Reading Assignment
Anderson, 3rd edition: Chapter 5 Section 5.1 – 5.3
All of the reading will be directly useful in the problems.

In this homework, we will apply Prandtl's lifting line to predicting the lift and induced drag of a tapered wing. The wing is symmetric about the root and the planform is shown below:
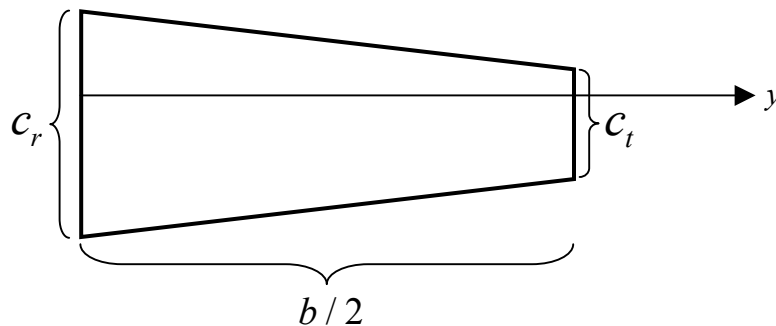
The airfoil of the wing is symmetric (uncambered). In this homework, we will specifically be looking at the cruise performance of the wing and considering how different taper ratios, $\sigma = c_t / c_r$ (in Problem #1), and geometric twist distributions, $\alpha_g(y)$ (in Problem #2) affect the induced drag.

Problem #1: Effect of Taper Ratio and Aspect Ratio (50%)

In this problem, we will analyze the impact of taper ratio and aspect ratio on an untwisted, $\alpha_g(y) = 0$. We will assume that the circulation distribution can be well approximated by the first two *odd* terms (i.e. n=1 and n=3) in the Fourier series expansion:

$$\Gamma(\theta) = 2bV_\infty \sum_{n=1}^{N} A_n \sin n\theta \quad \text{where} \quad y = -\frac{b}{2}\cos\theta$$

Note: we will only use odd terms as the even terms are asymmetric about $y = 0$. Since the wing is symmetric about $y = 0$ and we will not be considering any sideslip velocity, the circulation distribution must also be symmetric.

(a) Determine the 2×2 system of equations for A_1 and A_3 by enforcing Prandtl's lifting line equation at two spanwise locations along the wing. Specifically, since the wing is symmetric, enforce the lifting line equation at $\theta = \pi/4$ and $\pi/2$.

(b) Consider wings with taper ratio 0.6 and 1.0, and aspect ratio 5 and 10. For the four combinations of these two parameters, plot the span efficiency factor versus C_L and plot C_{D_i} versus C_L (for these two graphs, please plot all four wings on the same graph for easy

comparison). Which of the four wings has the lowest induced drag? Which of the four wings has the highest span efficiency factor? Which parameter, taper ratio or aspect ratio, has greater impact on the induced drag? Does the span efficiency depend on C_L?

(c) In the early stages of design, the planform area of the wing is usually constrained by the required lift at take-off. Thus, since the aircraft also has a target cruise velocity and weight, the cruise design is effectively for a fixed lift coefficient. For this homework, we will assume that this cruise lift coefficient is $C_L = 0.5$. At cruise conditions, plot the sectional lift coefficient distribution $c_l(y)$ versus $y/(b/2)$. Discuss where the largest $c_l(y)$ occurs on the four wings and explain the differences between the four wings.

(d) The local loading $L'(y)$ is useful to determine where the largest vertical loads are occurring and can then be used to determine the wing bending using a bending beam model. The local loading is usually plotted in a non-dimensional form by normalizing by the freestream dynamic pressure (q_∞) and the root chord (c_r). First, show that,

$$\frac{L'(y)}{q_\infty c_r} = c_l(y)\frac{c(y)}{c_r}.$$

Plot $c_l(y)\dfrac{c(y)}{c_r}$ versus $y/(b/2)$. Discuss where the largest loads occur on the four wings at cruise and explain the differences between the four wings.

Problem #2: The Effect of Geometric Twist (50%)

In this problem, we will consider only the wing with taper ratio of 0.6 and aspect ratio of 10. We will compare this wing without twist to a wing with twist. For the twisted wing, the root angle of attack will be set 5 degrees larger than the tip angle of attack, with the angle varying linearly from root to tip. Note: a wing with larger angle of attack at the root is said to have 'wash-out'. Wash-out is used to raise the loading near the root thereby reducing bending stresses and increasing the likelihood that stall will occur nearer the root first.

(a) As in Problem #1, determine the 2×2 system of equations for A_1 and A_3 by enforcing Prandtl's lifting line equation at $\theta = \pi/4$ and $\pi/2$. Describe what the differences are in this system of equations between no geometric twist and geometric twist.

(b) For the two wings, plot the span efficiency factor versus C_L and plot C_{D_i} versus C_L (for these two graphs, please plot both wings on the same graph for easy comparison). Which of the wings has the lowest induced drag? Which of the wings has the highest span efficiency factor? Does the span efficiency depend on C_L for either wing?

(c) Plot the sectional lift coefficient distribution and the loading distribution for cruise conditions. Describe and explain the differences of the distributions between the two wings.