Critical Mach Number

We can estimate the freestream Mach number at which the flow first accelerates above $M > 1$ (locally) using the Prandtl-Glauert scaling and isentropic relationships.

Recall from P-G:

On the airfoil surface:	$C_p(M_\infty) = \dfrac{C_p(M_\infty = 0)}{\sqrt{1 - M_\infty^2}}$

If we have $C_p(M_\infty = 0)$ say from an incompressible panel solution, we could then find C_p anywhere on the airfoil for higher M_∞ under the assumptions of P-G (linearized flow, $M_\infty < 1$).

We can also use isentropic relationships:

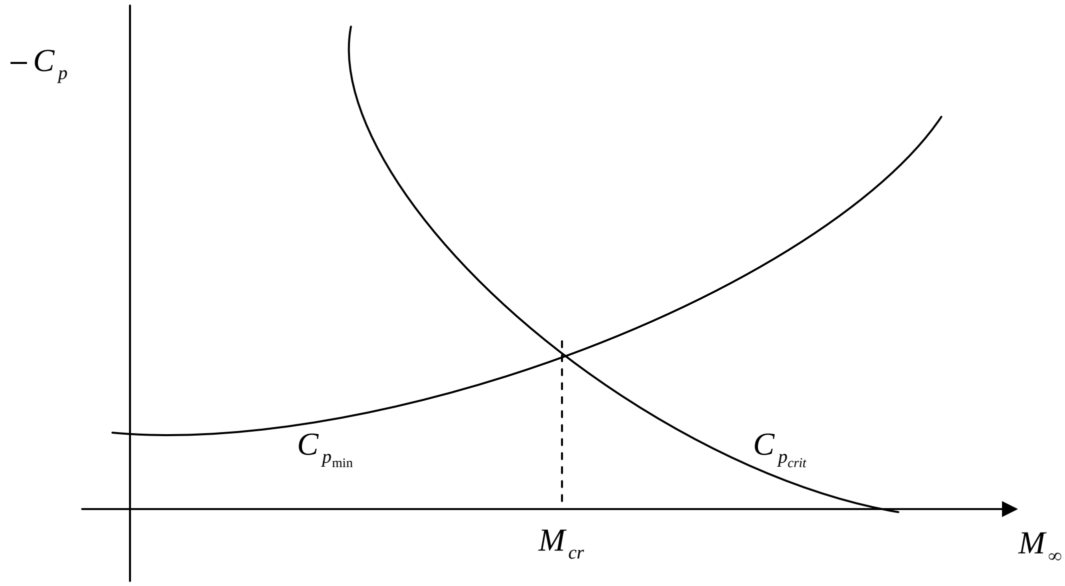
$$C_p = -\frac{2}{\gamma M_\infty^2}\left(\frac{p}{p_\infty} - 1\right)$$

$$\Rightarrow C_p = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{1 + \frac{1}{2}(\gamma - 1)M_\infty^2}{1 + \frac{1}{2}(\gamma - 1)M^2}\right)^{\frac{\gamma}{\gamma - 1}} - 1\right]$$

The C_p for $M = 1$ at a given M_∞ is:

$$C_{p_{crit}} = C_p(M = 1, M_\infty) = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{1 + \frac{1}{2}(\gamma - 1)M_\infty^2}{1 + \frac{1}{2}(\gamma - 1)}\right)^{\frac{\gamma}{\gamma - 1}} - 1\right]$$

This critical freestream M_∞ occurs when $C_{p_{P-G}}(M_{cr}) = C_{p_{crit}}(M_{cr})$.
This critical M_∞ can be found graphically or can be solved for with a root-finding method. Let's look at what happens graphically:



1. Find minimum C_p at $M_\infty = 0$
2. Plot $C_{p_{\min_{P-G}}}$ vs. M_∞
3. Plot $C_{p_{crit}}$ from isentropic relationships