Massachusetts Institute of Technology
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16.03/16.04 Unified Engineering III, IV
Spring 2004

Problem Set 10

Name: ________________________

Due Date: 4/21/04

	Time Spent (min)
CP11_12	
P3	
P4	
S10	
S11	
S12	
Study Time	

Announcements: Q5S will be on Friday, April 23.

CP11-12

The problems in this problem set cover lectures C11 and C12

1.

a. Define a recursive binary search algorithm.

b. Implement your algorithm as an Ada95 program.

c. What is the recurrence equation that represents the computation time of your algorithm?

d. What is the Big-O complexity of your algorithm? Show all the steps in the computation based on your algorithm.

Turn in a hard copy of your algorithm, recurrence equation, and Big-O analysis, and code listing, and an electronic copy of your code.

2. What is the Big-O complexity of:

a. Heapify function
b. Build_Heap function
c. Heap_Sort

Show all the steps in the computation of the Big-O complexity.

Note: the code for heap_sort, build_heap and heapify was shown in lecture C11 and has been distributed via email.

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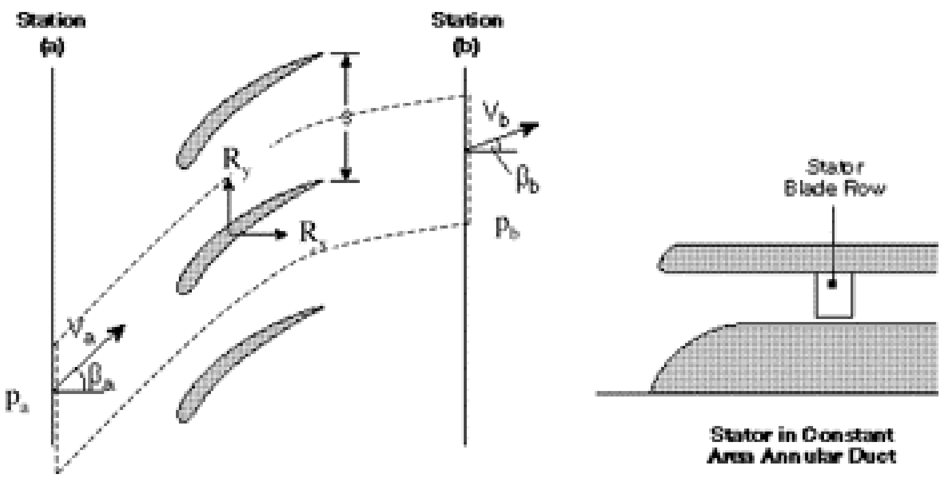
Problem P3. (Propulsion) (LO A & B)

An incompressible fluid flows steadily through a two-dimensional infinite row of fixed airfoils (e.g. a stator blade row). The blade row is contained in a constant area annulus, as shown on the right side of the figure below. The spacing between the airfoils is s. Assume that the velocities and pressures Va, Vb, pa, pb, are constant at stations (a) and (b), and that the flow angles are given by βa and βb.



a) Does the magnitude of the flow velocity increase/stay the same/decrease across the stator and why?

b) Using the control volume shown above (the upper and lower surfaces are streamlines), apply conservation of mass and momentum to determine the forces Rx and Ry that must be applied to the fluid (these are equal and opposite to the forces needed to keep each vane in place).

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Problem P4. (Propulsion) (L.O. D)

For this problem, use the aerodynamic, structural and battery parameters you have determined for your Dragonfly.

a) Calculate the speed that you would fly the Dragonfly if you wanted to maximize range with no additional payload.

b) Calculate the speed that you would fly the Dragonfly if you wanted to maximize endurance with no additional payload.

c) Assume the overall efficiency of the battery-motor-propeller combination is 0.15. As a function of the number of eggs carried (0 through 5) tabulate the velocity for maximum endurance, the power required, and the endurance.

Problem S10 (Signals and Systems)

This problem provides lots of practice using partial fraction expansions to determine inverse Laplace transforms. Please use the coverup method — it really is superior to other methods, and more reliable. Also, please check your answer, that is, verify that your expansion really is equivalent to the $G(s)$ given.

For each of the following Laplace transforms, find the inverse Laplace transform.

1. $G(s) = \dfrac{3s^2 + 3s - 10}{s^2 - 4}, \quad \text{Re}[s] > 2$
2. $G(s) = \dfrac{6s^2 + 26s + 26}{(s+1)(s+2)(s+3)}, \quad \text{Re}[s] > -1$
3. $G(s) = \dfrac{4s^2 + 11s + 9}{(s+1)^2(s+2)}, \quad \text{Re}[s] > -1$
4. $G(s) = \dfrac{4s^3 + 11s^2 + 5s + 2}{s^2(s+1)^2}, \quad \text{Re}[s] > 0$
5. $G(s) = \dfrac{s^3 + 3s^2 + 9s + 12}{(s^2+4)(s^2+9)}, \quad \text{Re}[s] > 0$

Problem S11 (Signals and Systems)

Consider an aircraft flying in cruise at 250 knots, so that

$$v_0 = 129 \text{ m/s}$$

Assume that the aircraft has lift-to-drag ratio

$$\frac{L_0}{D_0} = 15$$

Then the transfer function from changes in thrust to changes in altitude is

$$G(s) = \frac{2g}{mv_0} \frac{1}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)} \tag{1}$$

where the *natural frequency* of the phugoid mode is

$$\omega_n = \sqrt{2}\frac{g}{v_0} \tag{2}$$

the *damping ratio* is

$$\zeta = \frac{1}{\sqrt{2}(L_0/D_0)} \tag{3}$$

and $g = 9.82$ m/s is the acceleration due to gravity. The transfer function can be normalized by the constant factor $\frac{2g}{mv_0}$, so that

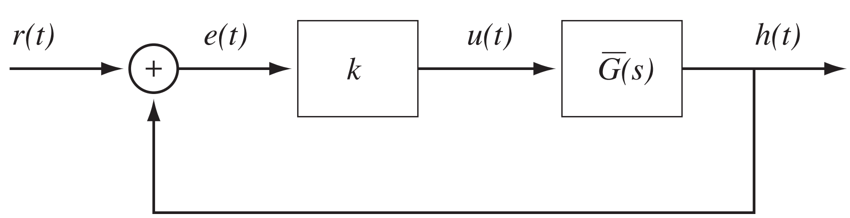
$$\bar{G}(s) = \frac{1}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)} \tag{4}$$

is the normalized transfer function, corresponding to normalized input

$$u(t) = \frac{2g}{mv_0}\delta T$$

1. Find and plot the impulse response corresponding to the transfer function $\bar{G}(s)$, using partial fraction expansion and inverse Laplace techniques. Hint: The poles of the system are complex, so you will have to do complex arithmetic.

2. Suppose we try to control the altitude through a feedback loop, as shown below



That is, the control input $u(t)$ (normalized throttle) is a gain k times the error, $e(t)$, which is the difference between the altitude $h(t)$ and the altitude reference $r(t)$. The transfer function from $r(t)$ to $h(t)$ can be shown to be

$$H(s) = \frac{1}{1 + kG(s)}$$

For the gain k in the range $[0, 0.1]$, plot the poles of the system in the complex plane. You should find that for any positive k, the complex poles are made less stable. What gain k makes the complex poles unstable, i.e., for what gain is the damping ratio zero?

3. For the gain k in the range $[-0.1, 0]$, plot the poles of the system in the complex plane. You should find that for any negative k, the real pole is unstable.

Note that neither positive gain or negative gain makes the system more stable than without feedback control. It is possible to do better with a dynamic gain, but this problem should give you an idea of why the phugoid dynamics are so hard to control with throttle only.

Problem S12 (Signals and Systems)

For each signal below, find the bilateral Laplace transform (including the region of convergence) by directly evaluating the Laplace transform integral. If the signal does not have a transform, say so.

1. $g(t) = \sin(at)\sigma(-t)$
2. $g(t) = te^{at}\sigma(-t)$
3. $g(t) = \cos(\omega_0 t)\, e^{-a|t|}$, for all t