEW	h = 2 I OPEN	h = 1 L CLOSED	h = 0 G CLOSED
B NEW	D NEW	h = 2 H OPEN	h = 1 K CLOSED
S NEW	A NEW	C NEW	h = 2 F OPEN

	OPEN List
1	(0,G)
2	(1,K) (1,L) (1,O)
3	(1,L) (1,O) (2,F) (2,H)
4	(2,F) (2,H) (2,I) (2,N)

```
if kold = h(X) then
  for each neighbor Y of X:
    if t(Y) = NEW or
      (b(Y) = X and h(Y) ? h(X) + c(X,Y)) or
      (b(Y) ? X and h(Y) > h(X) + c(X,Y)) then
      b(Y) = X; Insert(Y,h(X) + c(X,Y))
```

- Add new neighbors of L, then O on to the OPEN list
- Create backpointers.

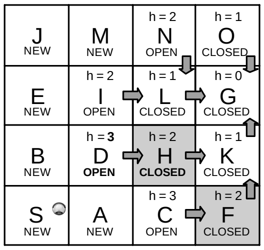
Use D* to Compute Initial Path

J NEW	M NEW	h = 2 N OPEN	h = 1 O CLOSED
E NEW	h = 2 I OPEN	h = 1 L CLOSED	h = 0 G CLOSED
B NEW	D NEW	h = 2 H OPEN	h = 1 K CLOSED
S NEW	A NEW	h = **3** C OPEN	h = 2 F CLOSED

	OPEN List
1	(0,G)
2	(1,K) (1,L) (1,O)
3	(1,L) (1,O) (2,F) (2,H)
4	(2,F) (2,H) (2,I) (2,N)
5	(2,H) (2,I) (2,N) (3,C)

- Continue until current state S is closed.

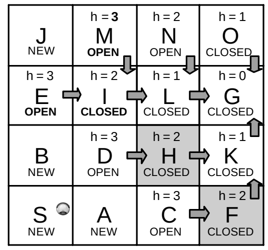
Use D* to Compute Initial Path



	OPEN List
1	(0,G)
2	(1,K) (1,L) (1,O)
3	(1,L) (1,O) (2,F) (2,H)
4	(2,F) (2,H) (2,I) (2,N)
5	(2,H) (2,I) (2,N) (3,C)
6	(2,I) (2,N) (3,C) (3,D)

- Continue until current state S is closed.

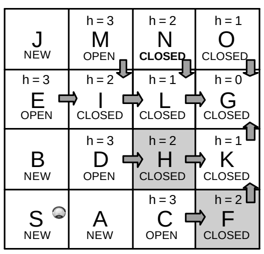
Use D* to Compute Initial Path



	OPEN List
1	(0,G)
2	(1,K) (1,L) (1,O)
3	(1,L) (1,O) (2,F) (2,H)
4	(2,F) (2,H) (2,I) (2,N)
5	(2,H) (2,I) (2,N) (3,C)
6	(2,I) (2,N) (3,C) (3,D)
7	(2,N) (3,C) (3,D) (3,E) (3,M)

- Continue until current state S is closed.

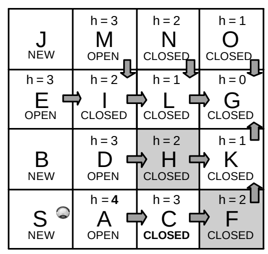
Use D* to Compute Initial Path



	OPEN List
1	(0,G)
2	(1,K) (1,L) (1,O)
3	(1,L) (1,O) (2,F) (2,H)
4	(2,F) (2,H) (2,I) (2,N)
5	(2,H) (2,I) (2,N) (3,C)
6	(2,I) (2,N) (3,C) (3,D)
7	(2,N) (3,C) (3,D) (3,E) (3,M)
8	(3,C) (3,D) (3,E) (3,M)

- Continue until current state S is closed.

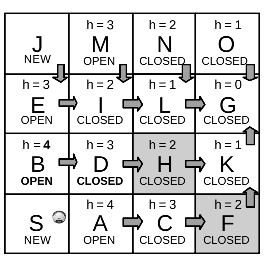
Use D* to Compute Initial Path



	OPEN List
1	(0,G)
2	(1,K) (1,L) (1,O)
3	(1,L) (1,O) (2,F) (2,H)
4	(2,F) (2,H) (2,I) (2,N)
5	(2,H) (2,I) (2,N) (3,C)
6	(2,I) (2,N) (3,C) (3,D)
7	(2,N) (3,C) (3,D) (3,E) (3,M)
8	(3,C) (3,D) (3,E) (3,M)
9	(3,D) (3,E) (3,M) (4,A)

- Continue until current state S is closed.

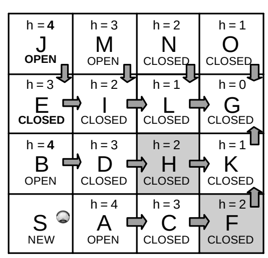
Use D* to Compute Initial Path



	OPEN List
1	(0,G)
2	(1,K) (1,L) (1,O)
3	(1,L) (1,O) (2,F) (2,H)
4	(2,F) (2,H) (2,I) (2,N)
5	(2,H) (2,I) (2,N) (3,C)
6	(2,I) (2,N) (3,C) (3,D)
7	(2,N) (3,C) (3,D) (3,E) (3,M)
8	(3,C) (3,D) (3,E) (3,M)
9	(3,D) (3,E) (3,M) (4,A)
10	(3,E) (3,M) (4,A) (4,B)

- Continue until current state S is closed..

Use D* to Compute Initial Path



	OPEN List
10	(3,E) (3,M) (4,A) (4,B)
11	(3,M) (4,A) (4,B) (4,J)
12	
13	
14	

- Continue until current state S is closed.

Use D* to Compute Initial Path

h = 4 J OPEN	h = 3 M CLOSED	h = 2 N CLOSED	h = 1 O CLOSED
h = 3 E CLOSED	h = 2 I CLOSED	h = 1 L CLOSED	h = 0 G CLOSED
h = 4 B OPEN	h = 3 D CLOSED	h = 2 H CLOSED	h = 1 K CLOSED
S NEW	h = 4 A OPEN	h = 3 C CLOSED	h = 2 F CLOSED

	OPEN List
10	(3,E) (3,M) (4,A) (4,B)
11	(3,M) (4,A) (4,B) (4,J)
12	(3,M) (4,A) (4,B)
13	
14	

- Continue until current state S is closed.

Use D* to Compute Initial Path

h = 4 J OPEN	h = 3 M CLOSED	h = 2 N CLOSED	h = 1 O CLOSED
h = 3 E CLOSED	h = 2 I CLOSED	h = 1 L CLOSED	h = 0 G CLOSED
h = 4 B OPEN	h = 3 D CLOSED	h = 2 H CLOSED	h = 1 K CLOSED
h = 5 S OPEN	h = 4 A CLOSED	h = 3 C CLOSED	h = 2 F CLOSED

	OPEN List
10	(3,E) (3,M) (4,A) (4,B)
11	(3,M) (4,A) (4,B) (4,J)
12	(3,M) (4,A) (4,B)
13	(4,B) (4,J) (5,S)
14	
15	

- Continue until current state S is closed.

Use D* to Compute Initial Path

h = 4 J OPEN	h = 3 M CLOSED	h = 2 N CLOSED	h = 1 O CLOSED
h = 3 E CLOSED	h = 2 I CLOSED	h = 1 L CLOSED	h = 0 G CLOSED
h = 4 B CLOSED	h = 3 D CLOSED	h = 2 H CLOSED	h = 1 K CLOSED
h = 5 S OPEN	h = 4 A CLOSED	h = 3 C CLOSED	h = 2 F CLOSED

	OPEN List
10	(3,E) (3,M) (4,A) (4,B)
11	(3,M) (4,A) (4,B) (4,J)
12	(3,M) (4,A) (4,B)
13	(4,B) (4,J) (5,S)
14	(4,J) (5,S)
15	

- Continue until current state S is closed.

Use D* to Compute Initial Path

h = 4 J CLOSED	h = 3 M CLOSED	h = 2 N CLOSED	h = 1 O CLOSED
h = 3 E CLOSED	h = 2 I CLOSED	h = 1 L CLOSED	h = 0 G CLOSED
h = 4 B CLOSED	h = 3 D CLOSED	h = 2 H CLOSED	h = 1 K CLOSED
h = 5 S OPEN	h = 4 A CLOSED	h = 3 C CLOSED	h = 2 F CLOSED

	OPEN List
10	(3,E) (3,M) (4,A) (4,B)
11	(3,M) (4,A) (4,B) (4,J)
12	(3,M) (4,A) (4,B)
13	(4,B) (4,J) (5,S)
14	(4,J) (5,S)
15	(5,S)

- Continue until current state S is closed.

D* Completed Initial Path

h = 4 J CLOSED	h = 3 M CLOSED	h = 2 N CLOSED	h = 1 O CLOSED
h = 3 E CLOSED	h = 2 I CLOSED	h = 1 L CLOSED	h = 0 G CLOSED
h = 4 B CLOSED	h = 3 D CLOSED	h = 2 H CLOSED	h = 1 K CLOSED
h = 5 S CLOSED	h = 4 A CLOSED	h = 3 C CLOSED	h = 2 F CLOSED

	OPEN List
10	(3,E) (3,M) (4,A) (4,B)
11	(3,M) (4,A) (4,B) (4,J)
12	(3,M) (4,A) (4,B)
13	(4,B) (4,J) (5,S)
14	(4,J) (5,S)
15	(5,S)
16	NULL

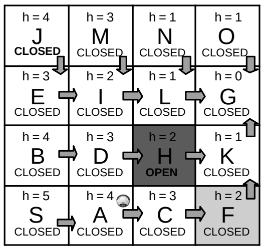
- Done: Current state S is closed, and Open list is empty.

Begin Executing Optimal Path

h = 4 J CLOSED	h = 3 M CLOSED	h = 2 N CLOSED	h = 1 O CLOSED
h = 3 E CLOSED	h = 2 I CLOSED	h = 1 L CLOSED	h = 0 G CLOSED
h = 4 B CLOSED	h = 3 D CLOSED	h = 2 H CLOSED	h = 1 K CLOSED
h = 5 S CLOSED	h = 4 A CLOSED	h = 3 C CLOSED	h = 2 F CLOSED

- Robot moves along backpointers towards goal
- Uses sensors to detect discrepancies along way.

Obstacle Encountered!



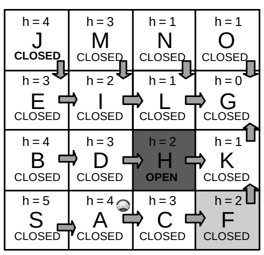
- At state A, robot discovers edge D to H is blocked off (cost 5,000 units).
- Update map and reinvoke D*

Running D* After Edge Cost Change

When edge cost c(X,Y) changes

- If X is marked Closed, then
  - Update h(X)
  - Mark X open and queue, key is new h(X).
- Run Process_State on queue
  - until path to current state is shown optimal,
  - or queue Open List is empty.

D* Update From First Obstacle

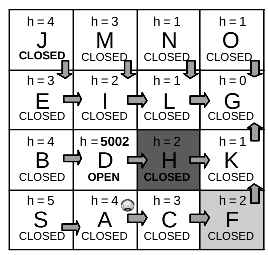


	OPEN List
1	(2,H)
2	
3	
4	

```
Function: Modify-Cost(X,Y,eval)
c(X,Y) = eval
if t(X) = CLOSED
   then Insert(X,h(X))
return Get-Kmin()
```

- Assign cost of 5,000 for D to H
- Propagate changes starting at H

D* Update From First Obstacle



	OPEN List
1	(2,H)
2	(3,D)
3	
4	

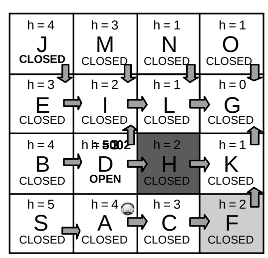
```
if kold = h(X) then
 for each neighbor Y of X:
  if t(Y) = NEW or
     (b(Y) = X and h(Y) ? h(X) + c(X,Y)) or
     (b(Y) ? X and h(Y) > h(X) + c(X,Y)) then
     b(Y) = X; Insert(Y,h(X) + c(X,Y))
```

- Raise cost of H's descendant D, and propagate.

Process_State: Raised State

- If X is a raise state its cost might be suboptimal.
- Try reducing cost of X using an *optimal neighbor* Y.
  - h(Y) = [h(X) before it was raised]
- Propagate X's cost to each neighbor Y
  - If Y is New, Then give it an initial path cost and propagate.
  - **If Y is a descendant of X, Then propagate ANY change.**
  - If X can lower Y's path cost,
    - Postpone: Queue X to propagate when optimal (reach current h(X))
  - If Y can lower X's path cost, and Y is suboptimal,
    - Postpone: Queue Y to propagate when optimal (reach current h(Y)).
  - ➔ Postponement avoids creating cycles.

D* Update From First Obstacle

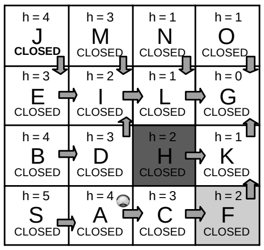


	OPEN List
1	(2,H)
2	(3,D)
3	
4	

```
if kold < h(X) then
 for each neighbor Y of X:
  if h(Y) = kold and h(X) > h(Y) + C(Y,X) then
    b(X) = Y; h(X) = h(Y) + c(Y,X);
```

- D may not be optimal, check neighbors for better path.
- Transitioning to I is better, and I's path is optimal, so update D.

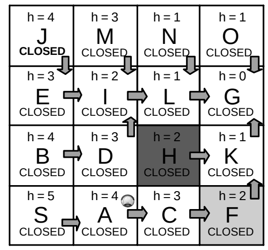
D* Update From First Obstacle



	OPEN List
1	
2	(3,D)
3	NULL
4	

- All neighbors of D have consistent h-values.
- No further propagation needed.

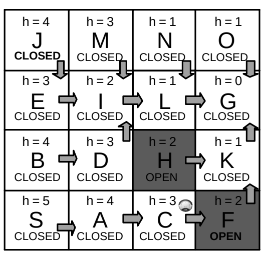
Continue Path Execution



	OPEN List
1	
2	(3,D)
3	NULL
4	

- A's path optimal.
- Continue moving robot along backpointers.

Second Obstacle!

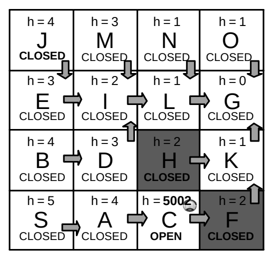


	OPEN List
1	(2,F) (2,H)
2	
3	
4	

Function: Modify-Cost(X,Y,eval)
c(X,Y) = eval
if t(X) = CLOSED
then Insert(X,h(X))
return Get-Kmin()

- At C robot discovers blocked edges C to F and H (cost 5,000 units).
- Update map and reinvoke D* until H(current position optimal).

D* Update From Second Obstacle

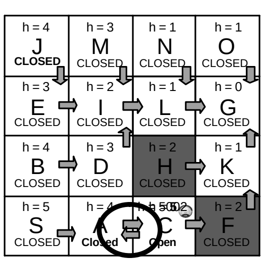


	OPEN List
1	(2,F) (2,H)
2	(3,C)
3	
4	

if kold = h(X) then
for each neighbor Y of X:
if t(Y) = NEW or
(b(Y) = X and h(Y) ? h(X) + c(X,Y)) or
(b(Y) ? X and h(Y) > h(X) + c(X,Y)) then
b(Y) = X; Insert(Y,h(X) + c(X,Y))

- Processing F raises descendant C's cost, and propagates.
- Processing H does nothing.

D* Update From Second Obstacle

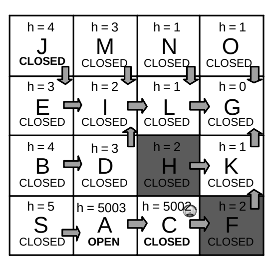


	OPEN List
1	(2,F) (2,H)
2	(3,C)
3	
4	

if kold < h(X) then
for each neighbor Y of X:
if h(Y) = kold and h(X) > h(Y) + C(Y,X) then
b(X) = Y; h(X) = h(Y) + c(Y,X);

- C may be suboptimal, check neighbors; → Better path through A!
- However, A may be suboptimal, and updating would create a loop!

D* Update From Second Obstacle



	OPEN List
1	(2,F) (2,H)
2	(3,C)
3	(4,A)
4	

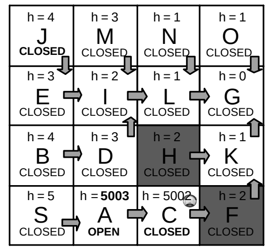
for each neighbor Y of X:
if t(Y) = NEW or
(b(Y) = X and h(Y) ? h(X) + c(X,Y))
then
b(Y) = X; Insert(Y,h(X) + c(X,Y))

- Don't change C's path to A (yet).
- Instead, propagate increase to A.

Process_State: Raised State

- If X is a raise state its cost might be suboptimal.
- **Try reducing cost of X using an *optimal neighbor* Y.**
  - h(Y) = [h(X) before it was raised]
- propagate X's cost to each neighbor Y
  - If Y is New, Then give it an initial path cost and propagate.
  - **If Y is a descendant of X, Then propagate ANY change.**
  - If X can lower Y's path cost,
    - Postpone: Queue X to propagate when optimal (reach current h(X))
  - **If Y can lower X's path cost, and Y is suboptimal,**
    - **Postpone:** Queue Y to propagate when optimal (reach current h(Y)).
  - ➔ Postponement avoids creating cycles.

D* Update From Second Obstacle

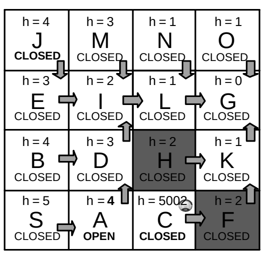


	OPEN List
1	(2,F) (2,H)
2	(3,C)
3	(4,A)
4	

```
if kold < h(X) then
for each neighbor Y of X:
  if h(Y) = kold and h(X) > h(Y) + C(Y,X) then
    b(X) = Y; h(X) = h(Y) + c(Y,X);
```

- A may not be optimal, check neighbors for better path.
- Transitioning to D is better, and D's path is optimal, so update A.

D* Update From Second Obstacle



	OPEN List
1	(2,F) (2,H)
2	(3,C)
3	(4,A)
4	

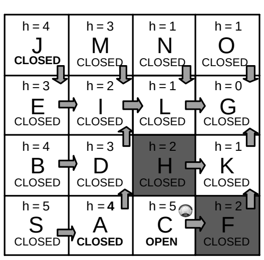
```
if kold < h(X) then
for each neighbor Y of X:
  if h(Y) = kold and h(X) > h(Y) + C(Y,X) then
    b(X) = Y; h(X) = h(Y) + c(Y,X);
```

- A may not be optimal, check neighbors for better path.
- Transitioning to D is better, and D's path is optimal, so update A.

Process_State: New or Lowered State

- Remove from Open list , state X with lowest k
- If X is a new/lowered state its path cost is optimal, Then propagate to each neighbor Y
  - If Y is New, give it an initial path cost and propagate.
  - If Y is a descendant of X, propagate any change.
  - Else, if X can lower Y's path cost, Then do so and propagate.

D* Update From Second Obstacle

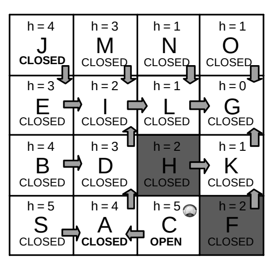


	OPEN List
1	(2,F) (2,H)
2	(3,C)
3	(4,A)
4	(5,C)

```
for each neighbor Y of X:
  if (b(Y) = X and h(Y) ? h(X) + c(X,Y)) then
    b(Y) = X; Insert(Y,h(X) + c(X,Y))
  else
    if b(Y) ? X and h(Y) > h(X) + c(X,Y) then
      Insert(X,h(X))
```

- A can improve neighbor C, so queue C.

D* Update From Second Obstacle

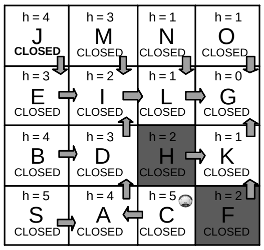


	OPEN List
1	(2,F) (2,H)
2	(3,C)
3	(4,A)
4	(5,C)
5	

```
if kold = h(X) then
for each neighbor Y of X:
  if t(Y) = NEW or
     (b(Y) = X and h(Y) ? h(X) + c(X,Y)) or
     (b(Y) ? X and h(Y) > h(X) + c(X,Y)) then
    b(Y) = X; Insert(Y,h(X) + c(X,Y))
```

- C lowered to optimal; no neighbors affected.
- Current state reached, so Process_State terminates.

Complete Path Execution



- Follow back pointers to Goal.
- No further discrepancies detected; goal achieved!

D* Pseudo Code

```
Function: Process-State()
 X = Min-State()
 if X=NULL then return -1
 kold = Get-Kmin(); Delete(X)
 if kold < h(X) then
  for each neighbor Y of X:
   if h(Y) = kold and h(X) > h(Y) + C(Y,X) then
    b(X) = Y; h(X) = h(Y) + c(Y,X);
 if kold = h(X) then
  for each neighbor Y of X:
   if t(Y) = NEW or
      (b(Y) = X and h(Y) ? h(X) + c(X,Y)) or
      (b(Y) ? X and h(Y) > h(X) + c(X,Y)) then
      b(Y) = X; Insert(Y,h(X) + c(X,Y))
else
 for each neighbor Y of X:
  if t(Y) = NEW or
     (b(Y) = X and h(Y) ? h(X) + c(X,Y)) then
     b(Y) = X; Insert(Y,h(X) + c(X,Y))
  else
     if b(Y) ? X and h(X) > h(X) + c(X,Y) then
        Insert(X,h(X))
     else
       if b(Y) ? X and h(X) > h(Y) + c(Y,X) and
          t(Y) = CLOSED and h(Y) > kold then
          Insert(Y,h(Y))
return Get-Kmin()
```

```
Function: Modify-Cost(X,Y,eval)
1:  c(X,Y) = eval
2:  if t(X) = CLOSED
       then Insert(X,h(X))
3:  return Get-Kmin()
```

```
Function: Insert(X, h_new)
1:  if t(X) = NEW
      then k(x) = h_new
2:  else if t(X) = OPEN
      then k(X) = min(k(X), h_new)
3:  else
      k(X) = min(h(X), h_new)
4:  h(X) = h_new;
5:  t(X) = OPEN
```

Recap: Continuous Optimal Planning

1. Generate global path plan from initial map.
2. Repeat until Goal reached, or failure.
   - Execute next step of current global path plan.
   - Update map based on sensor information.
   - Incrementally update global path plan from map changes.

➔ 1 to 3 orders of magnitude speedup relative to a non-incremental path planner.

Recap: Dynamic A*

- Supports search as a repetitive online process.
- Exploits similarities between a series of searches to solve much faster than from scratch.
- Reuses the identical parts of the previous search tree, while updating differences.
- Solutions guaranteed to be optimal.
- On the first search, behaves like traditional Dijkstra.