Massachusetts Institute of Technology

16.07 Dynamics

Problem Set 12

Out date: Nov. 21, 2007

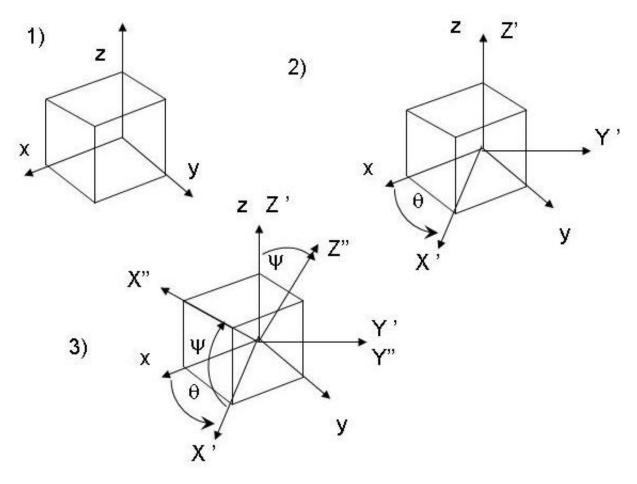Due date: Nov. 30, 2007

	Time Spent [minutes]
Problem 1	
Problem 2	
Study Time	

Turn in **each problem** *on* **separate sheets** *so that grading can be done in parallel*

MATLAB Problem 1 (20 points)

In problem set 2, you performed a coordinate transformation to align your coordinate system with the upper edge of a unit cube, the point 1,1,1. The transformation matrix was the product of the transformation matrices for each step in the process: counterclockwise rotation $\theta = \pi/4$ about z to obtain the $x'y'z'$; a clockwise rotation $\psi = \tan^{-1}\frac{1}{\sqrt{2}}$ about y' to obtain the x'', y'', z'' coordinate system. The final transformation matrix was

$$[T] = \begin{pmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta & \sin\psi \\ -\sin\theta & \cos\theta & 0 \\ -\cos\theta\sin\psi & -\sin\psi\sin\theta & \cos\psi \end{pmatrix} = \begin{pmatrix} .5774 & .5774 & .5774 \\ -0.7071 & 0.7071 & 0 \\ -0.4082 & -0.4082 & 0.8165 \end{pmatrix} \tag{1}$$

The inertia matrix for a cube about the origin in the x, y, z system is

$$Mb^2 \begin{pmatrix} 2/3 & -1/4 & -1/4 \\ -1/4 & 2/3 & -1/4 \\ -1/4 & -1/4 & 2/3 \end{pmatrix} \tag{2}$$

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a) Using MATLAB, apply the transformation matrix generated by the transformation to transform the inertia tensor/matrix of the cube. Show that the axes in the coordinate system x'', y'', z'' are principal axis. What is the inertia tensor in this coordinate system? What are the x'', y'', z'' vectors in terms of the original coordinate system x, y, z?

b) Now apply the MATLAB eigenvalue process to the inertia matrix of the cube in the original x, y, z system. This process results in the eigenvalues, which are the inertias about the 3 principal axes, and the vectors which constitute the principal axes. Compare your results with those of part a).

Since the 2 inertias in the plane perpendicular to the diagonal of the cube are equal, any axis in the plane is a principal axis. Therefore your numerical results could differ from your analytical results but would produce two axis directions in the plane normal to the cube diagonal.

c) Sketch your results in 3 dimensions, showing the cube in the x, y, z coordinate system. Sketch the 3 principal axes obtained from both a) and b).

MATLAB Problem 2 (20 Points)

The inertia tensor of the Apollo spacecraft in the docked "off to the moon" configuration with the lunar excursion module (LEM) and the Apollo command module spacecraft attached as shown, with all fuels and oxidizer tanks full, is

$$(I) = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix} = \begin{pmatrix} 75619.3 & 10412.6 & 11211.1 \\ 10412.6 & 712171. & -5022.98 \\ 11211.1 & -5022.98 & 715545. \end{pmatrix} \quad (3)$$

where x is the roll axis, y the pitch and z the yaw axis, as shown in the sketch. The units of the inertias are kgm^2.

a) Use MATLAB to run the eigenvalue program to determine the principal axis and the inertias about those principal axis. How are these principal axis oriented with respect to the body-fixed x, y, z (roll, pitch, yaw) coordinate system? What is the angle in degrees between the roll axis (x) and the nearest principal axis?
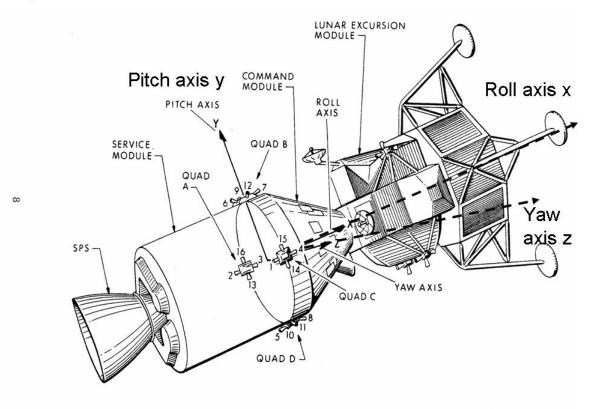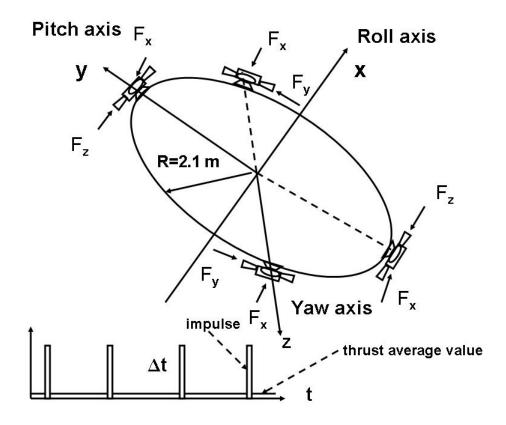

Fig. 1.1   Apollo Spacecraft

b) For an angular velocity vector ω about the roll axis (x) only, what is the magnitude and direction of the angular momentum vector. What are its components along the pitch, roll and yaw axes? What angle is degrees does it make with the roll axis?

c) On the flight to the moon, it was necessary to rotate the vehicle about the roll axis to ensure even solar heating. (This was called the barbecue mode.) If the desired roll rate was 1 revolution per 10 minutes, what steady moment/torque must be applied to maintain this motion?

d) Forces to the Apollo spacecraft are applied by clusters of 4 small reaction jets arranged as shown around the circumference of the vehicle at a radius of 2.1 meters; two jet configurations spaced equally on the z (yaw) axis; two jet configurations spaced equally about the y (pitch) axis. These jet configurations produce forces in the x, y, z directions. Opposing jets can produce moments/torques about the x axis, the y axis and the z axis. These reaction jets are not throttle-able but for the geometry shown and the characteristics of the jets, the impulsive torques created by a firing of opposing jets puts out an impulse of 75 Newton-meter-seconds. To achieve the torque required in part c), how often must opposing jets be fired?

e) An alternative to the strategy applied in c) and d) to create motion roughly aligned with the roll axis, to achieve even solar heating of the spacecraft on the way to the moon, consider applying an impulsive moment/torque of magnitude P only about the x axis, to produce a roll rate of 1 revolution per 10 minutes about the x axis. What are the components of this applied impulsive moment about the principal axes? What angular velocity ω result about each principal axis? What angle in degrees does the total angular velocity ω vector make with the nearest principal axis?

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Fall 2009

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