1 for $0 < x < L/5$:

$$\theta^2(x) = \frac{0.45\nu}{(5U_o\, x/L)^6}\int_0^x (5U_o\, \xi/L)^5\, d\xi$$

$$= 0.015\frac{\nu L}{U_o}$$

$$\therefore\ \theta(x) = 3.87\times10^{-5} L$$

$$\lambda = \frac{\theta^2}{\nu}\frac{dU_e}{dx} = 0.075 \rightarrow H = 2.36$$

for $L/5 < x < L$

$$\theta^2(x) = \theta^2(L/5) + \frac{0.45\nu}{U_o^6}\int_{L/5}^x U_o^5\, d\xi$$

$$= 0.015\left(\frac{\nu}{U_o L}\right)L^2 + 0.45\left(\frac{\nu}{U_o L}\right)\cdot L^2\left(\frac{x}{L} - 1/5\right)$$

$$\therefore\ \theta(x) = 2.12\times10^{-4}\left\{x/L - 1/6\right\}^{1/2} L$$

$$\lambda = 0 \longrightarrow H = 2.61$$

1b) Michel's Criterion

① $Re_\theta = Re_{\theta crit} \equiv 2.9\, Re_x^{0.4}$

for $0 < x < L/5$: $x/L = 16.9$ so no transition

$L/5 < x < L$: $x/L = 0.77$ (transition)

② $Re_\theta = Re_{\theta crit} \equiv 1.174\left[1 + \left(\frac{22,400}{Re_{x,tr}}\right)\right] Re_{x,tr}^{0.46}$

No transition point

Granville's Method

$$Re_\theta(x_{tr}) \approx Re_\theta(x_i) + 450 + 400\, e^{60\lambda_m}$$

$$\lambda_m = \frac{1}{x_{tr} - x_i} \int_{x_i}^{x_{tr}} \lambda(x)\, dx$$

for $x < L/5$, $\lambda(x) = \lambda_m = 0.075$. At initial instability $Re_\theta(x_i) \approx 2400$ for $H = 2.36$, $Re_\theta(x = L/5) = 387$ $\Rightarrow$ clearly no transition point for $x < L/5$.

for $x > L/5$, $\lambda = 0$, $H = 2.61$, $Re_\theta(x_i) = 201$ which is $< Re_\theta(x = L/5)$

$\therefore$ initial instability will occur at $x = L/5$.

$$Re_\theta(x_{tr}) = 387 + 450 + 400 = 1237$$

$$Re_\theta(x) = \left(\frac{\theta}{L}\right) \cdot Re_L$$

$$= 2.12 \times 10^{-4} \left\{ x/L - 1/6 \right\}^{1/2} \cdot 1 \times 10^{7}$$

$$\Rightarrow \frac{x_{tr}}{L} \approx 0.5$$

e^N Method (Envelop Method)

$$9 = \int_{Re_{\theta_0}}^{Re_\theta} \frac{dn}{dRe_\theta} \cdot dRe_\theta$$

for $0 < x < L/5$: $H = 2.36$, $Re_{\theta_0} = 2462$, $\frac{dn}{dRe_\theta} = 0.005$

At $x = L/5$, $Re_\theta = 387$ $\Rightarrow$ no instability, no transition

for $L/5 < x < L$: $H = 2.61$, $Re_{\theta_0} = 206$, $\frac{dn}{dRe_\theta} = 0.0117$

$$9 = (Re_{\theta_{tr}} - 387)\, 0.0117 \Rightarrow Re_{\theta_{tr}} = 1156 \rightarrow \frac{x_{tr}}{L} = 0.46$$

Rapid initial acceleration makes Michel's criterion dubious, since Re_θ for a given Re_x is lower than it would be for Blasius flow. The use of Re_x is inappropriate. Both Granville's method and e^N method are more suitable since they track the instability from $x = L/5$ where it really begins.

2a) Time averaging products: $\overline{uv} = \frac{1}{2T}\int_{-T}^{T} uv\,dt$

$$\overline{(\bar{u} + \tilde{u} + u')(\bar{v} + \tilde{v} + v')} = \bar{u}\bar{v} + \overline{\tilde{u}\tilde{v}} + \overline{u'v'}$$

Terms like $\overline{\tilde{u}v'}$, $\overline{\tilde{u}u'}$, etc. are zero, since $(\tilde{\ })$ and $(\ )'$ have no frequencies in common

Time Avg. equations:

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \qquad \begin{matrix}\bar{u}(x,y)\\ \bar{v}(x,y)\end{matrix} \text{ — dependant } \underline{\text{variables}}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = \bar{u}_e\frac{d\bar{u}_e}{dx} + \nu\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\partial}{\partial x}\left[\overline{\tilde{u}^2} + \underline{\underline{\overline{u'^2}}}\right] - \frac{\partial}{\partial y}\left[\overline{\tilde{u}\tilde{v}} + \underline{\underline{\overline{u'v'}}}\right]$$

terms which need to be modeled

2b) Phase-locked ensemble averaging: $\langle uv \rangle = \frac{1}{N}\sum u_i v_i$

$$\langle(\bar{u} + \tilde{u} + u')(\bar{v} + \tilde{v} + v')\rangle = (\bar{u} + \tilde{u})(\bar{v} + \tilde{v}) + \langle u'v' \rangle$$

Ensemble averaging has no effect on $(\bar{\ })$, $(\tilde{\ })$, or their products, since these quantities are always the same for all occurrences in the averaging summation

Ensemble averaged TSL equations:

$$\frac{\partial \bar{\tilde{u}}}{\partial y} + \frac{\partial \bar{\tilde{v}}}{\partial y} = 0 \qquad \begin{matrix}\bar{\tilde{u}}(x,y,t) = \bar{u} + \tilde{u}\\ \bar{\tilde{v}}(x,y,t) = \bar{v} + \tilde{v}\end{matrix} \text{ — dep. variables}$$

$$\frac{\partial \bar{\tilde{u}}}{\partial t} + \bar{\tilde{u}}\frac{\partial \bar{\tilde{u}}}{\partial x} + \bar{\tilde{v}}\frac{\partial \bar{\tilde{u}}}{\partial y} = \frac{\partial \bar{\tilde{u}}_e}{\partial t} + \bar{\tilde{u}}_e \frac{\partial \bar{\tilde{u}}_e}{\partial x} + \nu \frac{\partial^2 \bar{\tilde{u}}}{\partial y^2} - \frac{\partial}{\partial x}\left(\underline{\underline{\langle u'^2 \rangle}}\right) - \frac{\partial}{\partial y}\left(\underline{\underline{\langle u'v' \rangle}}\right)$$

The equations are unsteady, "turbulent" terms need modeling

2c) The ensemble-averaged equations are more suitable for periodic-unsteady flows, since "mean" flow (dep. variables) $(\bar{u} + \tilde{u})$ retain the inherent unsteadiness. Turbulence model is needed for $\langle u'^2 \rangle$ and $\langle u'v' \rangle$. The time averaged equations lump the unsteadiness into the additional stresses. This would require an "unsteadiness model".