> Integral Methods

B> Thwaites's Method

4.5> A) Integral K·E equation

B> Dissipation methods (2 eqn methods.

Reading : handouts.

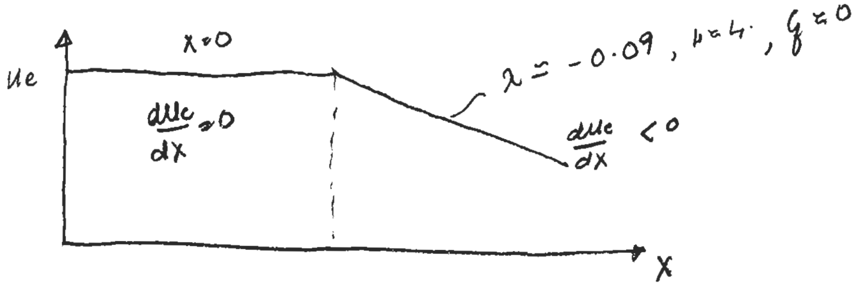
B> Limitation of Thwaites' method

$$\frac{d\theta}{dx} = \frac{C_f}{2} - (H+2)\frac{\theta}{u_e}\frac{du_e}{dx} = f\left(\theta, u_e, \frac{du_e}{dx}\right) \text{ by necessity}$$

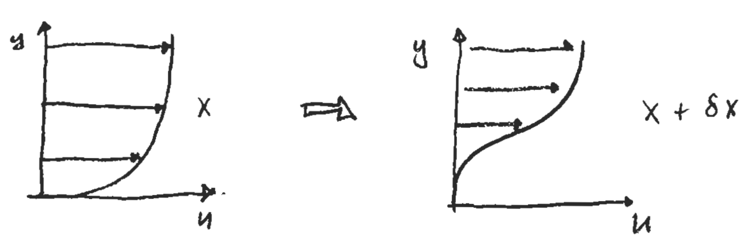
Empirical C_f and H relations limited to funcs. of $\theta, u_e, \frac{du_e}{dx}$

In general, too restrictive, inaccurate : Doesn't allow effects of upstream history, since H & C_f depend locally on λ

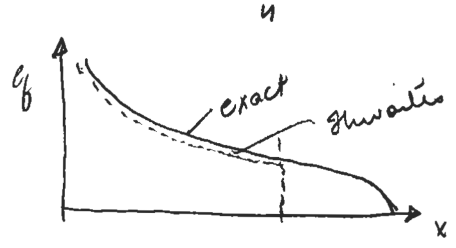
Ex



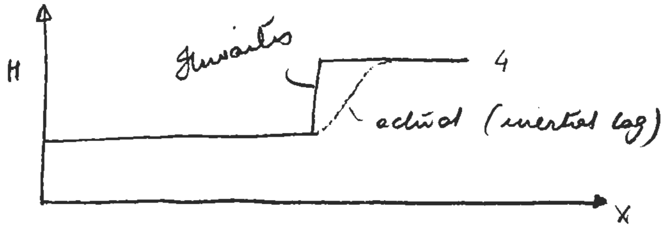
$\Rightarrow$ λ discontinuous $\Rightarrow$ H & C_f discontinuous.



no inertial lag of of velocity profile ($\frac{\partial u}{\partial x} \to \infty$ near wall)



instantaneously separates when $\lambda < 0$



H — kinematic
$\frac{du_e}{dx}$, λ — dynamic } weak assump

One equation methods uniquely tie H, G to $\frac{du_e}{dx}$

In gen'l " methods are inaccurate when $\frac{du_e}{dx}$ changes rapidly.

Better approach — kinematic ⟷ kinematic quantities

⇒ go to 2 equation methods.

4.5) Integral K.E equation

Basic diff. is G & H do not depend explicitly on $\frac{du_e}{dx}$
(unhooked from local pressure gradient)

Introduce K.E equation as 2nd equation

$$(u^2-u_e^2)\cdot \text{Continuity} + 2u\,[\text{momentum}]$$

$$(u^2-u_e^2)\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right]+2u\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-u_e\frac{du_e}{dx}-\frac{1}{\rho}\frac{\partial \tau}{\partial y}\right]=0$$

(note sign change)

or $$\frac{\partial}{\partial x}\left[(u_e^2-u^2)u\right]+\frac{\partial}{\partial y}\left[(u_e^2-u^2)v\right]+\frac{1}{\rho}\frac{\partial \tau}{\partial y}2u=0$$

Integrating in y gives

integration by parts.

$$\frac{d}{dx}\int_0^{y_e}\left[(u_e^2-u^2)u\right]dy-2\int_0^{y_e}\tau\frac{\partial u}{\partial y}dy=0$$

$$\Rightarrow \frac{d}{dx}\left(u_e^3\theta^*\right)=\frac{2D}{\rho}$$ — dimensional form

$$\frac{d\theta^*}{dx} + \frac{3\theta^*}{u_e}\cdot\frac{du_e}{dx} = 2C_D$$ ← non-dim form.

$$\theta^* = \int_0^{y_e}\left(1-(u/U_e)^2\right)(u/U_e)\,dy$$ — K·E thickness.

$$D = \int_0^{y_e} \tau \frac{\partial u}{\partial y}\,dy$$ — dissipation integral

$$C_D = \frac{1}{\rho U_e^3}\int_0^{y_e} \tau \frac{\partial u}{\partial y}\,dy$$ (dissipation coefficient) not drag

We can K·E shape parameter:

$$H^* \equiv \theta^*/\theta$$

write K·E equation in terms of H^*

$$\frac{dH^*}{dx} + (1-H)\frac{H^*}{u_e}\frac{du_e}{dx} = \frac{1}{\theta}\left[2C_D - H^* C_f/2\right]$$

→ also known K·E shape param't eqn.

- Applicable to laminar or turbulent flows (same with VKI eqn)

B) <u>Dissipation Methods.</u>

Hypothesis is:

$Re_\theta C_f = f(H)$ only ; $Re_\theta = \frac{u_e\theta}{\nu}$ (note Thwaites $Re_\theta C_f = f(\lambda)$)

$Re_\theta C_D = f(H)$

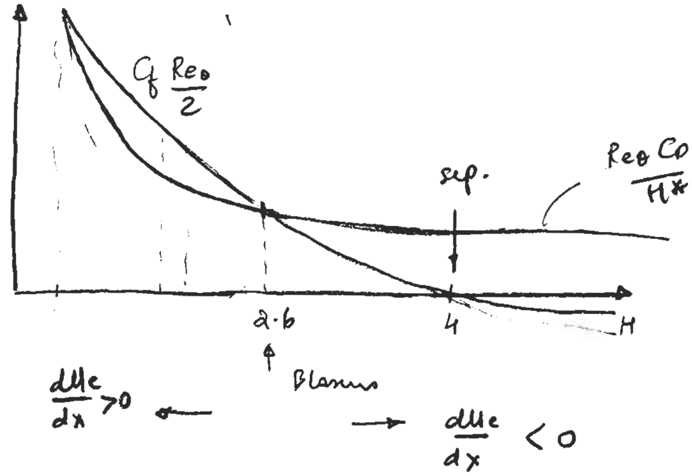
↑ dissip

$H^* = f(H)$ → $\frac{dH^*}{dx} = \frac{dH^*}{dH}\cdot\frac{dH}{dx}$

$$\Rightarrow \quad \frac{d\theta}{dx} = C_f/2 - (2+H)\,\theta/u_e\,\frac{du_e}{dx}$$

$$\frac{dH}{dx} = \frac{1}{dH^*/dH}\cdot\frac{H^*}{\theta}\left\{\frac{2C_D}{H^*} - C_f/2 + (H-1)\frac{\theta}{u_e}\frac{du_e}{dx}\right\}.$$

We have 2 simultaneous ODEs for θ & H which can be integrated given $u_e(x)$



Note: $(H-1)$ multiplies pressure gradient term – H drives itself
→ large H ⇒ b.l is more sensitive to pressure gradients

Final comment:

$\frac{d\theta}{dx} = \ldots$ → governs evolution of B.L thickness scale

$\frac{dH}{dx} = \ldots$ → governs evolution of profile shape.

Contrast between 1 & 2 eqn method

One	Two
$Re_\theta C_f = f\left(\theta, u_e, \frac{du_e}{dx}\right)$	$Re_\theta C_f = f(H)$
$H = f\left(\theta, u_e, \frac{du_e}{dx}\right)$	$Re_\theta C_D = f(H)$
	$H^* = f(H)$
Kinematic Quant. = f (Dyn. Quant)	Kin Quant = f (Kin. Quant)