Practice Problems

3. A unidirectional graphite/epoxy material has the following properties.

$$E_{11} = 130\,GPa$$

$$E_{22} = 10.5\,GPa$$

$$\nu_{12} = 0.28$$

$$G_{12} = 6\,GPa$$

The material is transversely isotropic, which means that

$$\nu_{12} = \nu_{13}$$

$$E_{22} = E_{33}$$

The applied stress is

$$\left.\begin{aligned} \sigma_{11} &= 60\,MPa \\ \sigma_{22} &= 30\,MPa \end{aligned}\right\} \text{given}$$

$\sigma_{33} = \sigma_{23} = \sigma_{13} = 0 \leftarrow$ since material is loaded

$\sigma_{12} = 0$ in the plane of its fibers.

To determine the strain components, let's write the stress-strain relations in terms of the strains.

$$\varepsilon_{11} = \frac{1}{E_1}\sigma_{11} - \frac{\nu_{12}}{E_1}\sigma_{22} - \frac{\nu_{13}}{E_1}\sigma_{33} \quad \text{— ①}$$

$$\varepsilon_{22} = -\frac{\nu_{12}}{E_1}\sigma_{11} + \frac{1}{E_2}\sigma_{22} - \frac{\nu_{23}}{E_2}\sigma_{33} \quad \text{— ②}$$

$$\varepsilon_{33} = -\frac{\nu_{13}}{E_1}\sigma_{11} - \frac{\nu_{23}}{E_2}\sigma_{22} + \frac{1}{E_3}\sigma_{33} \quad \text{— ③}$$

$$\varepsilon_{23} = \frac{1}{2G_{23}}\sigma_{23} \quad \text{— ④}$$

$$\varepsilon_{13} = \frac{1}{2G_{13}}\sigma_{13} \quad \text{— ⑤}$$

$$\varepsilon_{12} = \frac{1}{2G_{12}}\sigma_{12} \quad \text{— ⑥}$$

Now, plug the given elastic constants into equations ① ~ ⑥ to obtain strains.

$$① \rightarrow \quad \varepsilon_{11} = \frac{60\,MPa}{130\,GPa} - \frac{0.28}{130\,GPa}(30\,MPa)$$

$$= \underbrace{462 \times 10^{-6}}_{\sigma_{11}\text{ contribution}} - \underbrace{64.6 \times 10^{-6}}_{\sigma_{22}\text{ contribution}}$$

$$\Rightarrow \quad \varepsilon_{11} = 397 \times 10^{-6}$$

$$② \rightarrow \quad \varepsilon_{22} = -\frac{0.28}{130\,GPa}(60\,MPa) + \frac{30\,MPa}{10.5\,GPa}$$

$$= \underbrace{-129 \times 10^{-6}}_{\sigma_{11}\text{ contribution}} + \underbrace{2857 \times 10^{-6}}_{\sigma_{22}\text{ contribution}}$$

$$\Rightarrow \quad \varepsilon_{22} = 2728 \times 10^{-6}$$

$\nu_{13} = \nu_{12}$

$$③ \rightarrow \quad \varepsilon_{33} = -\frac{0.28}{130GPa}(60MPa) - \frac{\nu_{23}}{10.5GPa}(30MPa)$$

$$= \underbrace{-129 \times 10^{-6}}_{\sigma_{11}\ \text{contribution}} - \underbrace{2857 \times 10^{-6}\,\nu_{23}}_{\sigma_{22}\ \text{contribution}}$$

$$④ \rightarrow \quad \varepsilon_{23} = 0$$

$$⑤ \rightarrow \quad \varepsilon_{13} = 0$$

$$⑥ \rightarrow \quad \varepsilon_{12} = 0$$

To summarize:

$$\boxed{\begin{aligned} \varepsilon_{11} &= 397\ \mu\text{strain} \\ \varepsilon_{22} &= 2728\ \mu\text{strain} \\ \varepsilon_{33} &= -129. - 2857\,\nu_{23}\ \mu\text{strain} \end{aligned}}$$

4. The strains in the piece of aluminum can be obtained in the same manner as in previous problem

Elastic constants: $E = 10.3\ Msi = 10.3 Msi \times \frac{6.895 GPa}{1\ Msi}$

$$\Rightarrow E = 71 GPa$$

$$\nu = 0.30$$

Applied stress : $\sigma_{11} = 60MPa$

$\sigma_{22} = 30MPa$

$\sigma_{33} = \sigma_{23} = \sigma_{13} = \sigma_{12} = 0$

The stress-strain relations for the isotropic case are :

$$\varepsilon_{11} = \frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \quad \text{——— ①}$$

$$\varepsilon_{22} = -\frac{\nu}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \quad \text{——— ②}$$

$$\varepsilon_{33} = -\frac{\nu}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} + \frac{1}{E}\sigma_{33} \quad \text{——— ③}$$

$$\varepsilon_{23} = \frac{1}{2G_{23}}\sigma_{23} \quad \text{——— ④}$$

$$\varepsilon_{13} = \frac{1}{2G_{13}}\sigma_{13} \quad \text{——— ⑤}$$

$$\varepsilon_{12} = \frac{1}{2G_{12}}\sigma_{12} \quad \text{——— ⑥}$$

Plugging the elastic constants and applied stresses into equations ① through ⑥, we can find the strains.

$$① \longrightarrow \varepsilon_{11} = \frac{60MPa}{71GPa} - \frac{0.3}{71GPa}(30MPa)$$

$$= \underbrace{845\times10^{-6}}_{\sigma_{11}\text{ contribution}} - \underbrace{127\times10^{-6}}_{\sigma_{22}\text{ contribution}}$$

$$\Rightarrow \quad \varepsilon_{11} = 718 \times 10^{-6}$$

$$② \longrightarrow \quad \varepsilon_{22} = -\frac{0.3}{71\,GPa}(60\,MPa) + \frac{30\,MPa}{71\,GPa}$$

$$= \underbrace{-254 \times 10^{-6}}_{\sigma_{11} \text{ contribution}} + \underbrace{422 \times 10^{-6}}_{\sigma_{22} \text{ contribution}}$$

$$\Rightarrow \quad \varepsilon_{22} = 168 \times 10^{-6}$$

$$③ \longrightarrow \quad \varepsilon_{33} = -\frac{0.3}{71\,GPa}(60\,MPa) - \frac{0.3}{71\,GPa}(30\,MPa)$$

$$= \underbrace{-254 \times 10^{-6}}_{\sigma_{11} \text{ contribution}} - \underbrace{127 \times 10^{-6}}_{\sigma_{22} \text{ contribution}}$$

$$\Rightarrow \quad \varepsilon_{33} = -381 \times 10^{-6}$$

$$④ \longrightarrow \quad \varepsilon_{23} = 0$$

$$⑤ \longrightarrow \quad \varepsilon_{13} = 0$$

$$⑥ \longrightarrow \quad \varepsilon_{12} = 0$$

To summarize:

$$\boxed{\begin{aligned} \varepsilon_{11} &= 718 \text{ } \mu\text{strain} \\ \varepsilon_{22} &= 168 \text{ } \mu\text{strain} \\ \varepsilon_{33} &= -381 \text{ } \mu\text{strain} \end{aligned}}$$