Muddy Card Responses Lecture M16.

The mud marked * is from the equivalent lecture (also M16) in 2000 . I seem to have done a better job of explaining some of the derivations, as there were fewer questions on it this time through.

***I didn't understand the integral in the first PRS question.** The concept is that strain is the differential of displacement w.r.t position, so in order to obtain displacement from strain we need to integrate between the appropriate boundary conditions. In the PRS example the strain was one dimensional (just ε_{11}) so the tip displacement just result from the integral of that component of strain along the bar.

In strain elongation can it only have a displacement in one direction at a time in line with our example? No, in general there can be u_1, u_2 and u_3 components of displacement. The first PRS question was supposed to be a simple example that you could visualize.

***In your diagrams you consistently label ϕ_2 as the angle from the x_1 edge and ϕ_1 as the angle from the x_2 edge. Is this arbitrary, (i.e. does it affect the derivation)?** It is arbitrary (although I try to be consistent). I was doing it to allow you to keep track of where the terms came from in the derivation.

***Would you be able to clarify what you mean by "gradient"** $\left(u_2 + \frac{\partial u_2}{\partial x_1}\delta x_1\right)$? Since we are trying to obtain continuum definitions of stress and strain (i.e. they are quantities that can vary continuously across a structure, we need to allow them to vary!. The simplest way to do this is to choose a sufficiently small element size that the variation is linear i.e. $\left(\frac{\partial u_2}{\partial x_1}\right)$ is constant. Then the total increase in displacement across the element is given by the gradient times the length of the element, i.e. $\left(\frac{\partial u_2}{\partial x_1}\right)\delta x_1$. From there we apply the definition of strains (extensional components of strain are changes in length, shear strains are changes in angle) in order to obtain strain-displacement relationships.

***I'm still confused on how you get** $\varepsilon_{11} = \dfrac{\left(\delta x_1 + \left(u_1 + \frac{\partial u_1}{\partial x_1}\delta x_1 - u_1\right)\right) - \delta x_1}{\delta x_1} = \dfrac{\partial u_1}{\partial x_1}$. I hope that the arithmetic is straightforward. All we are doing is using our basic definition of extensional strain, namely that it is the ratio of the change in length to the initial (undeformed) length. We are applying this to an element which is sufficiently small that there is a linear variation of strain over the element. For an alternative view see CDL 4.10. Note that by subtracting out the u1 terms we are removing the rigid body translations from our definition of strain (i.e. deformations are distinguished from displacements). In the figure I drew a dashed square to indicate the undeformed, undisplaced unit element and the coloured, solid, square was the displaced and deformed equivalent element.

***Why do we want a symmetrical tensor? Won't this just confuse us when we get an assymetrical tensor? Also, is the symmetrical tensor really just two shears?** This will make more sense when we introduce the elasticity tensor, which is not necessarily symmetric, but which is much easier to deal with if the stress and strain tensors are symmetric. The stress tensor has to be symmetric due to equilibrium considerations, therefore it makes sense to define the strain tensor in a consistent manner.

***I don't really understand what a strain is? Just a translation and rotation.** This is an important point, strain is about changes in shape. Extensions and changes in angle of an element of material. Structures, made up of many elements of material change shape due to the integrated effects of strains distributed over the structure. If an element undergoes only a translation and rotation (i.e. it remains rigid) it does not deform, so there is no strain.

***I would think that the "x_1 edge" would have a perpendicular going in the x_1 direction?** I suppose it could be defined this way, but since the edge has a clear direction, along its direction, it makes more sense to me to define it in the manner I have done. I think that your confusion may arise from the fact that we are really looking at a 2 D picture for strain, so the edges are just lines. This contrasts to when we did stress we needed to consider the area of the faces that the stresses were acting on , in order to convert them to forces in order to apply equilbrium.

***You defined $\varepsilon_{ij} = \left(\frac{\ell_{def} - \ell_{undef}}{\ell_{undef}}\right)$ but $\varepsilon_{ij} = \left(\ell_{undef} - \ell_{def}\right)$ But they are related by the same equation, How?** This is not correct. $\left(\frac{\ell_{def} - \ell_{undef}}{\ell_{undef}}\right)$ is the basic definition of extensional strain. $\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)$ is the complete, small-strain, small-displacement definition of strain. Note if i=j then $\frac{\partial u_j}{\partial x_i} = \frac{\partial u_i}{\partial x_j}$ which allows us to recover the definitions of the extensional components of strain, i.e. $\varepsilon_{ii} = \frac{\partial u_i}{\partial x_i}$.

***If you are given a u_1 and u_2, how do you link σ and ε to know which component is due to what?** This is really the whole point of the small-strain, small-displacement definitions of strain. Given information about the displacement field, by plugging the equations defining the displacement field into the expression for the strains, and rotation, we can distinguish between deformations (strain) and rigid body motions.

***I don't see how** $\phi_1 = \dfrac{\left(u_1 + \dfrac{\partial u_1}{\partial x2}\delta x_2 - u_1\right)}{\delta x2}$**? I see how that is a ratio of the sides involved, but I don't see why there isn't a sin or cos or something in there. Is it because of the small angle approximation?** You are exactly correct. If the angles were large we would have $\tan^{-1}\phi_1 = \dfrac{\left(u_1 + \dfrac{\partial u_1}{\partial x2}\delta x_2 - u_1\right)}{\delta x2}$ but since for small angles, $\tan\theta \approx \theta$, we do not have to include the $\tan^{-1}$ term.

***How can the internal angles equalthe external angles for the rectangular element? (a drawing was attached)?. Lecture was very fast today.** The key point here is that the shear strain is defined by the change in angles, not the angle themselves. If the angle at the corner of the unit element was initially 90°, and the external angles change by ϕ_1 and ϕ_2 then the internal angle must change by the same total, i.e. $\phi_1 + \phi_2$. I am sorry that you found the lecture too fast, it was an important lecture and I tried to go at a pace sufficiently slow to ensure that the majority of the class could follow it.

Still unclear on how to draw Mohr's circle. The basic rules are given in the notes, step by step. Here are the facts stated another way
:
Extensional stresses (or strains) are plotted on the horizontal axis.
Shear stresses or strains are plotted on the vertical axis.
Each point on the circumference of the circle represents the normal stress acting on a face of an infinitessimal element (or parallel to an edge for strain), coupled with a shear stress acting on the same face (or shear strain acting on the same edge for strain).
A radius from the center of the circle to the circumference represents the direction corresponding to the normal to the face (for stress) or the direction of the edge (for strain)
Angles between directions on Mohr's circle are double what they are in physical space. i.e. a 90° angle between directions becomes 180° (a diameter) on Mohr's circle.
The circle is symmetric about the extensional axis (i.e. shear stress/strain = 0)
There are principal directions and stresses/strains, corresponding to the maximum and minimum extensional stresses/strains – i.e. directions in which there are no shears.

Mud, Mohr's circle. See above.

More Mohr's circle stuff. See above.

I still don't understand Mohr's circle. See above.

Can you explain the basics of what we know about strain (again)? At its heart strain is the continuum representation of deformation. There are two sorts of strain, extensional strain and shear strain. Elongational strain is a change in length relative to an initial length, shear strains are changes in angles. . By considering an infinitessimal element we can set up strain as a second order tensor, and this means that it can be treated mathematically in the same way as we treat stress (rotation of coordinates using Mohr's circle, diretion cosines etc.)

When/how do we use tensor notation? We will generally be using it for dealing with stress and strain in continuum mechanics. It makes expressing continuum equilbrium, constitutive behavior and compatibility conditions relatively straightforward.

What are the readings in Lardner? For this lecture, M16, Crandall, Dahl and Lardner 4.9-4.11. Note that CDL uses a different notation for strain – we will talk about this on Monday.

Why can't you get the radius with two points? In a sense you do only need two points to locate the circumference of Mohr's circle, so long as you know they are on a diameter. However, in a sense this is cheating, as if you know it is a diameter, that means that you have information about a third point, namely the center of the circle. The key point here (in this PRS question (number 3)) is that a strain gauge only measures exensional strain, it does not measure the shear strain, so you don't actually know the location of a point on the circumference of the circle, only the extensional component of it. This means that you will actually need a third value of the extensional strain to locate the circle circumference.

How did we find the radius beofre? With the strain? Why could we figure it out last lecture, but now the radius is variable? In previous examples I have given you two extensional stress and a shear stress acting on an infinitessimal element. This is typical of the sort of information you might obtain by considering the stress state due to known loads applied to a structure (consider the worm example, or the example of uniaxial tension covered in recitation). The example we looked at in the PRS question is more representative of how one would determine a state of strain experimentally - by applying sets of three strain gauges (rosettes). Since strain gauges only measure extensional strains, we have to obtain information about the shear strains indirectly.

I don't understand Ben's question. Can you go through it once more. I will do this at the start of lecture M17.

Just one more explanation of the last PRS. Fair enough.

Can you explain the last PRS again. Will do.

Muddy on your answer to Ben's question, (as expected). We will revisit this.

I had no idea how to do the last PRS question. Do take a look at it before our next lecture.

In the first PRS (the bar undergoing shear loading along its sides, resulting in a linear gradient of extensional stress) $\varepsilon = \dfrac{\ell_{deformed} - \ell_{undeformed}}{\ell_{undeformed}}$ **isn't** $\Delta\ell$ **just** $\varepsilon \times \ell_{undeformed}$ **so you don't need to integrate?** This is the key point. That would be true if the strain was uniform, such as in the bar of a truss. However, in PRS question 1, the strain was varying with position, x_1, which means that we have to integrate up ε over the length to obtain the change in length.

$\frac{\partial u_1}{\partial x_1} \to strain$ **Is this not simply the old definition of strain $\frac{\Delta \ell}{\ell}$. What is the physical meaning of this partial differential ($\frac{\partial u_1}{\partial x_1}$)?** We used the basic idea of strain being $\frac{\Delta \ell}{\ell}$ to obtain the extensional strain components, i.e. $\varepsilon_{mm} = \frac{\partial u_m}{\partial x_m}$. The basic idea, $\frac{\Delta \ell}{\ell}$, holds if the strain is uniaxail and uniform. We need the differential form to allow us to cope with non uniform strain which is multiaxial.

Whoa. Hmm. I think I'll get it once it sinks in, though I'm generally a bit confused. OK. A little more focus on the source or manifestation of your confusion will allow me to help.

I don't understand why the need of a gauge. This is a very important question. Here are some thoughts. In aerospace we are going to be designing some pretty complicated structures, and we are going to make assumptions and create models for how they carry load, and for that matter, what sort of loads they will have to carry. We need to do experiments to verify that these models/assumptions are correct. Strain gauges are a very convenient way to indirectly determine the loads that are being carried by the structure. In flight tests and structural tests aircraft (and spacecraft) may be instrumented with hundreds if not thousands of strain gauges. Strain gauges also have the important use of allowing us to determine material properties (as you did in the strain lab). If we develop a new material, we will need to determine its properties in order for it to be usful to us.

Class is too early. I am not a fan of whining. I firmly believe that one needs to take control of your own destitiny. To this end I think that you have at least three options: (1) Don't come to class. (2) Go to sleep earlier (3) transfer to an afternoon only major. You choose.

Is there any good tasting cough syrup? By defintion no, cough syrup is not allowed to taste good (this must be an FDA requirement)

No mud. But just to be cheeky, is there a circle drawing class for professors and TA's. No, but with years of practice I still can't draw a good circle on the blackboard.

15 cards signified no mud, or gave positive comments. Thankyou.