1a) $C_p \equiv \dfrac{p - p_\infty}{\frac{1}{2}\rho U_\infty^2}$, $p + \frac{1}{2}\rho U_e^2 = p_\infty + \frac{1}{2}\rho U_\infty^2 \longrightarrow C_p = 1 - \left(\dfrac{U_e}{U_\infty}\right)^2$

Given $(u_e/U_\infty) \simeq (x/c)^{\pm a} \Rightarrow C_p = 1 - (x/c)^{\pm 2a}$

$$C_L = L\Big/\tfrac{1}{2}\rho U_\infty^2 c = \frac{1}{\frac{1}{2}\rho U_\infty^2 c}\int_0^1 (p_L - p_u)\,dx = \int_0^1 (C_{pL} - C_{pu})\,d(x/c) = \frac{1}{1-2a} - \frac{1}{1+2a} = \frac{4a}{1-4a^2}$$

1b) Max. C_L will occur when upper surface is at separation, which corresponds to $a = 0.0904 \rightarrow C_L = 0.374$. This is independant of Re, since laminar separation is independant of Reynolds number.

1c) $\dfrac{\theta(x)}{c} = \theta_1\sqrt{\dfrac{\nu x}{u_e c^2}} = \dfrac{\theta_1}{\sqrt{Re_c}}\left(\dfrac{x}{c}\right)^{(1-m)/2}$ Top Surf: $m = -0.0904$, $\theta_1 = 0.868$; Bot Surf: $m = +0.0904$, $\theta_1 \approx 0.527$

$C_f(x) = 2\sqrt{\dfrac{\nu}{u_e x}}\cdot f_0'' = 2f_0''\dfrac{1}{\sqrt{Re_c}}\left(\dfrac{x}{c}\right)^{-(1+m)/2}$ Top Surf: $m = -0.0904$, $f_0'' = 0$; Bot Surf: $m = 0.0904$, $f_0'' \approx 0.48$

$$C_{D_{friction}} = \frac{1}{\frac{1}{2}\rho U_\infty^2 c}\int_0^c (\cancel{\tau_{w_u}}^{\,0} + \tau_{w_L})\,dx = 2f_0''\frac{1}{\sqrt{Re_c}}\int_0^1 (x/c)^{2a-(1+a)/2} = \frac{1.51}{\sqrt{Re_c}}$$
$$= \int_0^1 C_{fL}\left(\frac{u_e}{U_\infty}\right)^2 d(x/c)$$

$$C_{d_{PROF}} = C_{D_{FRICTION}} + C_{PRESSURE} = \left.\frac{\rho U_e^2\theta}{\frac{1}{2}\rho U_\infty^2 c}\right|_{\text{trailing edge}} = 2\left[\left(\frac{\theta}{c}\right)_u + \left(\theta/c\right)_L\right]_{\text{trailing edge}}$$
$$= \frac{2}{\sqrt{Re_c}}\left[0.868 + 0.567\right] = \frac{2.87}{\sqrt{Re_c}}$$

For $Re_c = 5000$, $C_{D_{FRICTION}} = 0.0213$ $C_L/C_{D_F} = 17.5$ (friction only)

$C_{D_{PROFILE}} = 0.0406$ $C_L/C_D = 9.2$ (with pressure)

$C_L/C_D \sim \sqrt{Re_c}$. Larger insects have an advantage.

2) We know that (from Falkner - Skan) $\Delta = \sqrt{\nu x / U_e} = \text{const.}\ x^{\frac{1-\beta_w}{2}}$ can produce similarity

Test if $\Delta \sim x^{\frac{1-\beta_w}{2}}$ holds for candidate definitions

2a) $\tau_w = \rho \frac{\mu u_e}{x} \cdot S_0 = \text{const.}\ x^{\beta_m + \beta_w - 1} = \text{const.}\ x^{\frac{3\beta_w - 1}{2}}$

$\therefore \frac{\mu u_e}{\tau_w} \sim x^{\beta_w} \cdot x^{\frac{1-3\beta_w}{2}} \sim x^{\frac{1-\beta_w}{2}}$ => can produce similarity

2b) $\delta^* = \delta_1^* \sqrt{\frac{\nu x}{u_e}} \sim x^{\frac{1-\beta_w}{2}}$ => can produce similarity

2c) $\theta + \delta^* = (\theta_1 + \delta_1^*) \sqrt{\frac{\nu x}{u_e}} \sim x^{\frac{1-\beta_w}{2}}$ => can produce similarity

2d) from Falkner - Skan solution

$$\eta_{99} = \frac{\delta_{99}}{\sqrt{\nu x / U_e}}$$

$\therefore$ for any given β_w

$\delta_{99} = \eta_{99} \sqrt{\frac{\nu x}{U_e}} \sim x^{\frac{1-\beta_w}{2}}$ => can produce similarity

Any quantity which $O\left(\sqrt{\frac{\nu x}{U_e}}\right)$ can also serve as a Δ definition