Lecture S12 Muddiest Points

General Comments

Today, we talked about the relationship between BIBO stability and the region of convergence. The neat result is that a system is BIBO stable if and only if the region of convergence of the Laplace transform of the impulse response contains the line $\text{Re}[s] = 0$.

Responses to Muddiest-Part-of-the-Lecture Cards

(17 cards)

1. *What is the region of convergence? (1 student)* It's the set of all values of s for which the Laplace transform integral converges.

2. *Why does it matter that the LT integral converges absolutely as opposed to just converges? (1)* Even though the LT might converge (but not absolutely) at the boundaries of the region of convergence, in the interior it will generally converge absolutely, because the integrand will be exponentially smaller (as t goes to either $+\infty$ or $-\infty$) than at the boundary of the r.o.c. That's good, because if the interior of the r.o.c. includes $\text{Re}[s] = 0$, then absolute convergence of the LT is the same as convergence of

$$\int_{-\infty}^{\infty} |g(t)|\, dt$$

3. *Stop proving everything. We believe you. (1)* But you need to understand why these things works. It's not satisfactory for me to just claim something is true — you shouldn't believe it unless I prove it.

4. *So a system is stable if and only if the input and output is BIBO. But you know input / output is BIBO if and only if area of impulse response is finite, which means the region of convergence of $G(s)$ includes $\text{Re}[s] = 0$. Is this correct? (1)* Very close. A system is BIBO stable if and only if the area of the absolute value of the impulse response of the impulse response, which is true if and only if the region of convergence of $G(s)$ (the LT of $g(t)$) includes $\text{Re}[s] = 0$.

5. *Why do we define BIBO [stability] to be what it is? (1)* Two reasons: (1) The definition is reasonable, and (2) the definition leads to a mathematically tractable result. Not every reasonable definition is mathematically tractable!

6. *So we can now determine if a system is stable. What is the real world application? (1)* Every control system in the real world must be designed to make the resulting closed-loop system stable.

7. *If the impulse response must converge absolutely for the system to be BIBO stable, are systems with impulse response that are convergent (but not absolutely) conditionally stable, or partially stable? What does it mean to be*

partially stable? *(1)* First, I think you mean the integral of the impulse response converges absolutely, not that the impulse response converges absolutely. Second, there are definitions that lead to the idea of neutral stability, which you called partial stability. Neutral stability is a tricky idea, though. For example, a system with a single pole at $s = 0$ is neutrally stable, but one with two poles ate $s = 0$ is not. It's easier to work with BIBO stability.

8. ***Can you explain once more the meaning of the graphs you draw that look like this:***
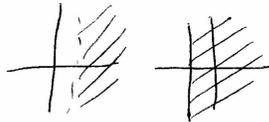
(1) The axes represent the complex plane, of the variable s. The vertical axis represents $\text{Re}[s] = 0$; the horizontal axis represents $\text{Im}[s] = 0$. The hashed region represents the region of convergence (of the Laplace transform of some signal, $g(t)$). Since G is stable if and only if the LT of $g(t)$ has r.o.c. that includes $\text{Re}[s] = 0$, the plot on the right indicates a stable system.

9. ***What is a pole? What is the $j\omega$ axis?*** *(1)* A pole is (roughly) where a function $G(s)$ is infinite. So the function $1/s$ has a pole at $s = 0$. As to the second question, any complex number s can be represented as $s = \sigma + j\omega$, where σ is the real part of s, and ω is the imaginary part. The $j\omega$ axis is the axis where $\sigma = 0$ — it's the vertical axis.

10. ***Still confused about why the system is stable if its region of convergence includes $\text{Re}[s] = 0$.*** *(1)* The answer is probably too long for a mud response. Briefly, a system is BIBO stable if and only the integral

$$\int_{-\infty}^{\infty} |g(t)|\, dt$$

is finite. The integral is finite if and only if the integral

$$\int_{-\infty}^{\infty} g(t)e^{-st}\, dt$$

converges absolutely for $\text{Re}[s] = 0$, since $|e^{-st}| = 1$ for $\text{Re}[s] = 0$.

11. ***If***

$$u(t) = \frac{|g(-t)|}{g(-t)}$$

isn't $u(t)$ undefined for $g(-t) = 0$? *(1)* Yes. If I had been a little more careful, I would have defined $u(t) = 0$ for that case. The proof is still basically correct.

12. **Why does**

$$\int_{-\infty}^{\infty} |g(t)e^{-j\omega t}|\, dt = \int_{-\infty}^{\infty} |g(t)|\, dt$$

(1) Because $|e^{-j\omega t}| = 1$.

13. **Why did you write the integrals today as**

$$\int_{0}^{\infty}$$

instead of

$$\int_{-\infty}^{\infty}$$

(1) Because I was dealing with causal systems. It doesn't really change anything.

14. **No mud. (4)** A few comments: **Please don't lower your voice when answering a question.** OK, will do. **I think board work is good, but 1/2 the time should be used for Q & A.** I can't promise it will be 50/50, but you make a good point, and we'll do some of each. **When working at the board in recitation, I would rather have a few minutes explanation (just of the setup) of the problem to make sure that I understand what my group has done and that it's correct.** We can try to do that, too.